Lesson 54: Matched Pair Designs and Hypothesis Tests

Example

Is the express lane faster?

For their second semester project in AP Statistics, Libby and Kathryn decided to investigate which line was faster in the supermarket: the express lane or the regular lane. To collect their data, they randomly selected 15 times during a week, went to the same store, and bought the same item. However, one of them used the express lane and the other used a regular lane. To decide which lane each of them would use, they flipped a coin. If it was heads, Libby used the express lane and Kathryn used the regular lane. If it was tails, Libby used the regular lane and Kathryn used the express lane. They entered their randomly assigned lanes at the same time, and each recorded the time in seconds it took them to complete the transaction.

Time in express lane (seconds)	Time in regular lane (seconds)
337	342
226	472
502	456
408	529
151	181
284	339
150	229
357	263
349	332
257	352
321	341
383	397
565	694
363	324
85	127

Construct a 95% confidence Interval for the mean difference.

Conduct a Hypothesis Test to test the claim that the express lane is faster at the alpha = 0.05 level.

Do either tests provide statistically significant results?

Daily Data Collection

My Claim: Rock music causes your heart rate to increase.

Report your values and test my claim at alpha = .05.

Design – Matched pair – each student is their own control.
Each flips a coin. Heads = rock first, quiet second. Tails = quiet first, rock second.
Treatment One – Start the first treatment for 60 seconds, then spend 60 more seconds with this treatment as you count your pulse. Record your pulse in the correct Column.
Treatment Two – Start the second treatment for 60 seconds, then spend 60 more seconds with this treatment as you count your pulse. Record your pulse in the correct Column.

	No Rock	With Rock	Difference		No Rock	With Rock	Difference
1				21			
2				22			
3				23			
4				24			
5				25			
6				26			
7				27			
8				28			
9				29			
10				30			
11				31			
12				32			
13				33			
14				34			
15				35			
16				36			
17				37			
18				38			
19				39			
20				40			

Hypotheses:

Statistics:

Conclusion:

The amount of pressure lost (in pounds per square inch) during the year for the air-filled and nitrogen-filled tires of each brand is shown in the table below.

Brand	Air	Nitrogen	Brand	Air	Nitrogen
BF Goodrich Traction T/A HR	7.6	7.2	Pirelli P6 Four Seasons	4.4	4.2
Bridgestone HP50 (Sears)	3.8	2.5	Sumitomo HTR H4	1.4	2.1
Bridgestone Potenza EL400	2.1	1.0	Yokohama Avid H4S	4.3	3.0
Bridgestone Potenza G009	3.7	1.6	BF Goodrich Traction T/A V	5.5	3.4
Bridgestone Potenza RE950	4.7	1.5	Bridgestone Potenza RE950	4.1	2.8
Continental Premier Contact H	4.9	3.1	Continental ContiExtreme Contact	5.0	3.4
Cooper Lifeliner Touring SLE	5.2	3.5	Continental ContiProContact	4.8	3.3
Dayton Daytona HR	3.4	3.2	Cooper Lifeliner Touring SLE	3.2	2.5
Faken Ziex ZE-512	4.1	3.3	General Exclaim UHP	6.8	2.7
Fuzion Hri	2.7	2.2	Hankook Ventus V4 H105	3.1	1.4
General Exclaim	3.1	3.4	Michelin Energy MXV4 Plus	2.5	1.5
Goodyear Assurance Tripletred	3.8	3.2	Michelin Pilot Exalto A/S	6.6	2.2
Hankook Optimo H418	3.0	0.9	Michelin Pilot HX MXM4	2.2	2.0
Kumho Solus KH16	6.2	3.4	Pirelli P6 Four Seasons	2.5	2.7
Michelin Energy MXV4 Plus	2.0	1.8	Sumitomo HTR+	4.4	3.7
Michelin Pilot XGT H4	1.1	0.7			

Construct a 95% confidence Interval for the mean difference.

Conduct a Hypothesis Test to see if a difference between Air and Nitrogen exists at the alpha = 0.05 level.

Do either tests provide statistically significant results?

HW 53 from section 9-3: 89, 94-97,99-104

Study for Quiz 16 over lessons 51-54
Reviews 48, 49, 18

Extra Problems

Q1

Worried about his prospects for the prom, Malcolm claims that girls at NWHS are a bit snobby and that the average number of girls that a guy must ask to the prom before getting a "yes" is 4. Doug disagrees. He doesn't think the girls are that snobby and that the average number of girls that a guy must ask out before getting a positive response is less than 4. An SRS from last year of 50 junior and senior guys found that the average number of girls that a guy asked out to the prom was 3.4. Assuming the standard deviation from the entire population is $\sigma = 2$, is there enough evidence to support Alex's claim (at the level $\alpha = .05$)?

Q2

Frank has been sensing that his car is not driving right. He takes his car to the mechanic who does some testing on the ignition timing. In order for Frank's car to run at optimum efficiency, the spark plugs need to ignite and spark, on the average every 1.3 seconds. Assume that the standard deviation of all spark plug firings is known to be $\sigma = 0.5$ seconds. The mechanic suspects that Frank's car spark plugs are not firing at this optimal interval. The mechanic took a random sample of 30 spark plug ignition firings from Frank's car and got the following data:

1.0	1.1	0.8	1.7	0.9	1.3	1.2	1.5	1.3	0.8
0.6	1.3	1.1	1.2	0.7	1.9	2.0	1.1	1.3	1.4
1.0	1.2	0.9	0.4	1.3	1.2	1.4	1.0	1.3	1.3

Based on this data, can we say that Frank's car problems stem from spark plug timing? (at the level $\alpha = .05$)?

Recommended AP Practice – Investigative Task

1. An exercise electrocardiogram (EKG) checks for changes in your heart during exercise and is useful in diagnosing coronary artery disease. An EKG has fewer potential side effects but is much less precise than thallium tomography. In one EKG study, 500 volunteers with known coronary artery disease and 500 volunteers with healthy arteries underwent EKG checks. The physicians administering and evaluating the tests did not know the physical condition of any volunteer. The following table gives the numbers of volunteers whom the physicians evaluated as "positive" for coronary disease.

Test for coronary disease

	Positive	Negative
Healthy volunteers	100	400
Volunteers with disease	305	195

(a) *Sensitivity* is defined as the probability of a positive test given that the subject has disease. What was the sensitivity of this study?

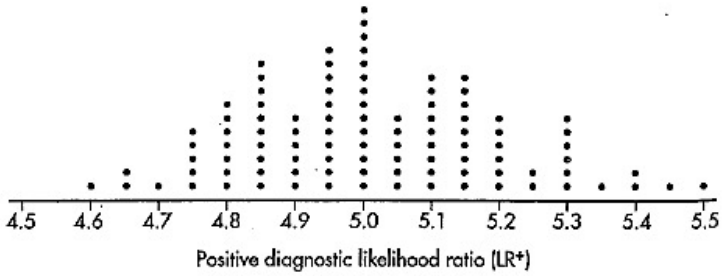
(b) *Specificity* is defined as the probability of a negative test given that the subject is healthy. What was the specificity of this study?

(c) A valuable tool for assessing the accuracy of such studies is the *positive diagnostic likelihood ratio* (LR^+) which gives the ratio of the probability a positive test result will be observed in a diseased person compared to the probability that the same result will be observed in a healthy person.

$$LR^+ = \frac{\text{sensitivity}}{\text{1-specificity}}$$

What was LR^+ in this study, and explain why the larger the value of LR^+, the more useful the test.

(d) Suppose in one such sample study, $LR^+ = 4.7$. To determine whether or not this is sufficient evidence that the population LR^+ is below the desired value of 5.0, 100 samples from a population with a known LR^+ of 5.0 are generated, and the resulting simulated values of LR^+ are shown in the dotplot:



Based on this dotplot and the sample $LR^+ = 4.7$, is there evidence that the population LR^+ is below the desired value of 5.0? Explain.