

Lesson 52: Error Types

Comparison to the American Judicial System

In the American judicial system we must first assume innocence and then must try to prove guilt. In court, we cannot prove innocence, we either have evidence "beyond a reasonable doubt" to 'prove' guilt, or we declare the person "not-guilty".

The same is true in statistics. With a hypothesis test, either one or two tailed, the null hypothesis is our assumption of innocence. If we wish to determine 'guilt' (the alternative hypothesis), we must first assume innocence (the null hypothesis). Either we have enough evidence to reject the null hypothesis (reject innocence) using a level of significance or we do not have enough evidence (fail to reject innocence).

We can never "accept" the null hypothesis since that is not what we set out to prove. Just like in court, when a jury fails to reject innocence (the null hypothesis) it doesn't say the person is found innocent, it says the person is found "not guilty".

Example

Julie was accused of shoplifting by the store manager. State the hypotheses:

Errors in Significance Testing

Type I Error: The null hypotheses is true, but is mistakenly rejected.

Type II Error: The null hypothesis is false, but is mistakenly NOT rejected.

		Population truth	
		Null is true	Null is false
Conclusion	Reject null	Type I error Probability of type I error = α	Correct decision
	Fail to reject null	Correct decision	Type II error Probability of type II error = β

Example

Explain a type I error for the shoplifting example

Explain a type II error for the shoplifting example

Which error type would be worse? Explain.

Power in Significance Testing

- Power is the probability of reaching the right conclusion when the alternative hypothesis is true.
- Power is the probability that the test will reject the null hypothesis when a specific alternate hypothesis is true.
- Power is the probability of avoiding a type II error (The null hypothesis is false, but is mistakenly NOT rejected)
- Power is a function in which the input is a specific alternate hypothesis and the output is a probability of rejecting the null hypothesis. The power changes as the alternate changes.
- The distance between the null hypothesis and the true parameter (p or μ) is sometimes called the effect. The greater the effect, the lower the chance of a type II error and the greater the power.
- Power = $1 - \beta$ where β = Probability of type II error
- Power = 0 means there is no chance of detecting the null hypothesis is false
- Power = 1 means the test is certain to detect the null hypotheses is false
- As the significance level, α -level, becomes smaller, it is tougher to reject the null hypothesis. This causes the probability of a type II error to increase and the power will decrease.
- As sample size increases, power increases because there is more evidence that the null hypothesis should be rejected.

Daily Data Collection

My Claim: The average number of showers taken in the last week is less than 7.
Report your value and test my claim at $\alpha = .05$.

Hypothesis

Data

Sample size	
Sample mean	
Sample st. dev.	
St. Dev. of the mean	
p-value	

Continued on the next page...

Graph

Conclusion

Describe a type 1 error:

Describe a type 2 error:

Would the conclusion change if the test were ran as a two-sided test?