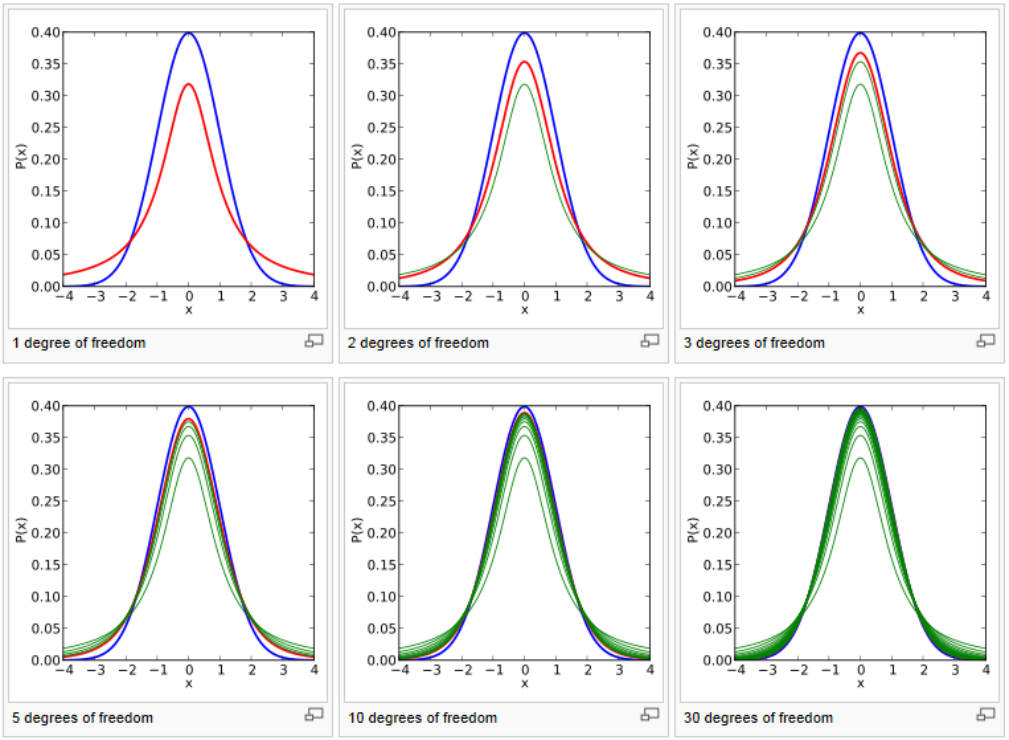
Lesson 48: Student's t-distribution

Previously, when constructing confidence intervals, we made the assumption that we knew the true population standard deviation, σ. When we do not know the population standard deviation, σ, we must estimate it from our sample using the sample standard deviation, s_x. However, when we do so, the test statistic, z, changes to a new test statistic called the **t-statistic**, and has a new distribution (density curve) associated with it.

	Standard Z-curve	Standard T-curve
Centered at	0	0
Area under	1	1
Shape	Symmetric Bell	Symmetric Bell
Width and Height	Always the same curve.	Curve changes based on the sample size. Low sample size creates a wider, shorter curve. High sample size creates a narrow, taller curve. As sample size increases, the shape of the t-curve approaches the shape of the z-curve.
Calculations	Areas left and right can be calculated based on the mean, standard deviation, and a value.	Areas left and right can be calculated based on the mean, standard deviation, a value, and the degrees of freedom. Degrees of freedom = n – 1



Calculations from tables

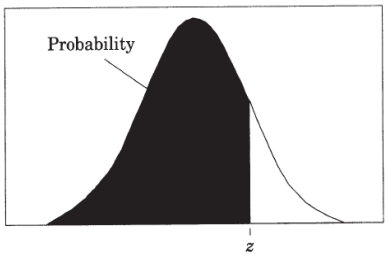
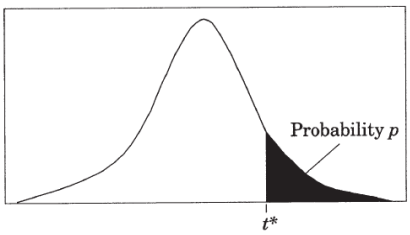
The Normal Curve (Z)	The Students T Curve (T)
 Table entry for z is the probability lying below z.	 Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.
Margins are z-values	Top margin = probability Left margin = degrees of freedom
Inside = probability	Inside value = T-value
Probability is area to the left	Probability is area to the right

Table A *(Continued)* Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879

Table B t distribution critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869

Note: both are in the AP STAT Program!

State the T-Values

Confidence Level = 90%, degrees of freedom = 8

Confidence Level = 60%, degrees of freedom = 12

Confidence Level = 98%, degrees of freedom = 25

Examples:

A principal claims the students spend at least 10 hours per week doing homework, on average. A random sample of 250 students selected at random finds the mean to be 10.2 and the standard deviation to 4.2 hours.

a. Construct a 95% confidence interval to see if the principal's claim is verified.

b. Why did we use a t-distribution instead of a z-distribution?

c. The counselor claims that students spend at least 16 hours per week doing homework, on average. Does the confidence interval verify her claim?

Daily Data Collection 1

Find a 95% confidence interval for the true mean value of
"The annual salary of your desired career"

Parts of the formula: $\bar{x}$ = _____ n = _____ s_x = _____

Degrees of freedom = _____ t = _____

1. Describe the distribution of $\bar{x}$

2. Find MOE = _____ Lower = _____ Upper = _____

3. Give the AP Statement

Daily Data Collection 2

Find a 90% confidence interval for the true mean value of
"The amount of your future annual salary that *should* be paid in taxes"

Parts of the formula: $\bar{x}$ = _____ n = _____ s_x = _____

Degrees of freedom = _____ t = _____

1. Describe the distribution of $\bar{x}$

2. Find MOE = _____ Lower = _____ Upper = _____

3. Give the AP Statement

Daily Data Collection 3

Find a 99% confidence interval for the true mean value of
"The amount of your future annual salary that *should* be saving/invested"

Parts of the formula: $\bar{x}$ = _____ n = _____ s_x = _____

Degrees of freedom = _____ t = _____

1. Describe the distribution of $\bar{x}$

2. Find MOE = _____ Lower = _____ Upper = _____

3. Give the AP Statement

HW 48 (Section 8-3): 57, 65, 67, 75-76