## Daily Data – Learning Statistics Through Investigating Data Chapter 6/7

## Lesson 41: Binomial and Geometric Distributions

**Binomial Distribution**

There are many experiments and situations that result in what are called *dichotomous* responses – responses for which there exist two possible choices (True / False, Yes / No, Defective / Non-defective, Male / Female, etc.). A simple example of such an experiment is that of tossing a coin, where there are only two possibilities, Heads or Tails. There are many other types of experiments similar to a coin toss where you are observing the "success" or "failure" of a certain outcome. Such experiments give us a probability distribution called a **binomial distribution**.

Requirements (Use this when asked if a situation is binomial):

1. Binary – there can only be 2 ways the event can turn out
2. Independent – events must be independent for each trial
3. Fixed – there must be a set number of trials
4. Probability of success is constant for all trials and the we are counting successes

Extra Rule – the sample size must be less than 10% of the entire population size.

The long process used in example 1 can be avoided by using the notion that this is a binomial probability distribution. There is a formula we can use to find the probability of any value in a binomial distribution:

$$P(x) = \frac{n!}{(n-x)!x!} \cdot p^x \cdot q^{n-x} \quad \textit{for } x = 0, 1, 2, 3, \ldots, n$$

n = # of trials x = # of successes among n trials

p = probability of success in any one trial q = probability of failure in any one trial (q = 1 – p)

This formula can be written as $\boldsymbol{P(x=k)=\binom{n}{k}p^k q^{n-k}}$ where the parenthesis is called n choose k. This is the combination formula for choosing k items from n total items. In combinations, order does NOT matter.

**Formulas for The binomial distribution:** $\mu = np$ $\sigma = \sqrt{np(1-p)}$

**<u>Example 1:</u>** Tom is about to take a five-question true/false quiz for which he is not prepared. He will be guessing on all five questions.

| X = # Correct | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| P(X) | | | | | | |

What is the probability that:

1) He gets all the answers correct?
2) He gets all the answers wrong?
3) He gets exactly three answers correct?
4) He will pass the quiz?

## The Geometric Distribution

While in a binomial distribution the random variable was the number of successes in a fixed number of trials, in a **geometric distribution** the random variable is the number of trials it takes to achieve a success.

**$P(X = n) = (1-p)^{n-1}p$ where p = probability of success**

Examples: 1) flip a coin until you get heads
2) Roll a die until you get a 6
3) Throw darts at a dartboard until you hit the bull's-eye

A geometric distribution must have the following properties:

1) Each trial in the experiment must have only two possible outcomes (success or failure)
2) The probability of success, p, doesn't change from trial to trial
3) The trials in the experiment are independent
4) The variable of interest is the number of trials required to reach the first success.

The mean, $\mu$, of a geometric distribution (the average number of times we can expect to repeat the trials before a success occurs) is simply 1/p where p is the probability of success.

**Formulas for The binomial distribution:** $\mu = \frac{1}{p}$ $\sigma = \sqrt{\frac{1-p}{p^2}}$

**Example 2:** Roll a 4-sided die until the number six appears and keep a record of how many rolls it took before a two was obtained.

| X = roll | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| P(X) | | | | | |

a. Find the probability that a 2 will come up on the first roll.

b. Find the probability that a 2 will come up on the second roll.

c. P(2 will not come up on the first roll) =

d. P(2 will not come up on the first two rolls) =

e. How many rolls can we expect, on average, to roll the die before getting the number 6?

f. Find the probability that it would take more than 5 rolls for us to get the number 2

## More Examples

Suppose Charlie manages to manipulate a coin in such a way that it lands on heads with a .7 probability and lands on tails with a .3 probability. Suppose John then flips the coin 20 times, what is the probability that it will land on heads for exactly 13 of the twenty flips? On average, how many flips will result in a heads? What is the standard deviation of this binomial distribution?

Suppose that the probability that any random freshman girl will agree to go with Sam to the senior prom is 0.1. Suppose Sam asks 20 random freshman girls to the prom, what is the probability that exactly 1 will say “yes”? That at least 1 will say “yes”? On average, how many freshman girls will agree to go with Sam to the prom? What is the standard deviation of this binomial distribution?

**HW 41 (Section 6-3): 73,75,77, 85, 95, 97, 99**