Lesson 40 – Random variables and expected value

Random Variable: A variable, usually represented by an *X,* which has a single numerical value (determined by chance) for each outcome of an experiment.

Examples: 1) *X* = The number of seniors who get into college early.
2) *X* = The number of defective tires on a car
3) *X* = A random number chosen between 0 and 1
4) *X* = The lifetime of a light bulb.

a) *Discrete Random Variable*: Has a finite or countable number of values. (Examples 1 and 2)

b) *Continuous Random variable*: Has infinitely many values and the values can be associated with a continuous scale so that there are no gaps or interruptions. (Examples 3 and 4)

Probability Distribution: Gives the probability of each value of a random variable.

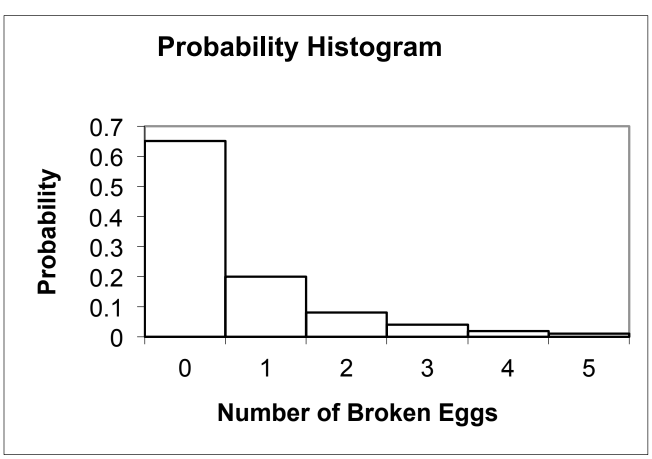
Example 1 - Eggs-ample: Suppose the random variable *X* is the number of broken eggs in a randomly selected carton of one dozen "store brand" eggs at a certain supermarket. Since the number of broken eggs is a discrete random variable, the probability distribution is a list or a table of the possible values of *X* and the corresponding probabilities

Number of Broken Eggs:	0	1	2	3	4	5
Probability:	.65	.20	.08	.04	.02	.01

Requirements for a probability distribution ***(legitimate):***

1) $\sum p(x) = 1$ says all the probabilities must add to 1
2) $0 \leq p(X) \leq 1$ for all values of x

Here is a probability Histogram that represents the above probability distribution:



$P(X > 3) =$

$P(X \geq 1) =$

$P(X < 2) =$

$P(X \geq 4) =$

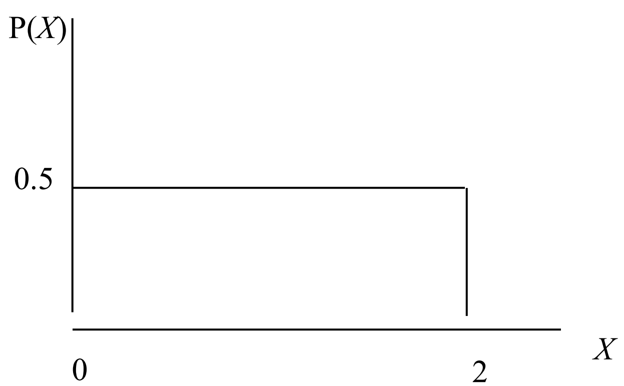
Example 2 - Tossing Coins:
What is the probability distribution of the discrete random variable *X* that counts the number of heads in three tosses of a coin?

Number of heads	0	1	2	3
Probability				

Let's create a probability histogram for this distribution.

For a **continuous random variable** we use *density curves*, not histograms, to graphically represent the distribution:

Example 3 - Random Number Example: *X* is a random number between 0 and 2. The distribution is a continuous distribution and is represented by the *uniform density curve* below:



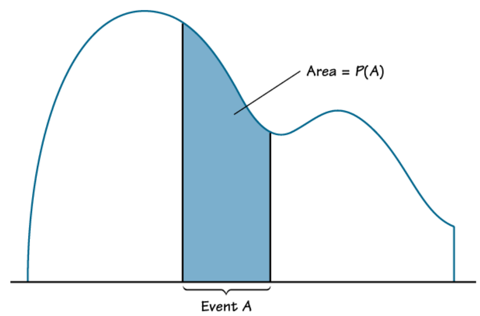
The probability of any event is the area under the density curve

$P(1 \le x \le 2) =$

$P(0 \le x \le 0.5) =$

$P(x \ge 0.3) =$

If the curve is not uniform, you would need to use geometry or calculus to find the probability of an event



And don't forget, the most familiar density curve is the standard normal curve. The Normal Distribution is a probability distribution.

The **Expected Value** of a random variable represents the **average value** of the outcomes. The expected value is the mean of a probability distribution. It is found by finding the value of:

Expected Value: **E =** $\sum(\boldsymbol{x} \bullet \boldsymbol{p}(\boldsymbol{x})) = \mathbf{1}$ **Mean:** $\mu = \sum x \cdot p(x)$

Example 4 - Looking back at the Probability distribution from example 1, we can calculate, on average, how many broken eggs there will be in a carton:

Number of Broken Eggs:	0	1	2	3	4	5
Probability:	.65	.20	.08	.04	.02	.01

This means that "on average" we can **expect** that _________ eggs will be broken in a randomly selected carton of eggs.

Example 5 - Getting the Flu: The probability that 0, 1, 2, 3, or 4 people will seek treatment for the flu during any given hour at an emergency room is show in the probability distribution below.

x	0	1	2	3	4
P(x)	.12	.25	?	.24	.06

a) What is the probability that 2 people will seek treatment for the flu during any given hour at an emergency room?

b.) What is the probability that at least 1 person will be treated for the flu in the next hour?

c.) What is the probability that 3 or more people will be treated for the flu in the next hour?

d.) What is the average number of people that an emergency room can expect to treat for the flu during any given hour?

HW 40 Section 6-1 part 1: 1, 5, 7, 9, 13

Lesson 40 – Day 2 – Water Activity

1. Purpose – can you tell the difference between bottled water and tap water?

   You will drink water from 3 samples. 1 of these is bottled water.

2. You must test them in the following order:

   First: Second: Third:

3. After testing each, circle the letter that you feel is the bottled water and show the teacher.
   Do NOT discuss your results with your classmates.

4. Make a chart of the number of people who stated each sample

Sample	A	B	C	Total
Frequency				
Relative Frequency				

The true bottled water sample is…

5. What percent correctly identified the bottled water?

6. Assume that no one can distinguish tap water from bottled water. If this is the case, then what is the probability that a person would guess correctly?

7. What would the percentage that can distinguish tap water from bottled water have to be in order for you to be convinced that the students are not guessing?

8. Make a histogram for the number of correct responses in class

<u>Expected Outcome in Games:</u>

- What you can expect in return for a $1 bet.
- Multiply each possible payout by the probability that it will happen. Then add all the values for the expected outcome.
- Expected value can also be used to determine whether or not a particular game is "fair" and therefore worth playing.
- Negative = favors the house/dealer
- Positive = favors the player

<u>Example:</u>
A player bets a dollar on yes or no and spins a spinner labeled Yes and Not. Yes pays out $3 with 25% of the board and No pays out 1$ with 75% of the board. What is the expected outcome for each bet?

Example - Patrick offers Christine to play a dice game whereby he will pay her $6 if she rolls a six, but Christine would have to pay Patrick $1 every time she rolls a number other than 6. Should Christine agree to play this game **over a long period of time?**

To find the expected value of a game multiply each payoff by its probability and then add.

Based on the above definition, the expected value for Christine is:

This means that for every roll of the die, Christine will earn, on average, $\frac{1}{6}$ dollars, which also means that Patrick will lose, on average $\frac{1}{6}$ dollars for every roll of the die.

If Patrick and Christine were to play this game 100 times, how much money (on average) can Christine expect to make? How much can Patrick expect to lose?

Fair Game: We say that a game is "fair" whenever its expected value is equal to zero. For example, you win a dollar every time you toss a coin and get heads, and lose a dollar every time you get tails.

Example - Zoot Suit Example: Would you be willing to play this game? You pay me $25 to play. I take a standard deck of cards. I Shuffle the cards well. You pick 1 card at random. The suit of that card becomes your winning suit. You do not put the card back. You pick 2 more cards at random. If those 2 cards are both the winning suit, I will pay you $500. If they are not, I pay you nothing.

Variance, and standard deviation of a probability distribution

Variance

1) $\sigma^2 = \sum (x-\mu)^2 \cdot p(x)$

2) $\sigma^2 = \sum [x^2 \cdot p(x)] - \mu^2$

Standard Deviation

3) $\sigma = \sqrt{\sum [x^2 \cdot p(x)] - \mu^2}$

(the square root of the variance)

Notes:

- The mean and standard deviation formulas are in the stat formula packet that you get on the AP
- The calculator will do this if you put the value in L1 and probabilities in L2

Example - Find the mean, variance, and standard deviation of the following probability distribution:

x	P(x)
0	.3
1	.1
2	.2
3	.1
4	.3

HW 40 Section 6-1 day 3: 14,18,19,23,25

Study for Quiz 12 (MIDTERM) over lessons 20-40