Probability Reference Sheet

Randomness - We call a phenomenon **random** if individual outcomes are uncertain but there is nonetheless a regular distribution of outcomes in a large number of repetitions.
Law of Large Numbers - A chance process has outcome that we cannot predict in a low number of repetitions. The Law of Large Numbers says that the proportion of times that a particular outcome occurs in a large number of repetitions will approach the true probability.
Probability - The **probability** of any outcome of a random phenomenon is the proportion of times the outcome would occur in a very long series of repetitions. Probability is always between 0 and 1. Probabilities can be expressed as a fraction, decimal, or as a percent.
Theoretical Probability - A probability found using the rules of probability of an event occurring.
Experimental Probability - A probability found by collecting data to find the probability of an event occurring.
Simulation - A simulation is an imitation of chance behavior. Many times this is carried out with random numbers.
Sample Space - The **sample space** S of a random phenomenon is the list of all possible outcomes.
Event - An **event** is any collection of outcomes in the sample space, i.e. a subset of the sample space.

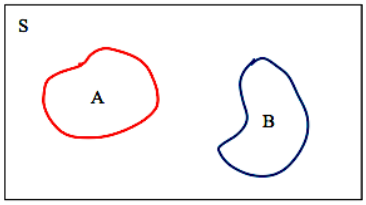
Probability Rules

Any Probability is a number between 0 and 1. An event with probability 0 never occurs, and an event with probability 1 occurs on every trial. Symbols: **$0 \le P(A) \le 1$**

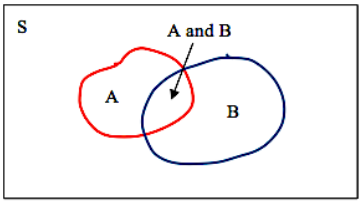
The sum of the probabilities of all possible outcomes must equal 1. Adding the sample space equals 1.
Symbols: $P(S) = 1$

Fundamental Counting Principle A.K.A Multiplication Principle
If you can do one task in *a* number of ways and a second task in *b* number of ways, then both tasks can be done in ***a x b*** number of ways.

Addition Rule for Mutually Exclusive Events (top diagram)
If A and B are "mutually exclusive" **P(A or B) = P(A) + P(B)**



Addition Rule for Any Two Events (bottom diagram)
For any two events A and B, **P(A or B) = P(A) + P(B) – P(A and B)**



symbols:	**$P(A \cup B) = P(A) + P(B) - P(A \cap B)$**

Multiplication rule for independent events
Two events A and B are independent if knowing that one occurs does not change the probability that the other occurs. If A and B are independent then the probability of A and B is the product of their individual probabilities.

Tables can be used to solve problems with two events. ***Easiest method***
Venn Diagrams can be used to solve probability problems involving two or more independent events.

Example – A player rolls two standard 6 sided dice. The chance of rolling an even or odd number on each dice is shown in a table and a Venn Diagram.

These events are independent

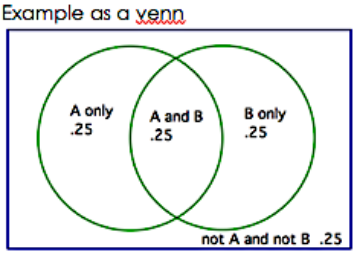
$P(A) = 0.5$ $P(B) = 0.5$

$P(\text{both even}) = P(\text{A and B even}) = P(A)*P(B) = 0.5 * 0.5 = 0.25$

$P(\text{A or B is event}) = P(A) + P(B) - P(\text{A and B}) = 0.5 + 0.5 - 0.25 = 0.75$

$P(\text{A is even and B is odd}) = P(\text{A is even})*P(\text{B is odd}) = 0.5 * 0.5 = 0.25$

Example as a venn



Example as a table

		Dice one		
		Even	Odd	Total
Dice Two	Even	.25	.25	.5
	Odd	.25	.25	.5
	Total	.5	.5	1

Probability Rules (continued)

The probability that an event does not occur is called the complement of the event and the probability is equal to 1 minus the probability that the event does occur. Symbols: $P(A^c) = 1 - P(A)$ *or* $P(\bar{A}) = 1 - P(A)$

General Multiplication Rule for Any Two Events and Conditional Probability

The joint probability that events A and B both happen can be found by $P(A \cap B) = P(A) \bullet P(B \mid A)$ OR→

Here P(B | A) is the conditional probability that B occurs, given the Information that A occurs.

My Preference:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Showing Independence

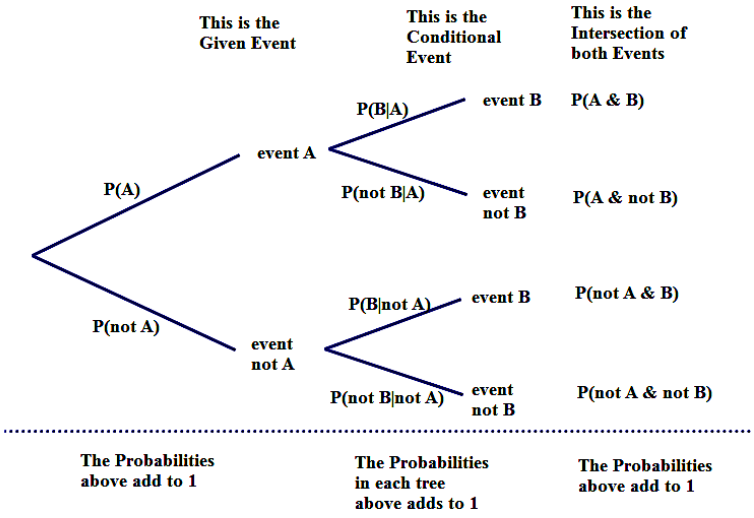
Method 1: Two events are independent if and only if **P(B | A) = P(B)** and **P(A | B) = P(A)**

Method 2: Two events are independent if and only if **P(A ∩ B) = P(A) • P(B)**

Tree Diagrams should be used in conditional situations.

Expected Outcome in Games:

What you can expect in return for a $1 bet. Multiply each possible payout by the probability that it will happen. Then add all the values for the expected outcome. We say that a game is "fair" whenever its expected value is equal to zero. Negative = favors the house/dealer. Positive = favors the player



Example:

Would you be willing to play this game? You pay me $25 to play. From a standard deck of cards you pick 1 card at random. The suit of that card becomes your winning suit. You do not put the card back. You pick 2 more cards at random. If those 2 cards are both the winning suit, I will pay you $500. If they are not, I pay you nothing.

The chance of winning on both cards is 22/425
The chance of losing (the complement) is 1 – 22/425 = 403/425

Expected Value: 500(22/425) – 25(403/425) = 2.18
This means a $1 bet expects to win $2.18.
You should play this game it is in your favor (positive)