

Lesson 31: Basics of Probability and Expectation

The mathematics of chance/randomness is called Probability.

Chance behavior is unpredictable in the short run, but has a regular and predictable pattern in the long run. For example, toss a coin –in the long run, about ½ will be heads $P(\text{heads}) = \frac{1}{2}$.

Samples are random – we are uncertain who will be chosen, but overall everyone has an equal chance of being selected.

Randomness

We call a phenomenon **random** if individual outcomes are uncertain but there is nonetheless a regular distribution of outcomes in a large number of repetitions.

Law of Large Numbers

A chance process has outcome that we cannot predict in a low number of repetitions. The Law of Large Numbers says that the proportion of times that a particular outcome occurs in a large number of repetitions will approach the true probability.

Probability

The **probability** of any outcome of a random phenomenon is the proportion of times the outcome would occur in a very long series of repetitions. Probability is always between 0 and 1. Probabilities can be expressed as a fraction, decimal, or as a percent.

Theoretical Probability

A probability found using the rules of probability of an event occurring.

Experimental Probability

A probability found by collecting data to find the probability of an event occurring.

Daily Data Collection – part 1

State the theoretical probability of rolling a 2 on a standard die.

Then find the experimental probability.

Theoretical probability: _____

25 volunteers will roll a standard die and report if a 2 was rolled. Circle each trial where a 2 is rolled

Trial	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Success count																									
Probability																									

Conclusion:

Daily Data Collection – part 2

State the theoretical probability of tossing an object at a target (basketball, ring toss, etc.).

Then find the experimental probability.

Theoretical probability: _____

Circle each trial where the target is hit.

Trial	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Success count																									
Probability																									

Conclusion:

Examples. The true probability of landing on “heads” for a coin toss is .5. A simulation can be run to test the probability by tossing the coin 10 times. Because this random process is unpredictable, there could be 8 heads and 2 tails. For this simulation, the $P(\text{Head}) = .8$ but if we repeat this simulation 100 times, the average of these simulations will show a value very close to .5.

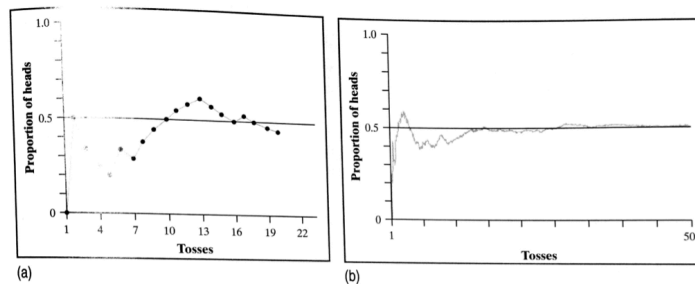


FIGURE 5.1 (a) The proportion of heads in the first 20 tosses of a coin. (b) The proportion of heads in the first 500 tosses of a coin.

Life Insurance

Probability and risk




































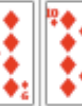




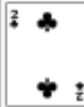







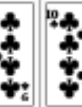



How do insurance companies decide how much to charge for life insurance? We can't predict whether a particular person will die in the next year. But the National Center for Health Statistics says that the proportion of men aged 20 to 24 years who die in any one year is 0.0015. This is the *probability* that a randomly selected young man will die next year. For women that age, the probability of death is about 0.0005. If an insurance company sells many policies to people aged 20 to 24, it knows that it will have to pay off next year on about 0.15% of the policies sold to men and on about 0.05% of the policies sold to women. Therefore, the company will charge about three times more to insure a man because the probability of having to pay is three times higher.

Common Situations

2 dice

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Example set of 52 poker playing cards

Suit	Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
Spades													
Hearts													
Diamonds													
Clubs													



		0	00
1-18	1st DOZEN	1	2
		4	5
		7	8
EVEN	2nd DOZEN	10	11
		13	14
		16	17
◆	3rd DOZEN	19	20
◆		22	23
		25	26
ODD		28	29
19-36		31	32
		34	35
		36	
		2 to 1	2 to 1