

Regression toward the mean:

In statistics, regression toward the mean is the phenomenon that if a variable is extreme on its first measurement, it will tend to be closer to the average on its second measurement—and, paradoxically, if it is extreme on its second measurement, it will tend to have been closer to the average on its first. To avoid making wrong inferences, regression toward the mean must be considered when designing scientific experiments and interpreting data.

Example 1 testing: A class of students takes two editions of the same test on two successive days. It has frequently been observed that the worst performers on the first day will tend to improve their scores on the second day, and the best performers on the first day will tend to do worse on the second day. The phenomenon occurs because student scores are determined in part by underlying ability and in part by chance. For the first test, some will be lucky, and score more than their ability, and some will be unlucky and score less than their ability. Some of the lucky students on the first test will be lucky again on the second test, but more of them will have (for them) average or below average scores. Therefore a student who was lucky on the first test is more likely to have a worse score on the second test than a better score. Similarly, students who score less than the mean on the first test will tend to see their scores increase on the second test.

Example 2 from the movie "Miracle" Speech by the coach to the 1980 US Hockey Team before a game versus the heavily favored Russian Team.

"If we played them 10 times, they might win 9 – but not on this day. Today is ours..."

History

The concept of regression comes from genetics and was popularized by Sir Francis Galton during the late 19th century with the publication of *Regression towards mediocrity in hereditary stature*.^[6] Galton observed that extreme characteristics (e.g., height) in parents are not passed on completely to their offspring. Rather, the characteristics in the offspring regress towards a mediocre point (a point which has since been identified as the mean). By measuring the heights of hundreds of people, he was able to quantify regression to the mean, and estimate the size of the effect. Galton wrote that, "the average regression of the offspring is a constant fraction of their respective mid-parental deviations". This means that the difference between a child and its parents for some characteristic is proportional to its parents' deviation from typical people in the population. If its parents are each two inches taller than the averages for men and women, on average, it will be shorter than its parents by some factor (which, today, we would call one minus the regression coefficient) times two inches. For height, Galton estimated this coefficient to be about 2/3: the height of an individual will measure around a midpoint that is two thirds of the parents' deviation from the population average.

The effect can also be exploited for general inference and estimation. The hottest place in the country today is more likely to be cooler tomorrow than hotter, as compared to today. The best performing mutual fund over the last three years is more likely to see relative performance decline than improve over the next three years. The most successful Hollywood actor of this year is likely to have less gross than more gross for his or her next movie. The baseball player with the greatest batting average by the All-Star break is more likely to have a lower average than a higher average over the second half of the season.

The calculation and interpretation of "improvement scores" on standardized educational tests in Massachusetts probably provides another example of the regression fallacy. In 1999, schools were given improvement goals. For each school, the Department of Education tabulated the difference in the average score achieved by students in 1999 and in 2000. It was quickly noted that most of the worst-performing

schools had met their goals, which the Department of Education took as confirmation of the soundness of their policies. However, it was also noted that many of the supposedly best schools in the Commonwealth, such as Brookline High School (with 18 National Merit Scholarship finalists) were declared to have failed. As in many cases involving statistics and public policy, the issue is debated, but “improvement scores” were not announced in subsequent years and the findings appear to be a case of regression to the mean.

The psychologist Daniel Kahneman, winner of the 2002 Nobel Prize in economics, pointed out that regression to the mean might explain why rebukes can seem to improve performance, while praise seems to backfire. “I had the most satisfying Eureka experience of my career while attempting to teach flight instructors that praise is more effective than punishment for promoting skill-learning. When I had finished my enthusiastic speech, one of the most seasoned instructors in the audience raised his hand and made his own short speech, which began by conceding that positive reinforcement might be good for the birds, but went on to deny that it was optimal for flight cadets. He said, “On many occasions I have praised flight cadets for clean execution of some aerobatic maneuver, and in general when they try it again, they do worse. On the other hand, I have often screamed at cadets for bad execution, and in general they do better the next time. So please don’t tell us that reinforcement works and punishment does not, because the opposite is the case.” This was a joyous moment, in which I understood an important truth about the world: because we tend to reward others when they do well and punish them when they do badly, and because there is regression to the mean, it is part of the human condition that we are statistically punished for rewarding others and rewarded for punishing them. I immediately arranged a demonstration in which each participant tossed two coins at a target behind his back, without any feedback. We measured the distances from the target and could see that those who had done best the first time had mostly deteriorated on their second try, and vice versa. But I knew that this demonstration would not undo the effects of lifelong exposure to a perverse contingency.”

UK law enforcement policies have encouraged the visible siting of static or mobile speed cameras at accident blackspots. This policy was justified by a perception that there is a corresponding reduction in serious road traffic accidents after a camera is set up. However, statisticians have pointed out that, although there is a net benefit in lives saved, failure to take into account the effects of regression to the mean results in the beneficial effects being overstated. It is thus claimed that some of the money currently spent on traffic cameras could be more productively directed elsewhere.

Statistical analysts have long recognized the effect of regression to the mean in sports; they even have a special name for it: the “Sophomore Slump”. For example, Carmelo Anthony of the NBA’s Denver Nuggets had an outstanding rookie season in 2004. It was so outstanding, in fact, that he couldn’t possibly be expected to repeat it: in 2005, Anthony’s numbers had dropped from his rookie season. The reasons for the “sophomore slump” abound, as sports are all about adjustment and counter-adjustment, but luck-based excellence as a rookie is as good a reason as any.

Regression to the mean in sports performance may be the reason for the “Sports Illustrated Cover Jinx” and the “Madden Curse”. John Hollinger has an alternate name for the phenomenon of regression to the mean: the “fluke rule”, while Bill James calls it the “Plexiglas Principle”.

Because popular lore has focused on “regression toward the mean” as an account of declining performance of athletes from one season to the next, it has usually overlooked the fact that such regression can also account for improved performance. For example, if one looks at the batting average of Major League Baseball players in one season, those whose batting average was above the league mean tend to regress downward toward the mean the following year, while those whose batting average was below the mean tend to progress upward toward the mean the following year.

The Gambler's Fallacy

The gambler's fallacy is the mistaken belief that if something happens more frequently than normal during some period, then it will happen less frequently in the future (presumably as a means of balancing nature). In situations where what is being observed is truly random (i.e. independent trials of a random process), this belief, though appealing to the human mind, is false. This fallacy can arise in many practical situations although it is most strongly associated with gambling where such mistakes are common among players.

Example: If a coin is flipped and lands "heads" 5 times in a row, are you more likely to bet "tails"?