Lesson 33: Probability Rules

Sample Space

The **sample space** S of a random phenomenon is the list of all possible outcomes.

Event

An **event** is any collection of outcomes in the sample space, i.e. a subset of the sample space.

Example 1: Show the sample space for rolling a 6 sided die and flipping a coin.

Questions with sample events:

What is the probability of the result being 4H?

What is the probability of the result being even?

What is the probability of the result having a 3 or T?

What is the probability of the result have a H and NOT a 4?

Example 2: Show the sample space for the genders of 3 children.

Questions with sample events:

P(2 boys) =

P(more than 1 girl) =

P(no boys) =

P(at least 1 boy) =

Fundamental Counting Principle A.K.A Multiplication Principle

If you can do one task in *a* number of ways and a second task in *b* number of ways, then both tasks can be done in $a \times b$ number of ways.

Examples:

1. The standard Connecticut license plate is three letters, followed by 3 numbers. How many license plates can you form using these specifications?

2. How many 5 digit zip codes are possible?

3. A family decides to have 5 kids, how many different birth orders can there be?

Example for learning the rules below: Show the sample space for drawing two digits from the set 1, 2, 3, 4 without replacement to create a two-digit number.

Probability Rules

Rule #1 Any Probability is a number between 0 and 1. An event with probability 0 never occurs, and an event with probability 1 occurs on every trial.

Symbols: $0 \le P(A) \le 1$

Example: P(first digit is a 3) = .25 This must be between 0 and 1.
P(event) = 1.2 is impossible.

Rule #2 The sum of the probabilities of all possible outcomes must equal 1.

Symbols: P(S) = 1

Example: 12 possible events. Probability of each is 1/12. 12 • 1/12 = 1

Rule #3 Addition rule for mutually exclusive events

If two events have no outcomes in common are called mutually exclusive and the probability that one event or the other event occurs is the sum of their individual probabilities.
Another word for mutually exclusive is disjoint.

Symbols: If A and B are "mutually exclusive" Then P(A or B) = P(A) + P(B)
∪ means "or", so P(A or B) can also be written P(A ∪ B)

Example: Two events that cannot occur at the same time:
P(first digit is 1) = .25
P(first digit is 2) = .25

P(first digit is 1 or 2) = .25 + .25 = .5

Rule #4 The probability that an event does not occur is called the complement of the event and the probability is equal to 1 minus the probability that the event does occur.

Symbols: $P(A^c) = 1 - P(A)$ or $P(\overline{A}) = 1 - P(A)$

Example: P(second digit is 3) = .25 so P(send digit is NOT 3) = 1 – .25 = .75
Event A = 24 and P(A) = 1/12
P(not A) = 1 – 1/12 = 11/12

Rule #5 Multiplication rule for independent events

Two events A and B are independent if knowing that one occurs does not change the probability that the other occurs. If A and B are independent then the probability of A and B is the product of their individual probabilities.

Symbols: If A and B are independent, then P(A and B) = P(A)•P(B)
∩ means "and" , so P(A and B) can also be written P(A ∩ B)

Example 1: Event A = first digit is a 4
Event B = second digit is a 3.
These are NOT independent because A occurring changes the probability of B occurring. Therefore this rule cannot be applied.

Example 2: Event A = heads on a coin
Event B = roll a 4 on a die.
These are independent so P(H and 4) = ½ • 1/6 = 1/12

Notes:
The multiplication rule applies only to independent events; you cannot use it if events are not independent.
Do not confuse disjointness and independence. Disjoint events cannot be independent because both events cannot occur at the same time.

Distance Learning

Applying probability rules

Distance-learning courses are rapidly gaining popularity among college students. Randomly select an undergraduate student who is taking a distance-learning course for credit, and record the student's age. Here is the probability model:[5]

Age group (yr):	18 to 23	24 to 29	30 to 39	40 or over
Probability:	0.57	0.17	0.14	0.12

PROBLEM:

(a) Show that this is a legitimate probability model.

(b) Find the probability that the chosen student is not in the traditional college age group (18 to 23 years).

CHECK YOUR UNDERSTANDING

Choose an American adult at random. Define two events:

A = the person has a cholesterol level of 240 milligrams per deciliter of blood (mg/dl) or above (high cholesterol)
B = the person has a cholesterol level of 200 to 239 mg/dl (borderline high cholesterol)

According to the American Heart Association, $P(A) = 0.16$ and $P(B) = 0.29$.

1. Explain why events A and B are mutually exclusive.
2. Say in plain language what the event "A or B" is. What is $P(A \text{ or } B)$?
3. If C is the event that the person chosen has normal cholesterol (below 200 mg/dl), what's $P(C)$?

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