

Lesson 44: Central Limit Theorem

★ ★ ★ The Central Limit Theorem ★ ★ ★

Review - Sampling Distribution of a Sample Mean \bar{x} from a Normal Population

Draw an SRS of size n from a population that has a normal distribution with mean μ and standard deviation σ .

Then the sample mean \bar{x} has a normal distribution with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$.

Since the mean of the sampling distribution is equal to μ , this makes \bar{x} an unbiased estimator of μ .

Sampling Distribution of a Sample Mean \bar{x} from a Non-Normal Population

Draw an SRS of size n from a population where the shape is unknown or if the shape is skewed that has a normal distribution with mean μ and standard deviation σ .

The distribution of the sample means will still be close to normal as long as the sample size is large enough. The rule of thumb: we expect a normal distribution if $n \geq 30$.

This idea is called the Central Limit Theorem.

Draw an SRS of size n from any population whatsoever with mean μ and finite standard deviation σ . When n is large the sampling distribution of the sample mean \bar{x} is close to the normal distribution with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$.

Example:

The number of flaws per square yard in a type of carpet material varies with mean 1.6 flaws per square yard and a standard deviation of 1.2 flaws per square yard. The population distribution cannot be normal because a count takes only whole number values. An inspector studies 200 square yards of the material, records the number of flaws found in each square yard, and calculates \bar{x} , the mean number of flaws per square yard inspected.

Find the standard deviation of \bar{x}

Describe the shape for the distribution of \bar{x}

What is the probability the mean number of flaws exceeds 2 per square yard for a randomly selected square yard of material?

Daily Data Collection

Every person in class will roll two standard dice and record the sum. The class will average all sums.
 When the teacher says "group up" everyone will randomly form groups of 5.
 Each group will report the average value on their cards for their group of 5. Repeat until 100
 sample means are recorded and graphed below.

Population/Class mean = _____

Population/Class standard deviation = _____

Plot all sums

Count											
	2	3	4	5	6	7	8	9	10	11	12

What is the shape of the distribution for the sums?

OLD – If a random person is selected, what is the probability their sum is less than 6?

NOW THE CLASS WILL BEGIN GROUPING UP WITH SIZE 5

$n =$ _____

$\mu_{\bar{x}} =$ _____

$\sigma_{\bar{x}} =$ _____

Plot all sample means

Count											
	2	3	4	5	6	7	8	9	10	11	12

What is the shape of the distribution for the means?

Does the Central Limit Theorem Apply?

If a random group of 5 is selected, what is the probability their average is less than 6?

A Twist of Old and New:

Nationwide, the average GPA of members of the Delta Tau Chi Fraternity is normally distributed with mean of 2 and standard deviation of .5

- A. What is the probability that a randomly selected member of ΔTX has a GPA over 2.5?
- B. Jack is a member of ΔTX and feels the numbers are inaccurate. He surveys a SRS of 50 ΔTX at Ohio Universities. What is the probability that the sample mean is over 2.5?
- C. What is the probability that less than 10 ΔTX members in the survey have a GPA over 2.5?
- D. Jill is a member of Delta Delta Delta (May we help you, help you, help you?) which has a nationwide GPA that is normally distributed with mean 2.2 and standard deviation of .7. Jill surveys an SRS of 40 $\Delta\Delta\Delta$ at Ohio Universities. What is the probability that the sample mean for Jack is greater than the sample mean for Jill?

HW 44 (Section 7-3): 56, 59, 61, 63-68

**Study for Quiz 13
Lessons 41-44, review 40**