

Lesson 43: Distribution of the sample mean

Mean and Standard Deviation of a Sample Mean

Suppose that \bar{x} is the mean of an SRS of size n drawn from a large population with mean μ and standard deviation σ .

The mean of the sampling distribution of \bar{x} is $\mu_{\bar{x}} = \mu$

The standard deviation is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Note: in order to be able to use the standard deviation formula, the sample size must be less than 10% of the population size. So N must be at least 10 times larger than n . so $10n \leq N$

On the AP Formula Page:

If \bar{x} is the mean of a random sample of size n from an infinite population with mean μ and standard deviation σ , then:

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Sampling Distribution of a Sample Mean \bar{x} from a Normal Population

Draw an SRS of size n from a population that has a normal distribution with mean μ and standard deviation σ .

Then the sample mean \bar{x} has a normal distribution with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$.

Since the mean of the sampling distribution is equal to μ , this makes \bar{x} an unbiased estimator of μ .

Summary:

- 1.) Averages are less variable than individual observations. [because σ is divided by \sqrt{n}]
- 2.) Averages are more normal than individual observations.

If we look at a histogram of averages, we will get a histogram that is more normal and less spread out than a histogram of individual observations. Data is much easier to work with if it is normal and has a small spread, so it is to our advantage to look at a distribution of averages.

Examples:

Suppose the heights of young women are normally distributed with $\mu = 64.5$ inches and $\sigma = 2.5$ inches. What is the probability that the mean height of an SRS of 10 young women is greater than 66.5 inches?

Suppose the heights of young women are normally distributed with $\mu = 64.5$ inches and $\sigma = 2.5$ inches. What is the probability that one randomly selected woman has a height greater than 66.5 inches?

HW 43 (Section 7-3): 49-51, 53-55