

Lesson 14: Empirical Rule

Daily Data Collection

Shoe sizes of males and females.

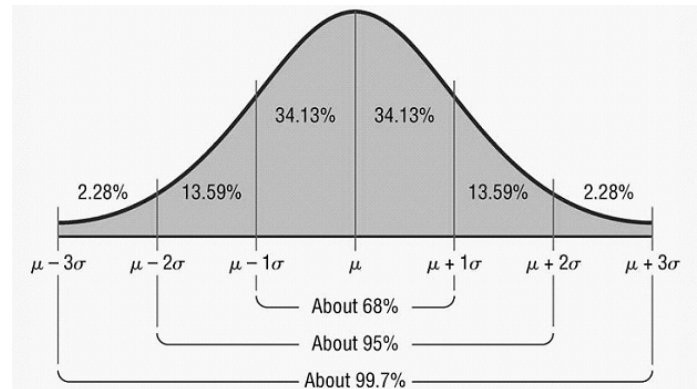
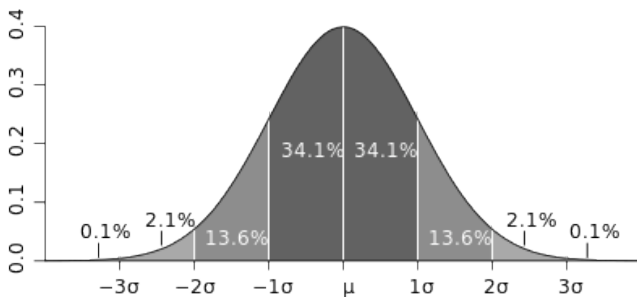
Normal Distributions and the Empirical Rule (68-95-99.7 Rule)

All Normal distributions have the same overall shape. Any differences can be explained by μ and σ .



In a normal distribution with mean μ and standard deviation σ :

- 68% of the observations fall within 1 standard deviation of the mean.
- 95% of the observations fall within 2 standard deviations of the mean.
- 99.7% of the observations fall within 3 standard deviations of the mean.



Calculus connection:

On the normal curve, at a distance of σ on either side of the mean μ , are two points. These points are called inflection points and they signify where the curve changes concavity. The normal curve changes from concave up to concave down at these points. To find the inflection points, you would need to find the second derivative of the function.

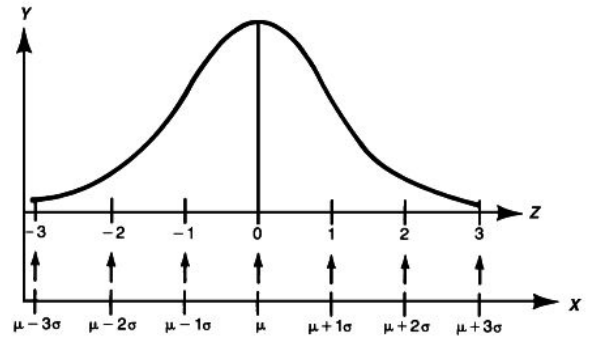
$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}.$$

Equation:

Standard Normal Curve

The normal curve with mean = 0 and standard deviation = 1 is called the **standard normal curve**.

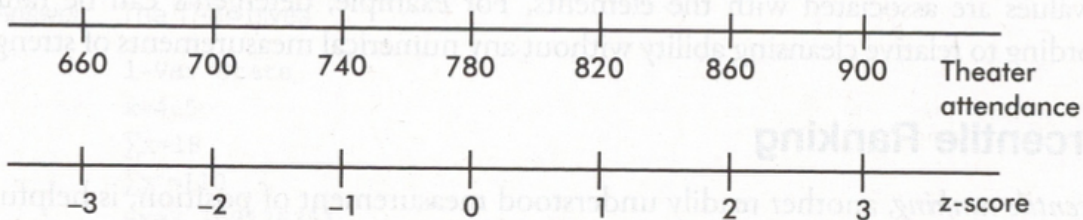
Reminder: Area under the curve = 1



The Translation of X to Z by the Transformation $Z = (X - \mu) / \sigma$

Figure 3

Suppose the attendance at a movie theater averages 780 with a standard deviation of 40. Adding multiples of 40 to and subtracting multiples of 40 from the mean 780 gives



Sketch this graph:

Class Data:

Find the mean and standard deviation for the shoe size of Males and Females. Sketch the curves for each and mark the points of the empirical rule.

Guided Practice:

Earlier, we saw that the distribution of Iowa Test of Basic Skills (ITBS) vocabulary scores for seventh-grade students in Gary, Indiana, is close to Normal. Suppose that the distribution is exactly Normal with mean $\mu = 6.84$ and standard deviation $\sigma = 1.55$. (These are the mean and standard deviation of the 947 actual scores.)

PROBLEM:

- (a) Sketch a Normal density curve for this distribution of test scores. Label the points that are one, two, and three standard deviations from the mean.
- (b) What percent of the ITBS vocabulary scores are less than 3.74? Show your work.
- (c) What percent of the scores are between 5.29 and 9.94? Show your work.

CHECK YOUR UNDERSTANDING

The distribution of heights of young women aged 18 to 24 is approximately $N(64.5, 2.5)$.

1. Sketch a Normal density curve for the distribution of young women's heights. Label the points one, two, and three standard deviations from the mean.
2. What percent of young women have heights greater than 67 inches? Show your work.
3. What percent of young women have heights between 62 and 72 inches? Show your work.