Games of The Price is Right: Plinko

Just yesterday, The Price is Right celebrated its 40th anniversary. It's certainly among the most well known game shows of all time, and its placement during daytime TV makes it a wonderful time sink for children. Through this, I suspect that The Price is Right is many children's first exposure to some fairly deep probability and statistics.

I've been watching The Price is Right since very early in my childhood, so it's always very painful to see people on it who aren't familiar with the strategies of different games, or even just of some of the basic probabilities underlying those games. In short, The Price is Right is full of examples of and opportunities for the use of practical statistics.

In thinking of where to start with something like this I couldn't help but pass up perhaps the most well known game on The Price is Right, Plinko. If you're not familiar, Plinko is a game where contestants can earn a number of little round disks (think hockey pucks but a little larger diameter, maybe a touch thinner) which they then drop down a pegboard with different valued gates at the bottom. It looks like this:

And can be graphically represented as this:

	5		4		3		2		1		2		3		4		5	
		o		o		o		o		o		o		o		o		
	o		o		o		o		o		o		o		o		o	
		o		o		o		o		o		o		o		o		
	o		o		o		o		o		o		o		o		o	
		o		o		o		o		o		o		o		o		
	o		o		o		o		o		o		o		o		o	
		o		o		o		o		o		o		o		o		
	o		o		o		o		o		o		o		o		o	
		o		o		o		o		o		o		o		o		
	o		o		o		o		o		o		o		o		o	
		o		o		o		o		o		o		o		o		
	o		o		o		o		o		o		o		o		o	
		o		o		o		o		o		o		o		o		
	100		500		1K		0		10K		0		1K		500		100	

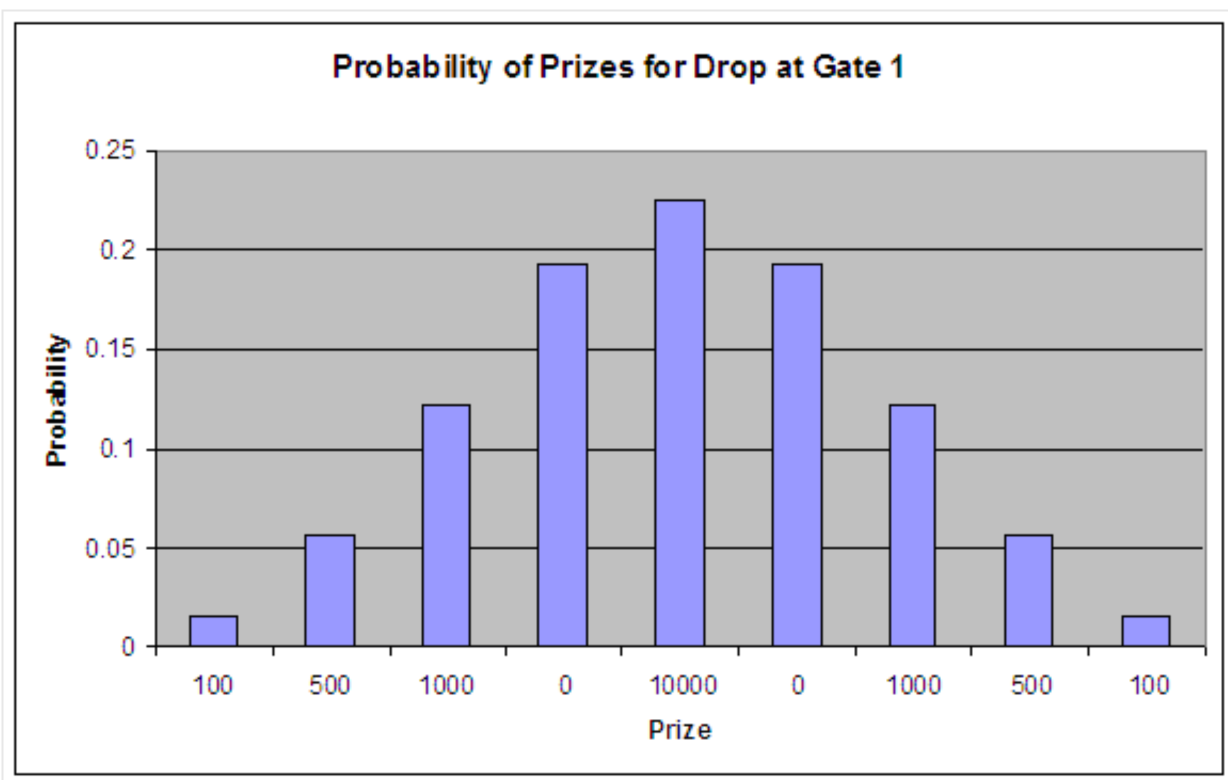
The green areas up top are the the different gates that contestants can drop the puck through at the beginning of the game. I've numbered them starting at the center, and if we assume that the board doesn't have any problems the gates should produce symmetric results about the center.

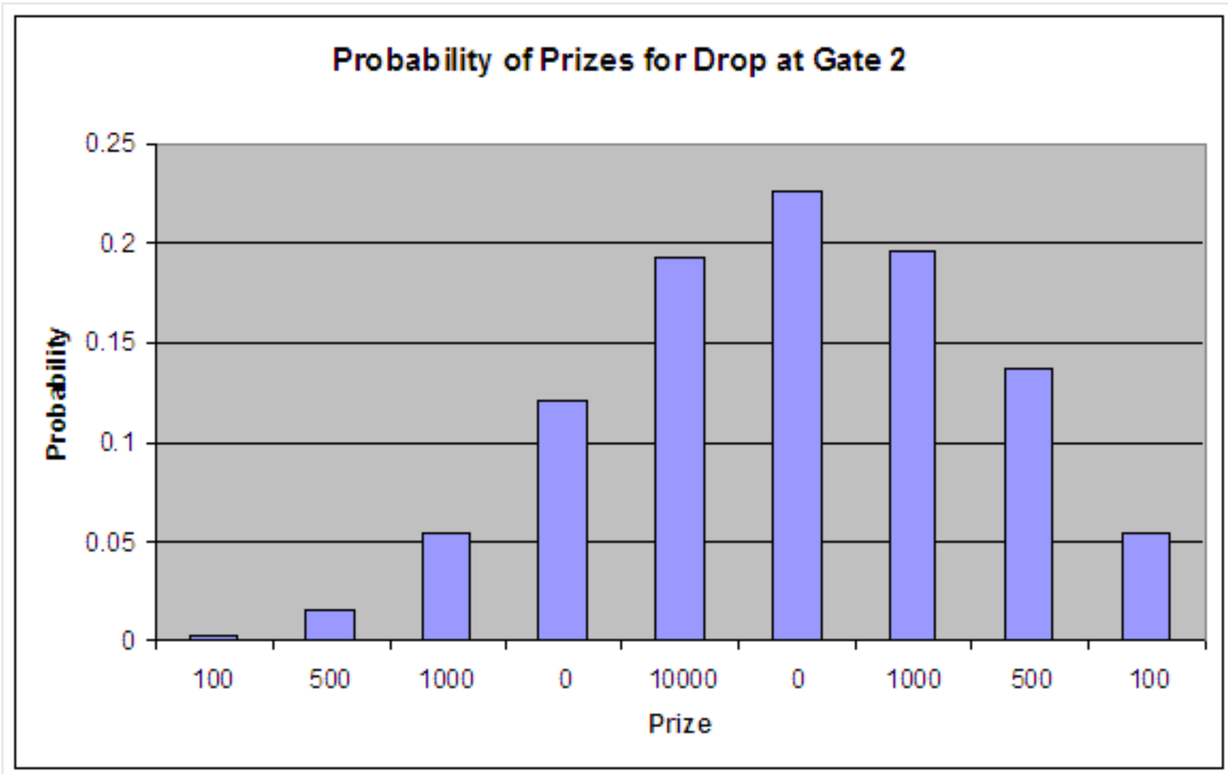
The idea of the pegboard is pretty simple, and we can make some assumptions about the way the puck travels down it that should help approximate what's happening.

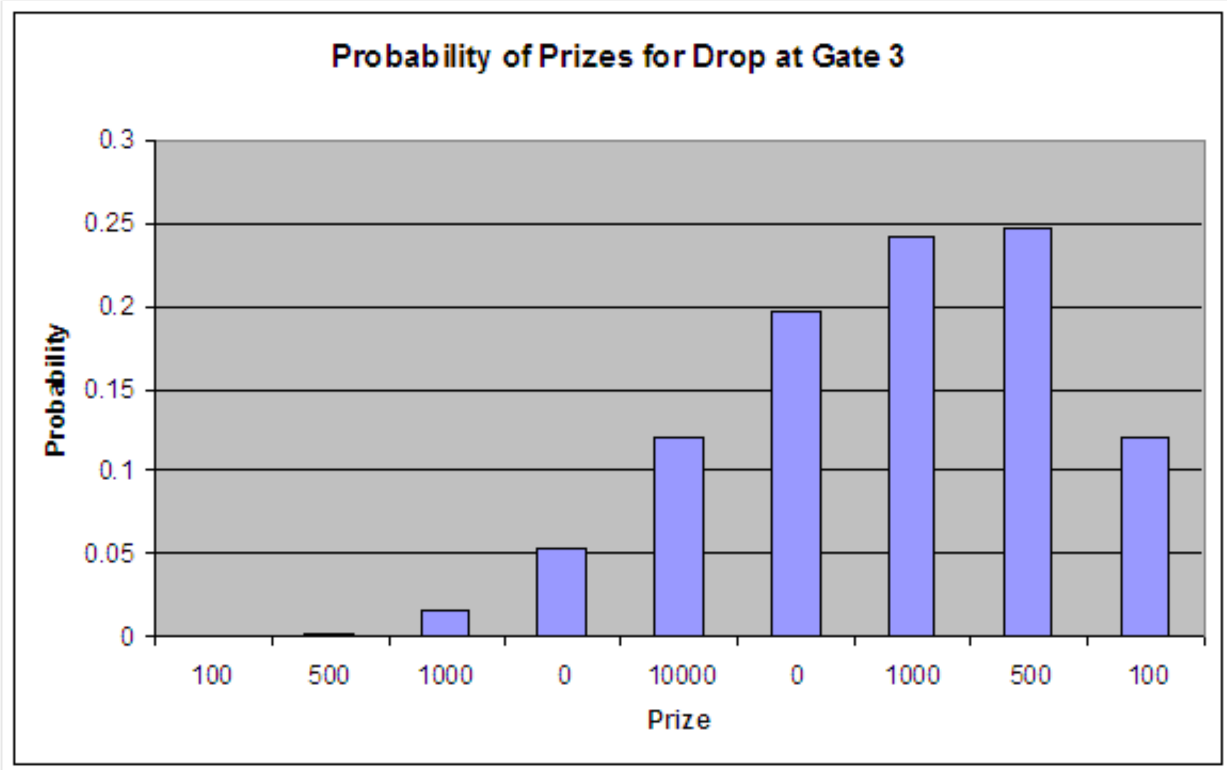
Now, if we knew a lot more about the board we could get a lot more accurate. We'd need to know the coefficient of friction between the puck and board, the slope of the board, the elasticity of the collision between the puck and pegs, the relative size of the puck and the pegs, and the starting velocity of the puck (the idea is to drop it from stationary, but a contestant could easily impart some force when letting it go).

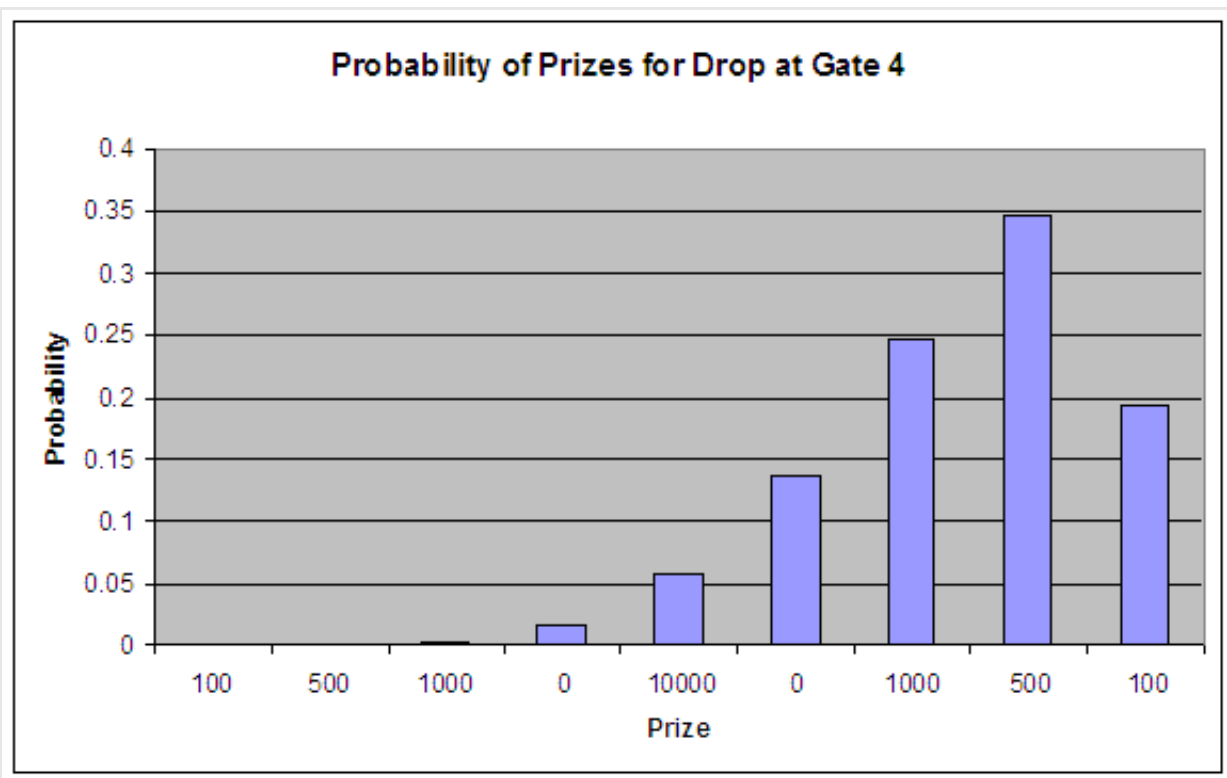
The big assumption that I'm going to make here is that when the puck hits a peg it has a 50/50 chance of falling to either the left or the right. For any single collision that is certainly at least a little off, but over the course of the path through the board I suspect those random differences should more or less cancel out. I think this is also the spirit of what a pegboard is meant to do.

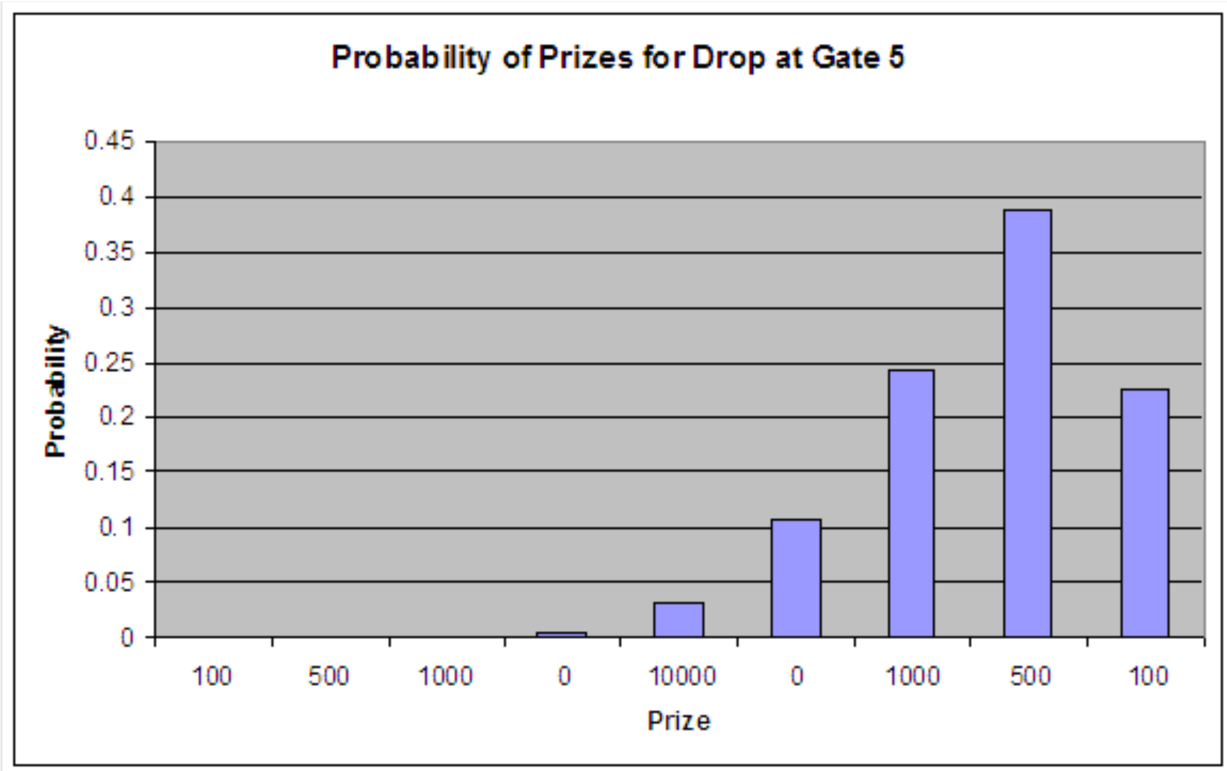
With this information it's pretty easy to calculate the cascading odds of the puck being in any given position on the board based on the gate in which it starts, and the probability that it will fall in any of the prize gates given that same starting gate. These odds make for some good graphs:




Probability of Prizes for Drop at Gate 2
Probability
0.25
0.2
0.15
0.1
0.05
0
100
500
1000
0
10000
0
1000
500
100
Prize


Probability of Prizes for Drop at Gate 3
0.3
0.25
0.2
0.15
0.1
0.05
0
Probability
100
500
1000
0
10000
0
1000
500
100
Prize


Probability of Prizes for Drop at Gate 4
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Probability
100
500
1000
0
10000
0
1000
500
100
Prize



Keep note that the y-axis does change on these graphs, and that the graphs also only reflect the right side of the board. The graphs for the left side would be symmetric but skewed in the other direction.

The first thing that should be apparent is the normal-like nature of the first graph, and the increasingly skewed nature of the following graphs as the puck starts out closer and closer to the sidewall. Something that makes sense but I hadn't thought about is the fact that in later graphs (gate 3 onward) the $100 prize is always lower than the prize reflected across the mode.

We can use the gate 5 graph as an example - as the graph starts to push up against the right sidewall the odds don't pile up at the wall but instead spread out to the left. This is partly because any time the puck gets to the wall it has no option except kicking back to the left, whereas a puck in the center of the board can fall either right or left.

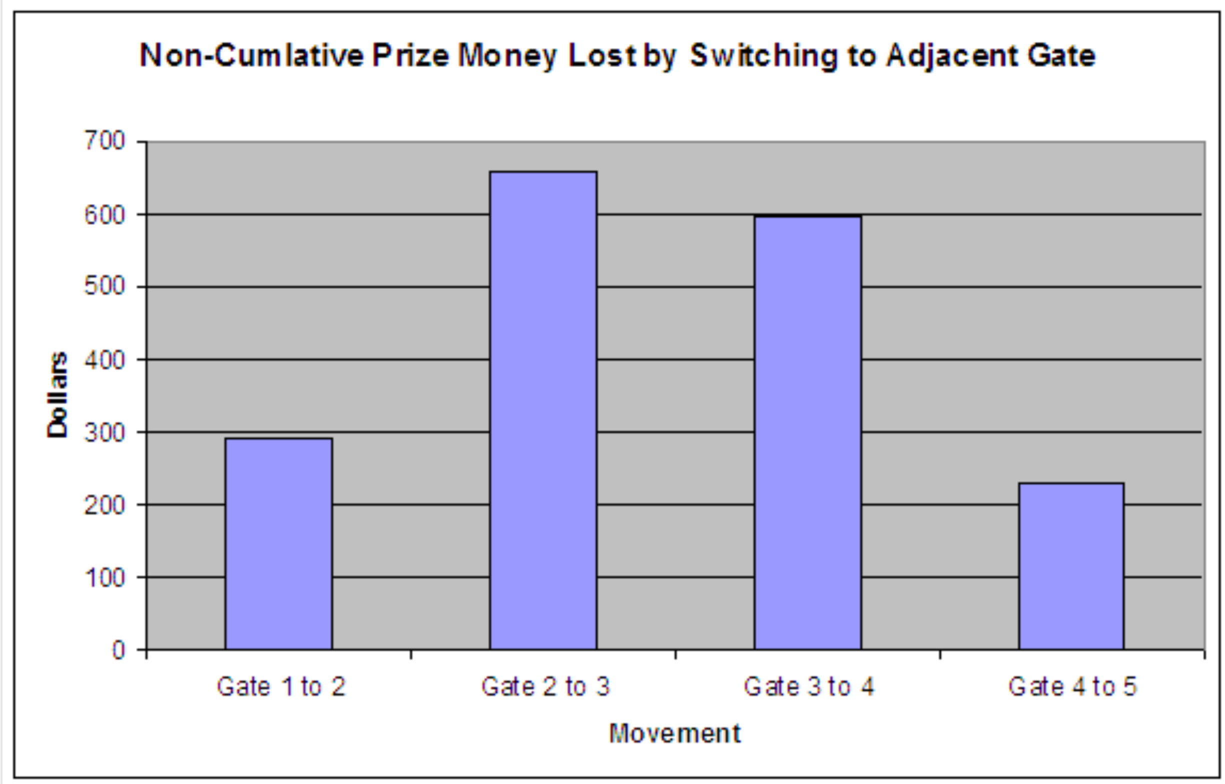
What I've wondered for a long time watching Plinko is which gate has the largest average payoff. I've always suspected it to the be the dead center (gate 1), but watching price is right there are a lot of people who place the puck in plenty of different places. They even look to the audience for guidance, as if the entire audience is likely to reach consensus on the issue.

Now that we have the probabilities for each of the prize values for each of the gates it is now a simple case of calculating the expected values for each of the gates. This is simply the sum of the products of each probability and each prize value, and will tell us the average amount of prize money that can be expected for a large number of drops from the same gate.

On The Price is Right you start with one drop, but have the opportunity to win four more, giving you the capability of five drops and a total potential prize of $50,000. Not shockingly, no one has ever come even close to this $50,000 value. Here's the expected values for each of the gates given the number of pucks the contestant is able to acquire:

Chances	1	2	3	4	5
Gate 1	2557.91	5115.82	7673.73	10231.64	12789.55
Gate 2	2265.92	4531.84	6797.76	9063.68	11329.6
Gate 3	1605.86	3211.72	4817.58	6423.44	8029.3
Gate 4	1009.08	2018.16	3027.24	4036.32	5045.4
Gate 5	780.37	1560.74	2341.11	3121.48	3901.85

These expected values confirm my long-held suspicion: the statistical advantage goes to the contestant who drops their pucks in the center gate. For contestants able to get all five pucks and drop all five of them in the center gate, prize money should average $12,789.55. Interestingly, using the gates just off of center (gate 2) doesn't lose you as much money as I would have thought - $291.99 for a single puck and $1459.95 for all five. It also doesn't lose as much money as moving to the next gate, as can be seen if we examine the difference between expected values of adjacent gates (think of it as the first derivative):



If you've made the mistake of deciding to move away from the center spot, the non-cumulative magnitude of the mistake is even worse if you move from gate 2 to gate 3. There's not much more to be gained dwelling on the magnitudes of these mistakes, and the result should be pretty clear:

For the biggest payoff in Plinko, use the center-most gate.