Pre-Cal Worksheet 12-1 A Name _____ Pd_

Find the next four terms in each arithmetic sequence.

6. 6, 11, 16, ...

7. $-15, -7, 1, \ldots$

8. $a - 6, a - 2, a + 2, \ldots$

18. $-18, -7, 4, \ldots$

21. $b, b + 4, b + 8, \ldots$

24. $5 + k, 5, 5 - k, \ldots$

19. 3, 4.5, 6, ...

22. $-x, 0, x, \ldots$

25. $2a - 5, 2a + 2, 2a + 9, \ldots$

9. Find the 17th term in the sequence for which $a_1 = 10$ and $d = -3$.

10. Find n for the sequence for which $a_n = 37$, $a_1 = -13$, and $d = 5$.

11. What is the first term in the sequence for which $d = -2$ and $a_7 = 3$?

12. Find d for the sequence for which $a_1 = 100$ and $a_{12} = 34$.

For Exercises 27-34, assume that each sequence or series is arithmetic.

27. Find the 25th term in the sequence for which $a_1 = 8$ and $d = 3$.

28. Find the 18th term in the sequence for which $a_1 = 1.4$ and $d = 0.5$.

29. Find n for the sequence for which $a_n = -41$, $a_1 = 19$, and $d = -5$.

30. Find n for the sequence for which $a_n = 138$, $a_1 = -2$, and $d = 7$.

31. What is the first term in the sequence for which $d = -3$, and $a_{15} = 38$?

For Exercises 35-49, assume that each sequence or series is arithmetic.

35. What is the eighth term in the sequence $-4 + \sqrt{5}, -1 + \sqrt{5}, 2 + \sqrt{5}, \ldots$?

36. What is the twelfth term in the sequence $5 - i, 6, 7 + i, \ldots$?

37. Find the 33rd term in the sequence 12.2, 10.5, 8.8, ...

38. Find the 79th term in the sequence $-7, -4, -1, \ldots$.

69. SAT/ACT Practice If $a - 4b = 15$ and $4a - b = 15$, then $a - b = ?$

A 3 B 4 C 6 D 15 E 30

Pre-Cal Worksheet 12-1 B Name _____ Pd_

13. Write a sequence that has two arithmetic means between 9 and 24.
14. What is the sum of the first 35 terms in the series $7 + 9 + 11 + \cdots$?
15. Find n for a series for which $a_1 = 30$, $d = -4$, and $S_n = -210$.

39. Write a sequence that has one arithmetic mean between 12 and 21.
40. Write a sequence that has two arithmetic means between -5 and 4.
41. Write a sequence that has two arithmetic means between $\sqrt{3}$ and 12.
42. Write a sequence that has three arithmetic means between 2 and 5.
43. Find the sum of the first 11 terms in the series $\frac{3}{2} + 1 + \frac{1}{2} + \ldots$.
44. Find the sum of the first 100 terms in the series $-5 - 4.8 - 4.6 - \ldots$.

45. Find the sum of the first 26 terms in the series $-19 - 13 - 7 - \ldots$.
46. Find n for a series for which $a_1 = -7$, $d = 1.5$, and $S_n = -14$.
47. Find n for a series for which $a_1 = -3$, $d = 2.5$, and $S_n = 31.5$.
48. Write an expression for the nth term of the sequence 5, 7, 9, ...
49. Write an expression for the nth term of the sequence 6, -2, -10, ...

Find the next 4

17. $5, -1, -7, \ldots$

20. $5.6, 3.8, 2, \ldots$

23. $5n, -n, -7n, \ldots$

26. Determine the common difference and find the next three terms of the arithmetic sequence $3 + \sqrt{7}, 5, 7 - \sqrt{7}, \ldots$.

32. What is the first term in the sequence for which $d = \frac{1}{3}$ and $a_7 = 10\frac{2}{3}$?

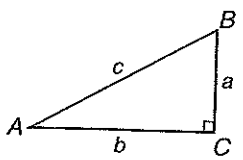
33. Find d for the sequence in which $a_1 = 6$ and $a_{14} = 58$.

34. Find d for the sequence in which $a_1 = 8$ and $a_{11} = 26$.

16. **Theater Design** The right side of the orchestra section of the Nederlander Theater in New York City has 19 rows, and the last row has 27 seats. The numbers of seats in each row increase by 1 as you move toward the back of the section. How many seats are in this section of the theater?

50. **Keyboarding** Antonio has found that he can input statistical data into his computer at the rate of 2 data items faster each half hour he works. One Monday, he starts work at 9:00 A.M., inputting at a rate of 3 data items per minute. At what rate will Antonio be inputting data into the computer by lunchtime (noon)?

65. Solve triangle ABC if $B = 19°32'$ and $c = 4.5$. Round angle measures to the nearest minute and side measures to the nearest tenth. *(Lesson 5-5)*



Pre-Cal Worksheet 12-2A Name _____ Pd___

Determine the common ratio and find the next three terms of each geometric sequence.

7. $\frac{2}{3}, 4, 24, \ldots$

8. $2, 3, \frac{9}{2}, \ldots$

9. $1.8, -7.2, 28.8, \ldots$

Determine the common ratio and find the next three terms of each geometric sequence.

16. $10, 2, 0.4, \ldots$

19. $\frac{3}{4}, \frac{3}{10}, \frac{3}{25}, \ldots$

22. $9, 3\sqrt{3}, 3, \ldots$

17. $8, -20, 50, \ldots$

20. $-7, 3.5, -1.75, \ldots$

23. $i, -1, -i, \ldots$

For Exercises 10–14, assume that each sequence or series is geometric.

10. Find the seventh term of the sequence $7, 2.1, 0.63, \ldots$.

11. If $r = 2$ and $a_5 = 24$, find the first term of the sequence.

12. Find the first three terms of the sequence for which $a_4 = 2.5$ and $r = 2$.

For Exercises 26–40, assume that each sequence or series is geometric.

26. Find the fifth term of a sequence whose first term is 8 and common ratio is $\frac{3}{2}$.

27. Find the sixth term of the sequence $\frac{1}{2}, -\frac{3}{8}, \frac{9}{32}, \ldots$.

28. Find the seventh term of the sequence $40, 0.4, 0.004, \ldots$.

29. Find the ninth term of the sequence $\sqrt{5}, \sqrt{10}, 2\sqrt{5}, \ldots$.

30. If $r = 4$ and $a_6 = 192$, what is the first term of the sequence?

31. If $r = -\sqrt{2}$ and $a_5 = 32\sqrt{2}$, what is the first term of the sequence?

32. Find the first three terms of the sequence for which $a_5 = -6$ and $r = -\frac{1}{3}$.

54. If $\csc \theta = 3$ and $0° \leq \theta \leq 90°$, find $\sin \theta$. *(Lesson 7-1)*

55. **Weather** The maximum normal daily temperatures in each season for Lincoln, Nebraska, are given below. Write a sinusoidal function that models the temperatures, using $t = 1$ to represent winter. *(Lesson 6-6)*

Normal Daily Temperatures for Lincoln, Nebraska

Winter	Spring	Summer	Fall
36°	61°	86°	65°

Source: Rand McNally & Company

56. Given $A = 43°$, $b = 20$, and $a = 11$, do these measurements determine one triangle, two triangles, or no triangle? *(Lesson 5-7)*

57. **SAT Practice** **Grid-In** If n and m are integers, and $-(n^2) \leq -\sqrt{49}$ and $m = n + 1$, what is the least possible value of mn?

Pre-Cal Worksheet 12-2 B Name _____ Pd_

13. Write a sequence that has two geometric means between 1 and 27.

14. Find the sum of the first nine terms of the series $0.5 - 1 + 2 - \cdots$.

34. Write a sequence that has three geometric means between 256 and 81.

35. Write a sequence that has two geometric means between -2 and 54.

36. Write a sequence that has one geometric mean between $\frac{4}{7}$ and 7.

37. What is the sum of the first five terms of the series $\frac{5}{3} + 5 + 15 + \cdots$?

38. What is the sum of the first six terms of the series $65 + 13 + 2.6 + \cdots$?

39. Find the sum of the first ten terms of the series $1 - \frac{3}{2} + \frac{9}{4} - \cdots$.

40. Find the sum of the first eight terms of the series $2 + 2\sqrt{3} + 6 + \cdots$.

State R and the next 2 terms

18. $\frac{2}{9}, \frac{2}{3}, 2, \ldots$

21. $3\sqrt{2}, 6, 6\sqrt{2}, \ldots$

24. $t^8, t^5, t^2, \ldots$

25. The first term of a geometric sequence is $\frac{a}{b^2}$, and the common ratio is $\frac{b}{a^2}$. Find the next five terms of the geometric sequence.

33. Find the first three terms of the sequence for which $a_5 = 0.32$ and $r = 0.2$.

15. **Investment** Mika Rockwell invests in classic cars. He recently bought a 1978 convertible valued at $20,000. The value of the car is predicted to appreciate at a rate of 3.5% per year. Find the value of the car after 10, 20, and 40 years, assuming that the rate of appreciation remains constant.

41. **Biology** A cholera bacterium divides every half-hour to produce two complete cholera bacteria.
    a. If an initial colony contains a population of b_0 bacteria, write an equation that will determine the number of bacteria present after t hours.
    b. Suppose a petri dish contains 30 cholera bacteria. Use the equation from part **a** to determine the number of bacteria present 5 hours later.
    c. What assumptions are made in using the formula found in part **a**?

42. **Critical Thinking** Consider the geometric sequence with $a_4 = 4$ and $a_7 = 12$.
    a. Find the common ratio and the first term of the sequence.
    b. Find the 28th term of the sequence.

43. **Consumerism** High Tech Electronics advertises a weekly installment plan for the purchase of a popular brand of big screen TV. The buyer pays $5 at the end of the first week, $5.50 at the end of the second week, $6.05 at the end of the third week, and so on for one year.
    a. What will the payments be at the end of the 10th, 20th, and 40th weeks?
    b. Find the total cost of the TV.
    c. Why is the cost found in part **b** not entirely accurate?

44. **Statistics** A number x is said to be the *harmonic mean* of y and z if $\frac{1}{x}$ is the average of $\frac{1}{y}$ and $\frac{1}{z}$.
    a. Find the harmonic mean of 5 and 8.
    b. 8 is the harmonic mean of 20 and another number. What is the number?

Pre-Cal Worksheet 12-3 A Name _____ Pd__

Find each limit, or state that the limit does not exist and explain your reasoning.

5. $\lim_{n\to\infty} \frac{1}{5^n}$

6. $\lim_{n\to\infty} \frac{5-n^2}{2n}$

7. $\lim_{n\to\infty} \frac{3n-6}{7n}$

Find each limit, or state that the limit does not exist and explain your reasoning.

14. $\lim_{n\to\infty} \frac{7-2n}{5n}$

15. $\lim_{n\to\infty} \frac{n^3-2}{n^2}$

16. $\lim_{n\to\infty} \frac{6n^2+5}{3n^2}$

17. $\lim_{n\to\infty} \frac{9n^3+5n-2}{2n^3}$

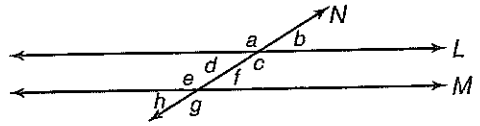
18. $\lim_{n\to\infty} \frac{(3n+4)(1-n)}{n^2}$

19. $\lim_{n\to\infty} \frac{8n^2+5n+2}{3+2n}$

After you work each problem, record your answer on the answer sheet provided or on a piece of paper.

Multiple Choice

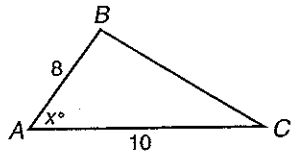
1. In the figure below, line L is parallel to line M. Line N intersects both L and M, with angles a, b, c, d, e, f, g, and h as shown. Which of the following lists includes all of the angles that are supplementary to $\angle a$?



A b, d, f, h B c, e, g C b, d, c

D e, f, g, h E d, c, h, g

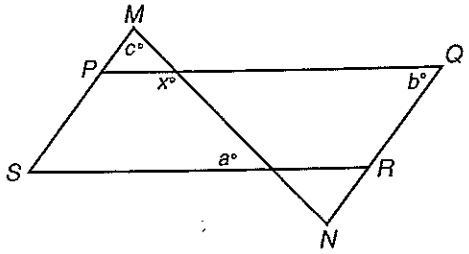
2. In the figure below, what is the area of $\triangle ABC$ in terms of x?



A $10 \sin x$ B $40 \sin x$ C $80 \sin x$

D $40 \cos x$ E $80 \cos x$

3. If $PQRS$ is a parallelogram and $\overline{MN}$ is a line segment, then x must equal



A $180 - b$ B $180 - c$ C $a + b$

D $a + c$ E $b + c$

4. If a rectangular swimming pool has a volume of 16,500 cubic feet, a depth of 10 feet, and a length of 75 feet, what is the width of the pool, in feet?

A 22 B 26 C 32 D 110 E 1650

5. $\frac{1}{10^{100}} - \frac{1}{10^{99}} =$

A $\frac{-9}{10^{100}}$ B $\frac{-1}{10^{100}}$ C $\frac{1}{10^{100}}$ D $\frac{1}{10}$ E $\frac{9}{10}$

Pre-Cal Worksheet 12-3 B Name _____ Pd__

Find the sum of each infinite series, or state that the sum does not exist and explain your reasoning.

10. $-6 + 3 - \frac{3}{2} + \cdots$

11. $\frac{3}{4} + \frac{1}{4} + \frac{1}{12} + \cdots$

12. $\sqrt{3} + 3 + \sqrt{27} + \cdots$

Find the sum of each series, or state that the sum does not exist and explain your reasoning.

31. $16 + 12 + 9 + \cdots$

34. $6 + 5 + 4 + \cdots$

37. $\frac{6}{5} + \frac{4}{5} + \frac{8}{15} + \cdots$

32. $5 + 7.5 + 11.25 + \cdots$

35. $\frac{1}{8} + \frac{1}{4} + \frac{1}{2} + \cdots$

38. $\sqrt{5} + 1 + \frac{\sqrt{5}}{5} + \cdots$

33. $10 + 5 + 2.5 + \cdots$

36. $-\frac{2}{3} + \frac{1}{9} - \frac{1}{54} + \cdots$

39. $8 - 4\sqrt{3} + 6 - \cdots$

20. $\lim_{n\to\infty} \frac{4 - 3n + n^2}{2n^3 - 3n^2 + 5}$

21. $\lim_{n\to\infty} \frac{n}{3^n}$

22. $\lim_{n\to\infty} \frac{(-2)^n n}{4 + n}$

3. Find the limit of the sequence described by the general expression $\frac{5n + (-1)^n}{n^2}$, or state that the limit does not exist. Explain your reasoning.

13. **Entertainment** Pete's Pirate Ride operates like the bob of a pendulum. On its longest swing, the ship travels through an arc 75 meters long. Each successive swing is two-fifths the length of the preceding swing. If the ride is allowed to continue without intervention, what is the total distance the ship will travel before coming to rest?

40. **Physics** A basketball is dropped from a height of 35 meters and bounces $\frac{2}{5}$ of the distance after each fall.

a. Find the first five terms of the infinite series representing the vertical distance traveled by the ball.

b. What is the total vertical distance the ball travels before coming to rest? (*Hint:* Rewrite the series found in part **a** as the sum of two infinite geometric series.)

41. **Critical Thinking** Consider the sequence whose nth term is described by $\frac{n^2}{2n+1} - \frac{n^2}{2n-1}$.

a. Explain why $\lim\limits_{n\to\infty}\left(\frac{n^2}{2n+1} - \frac{n^2}{2n-1}\right) \neq \lim\limits_{n\to\infty}\frac{n^2}{2n+1} - \lim\limits_{n\to\infty}\frac{n^2}{2n-1}$.

b. Find $\lim\limits_{n\to\infty}\left(\frac{n^2}{2n+1} - \frac{n^2}{2n-1}\right)$.

42. **Engineering** Francisco designs a toy with a rotary flywheel that rotates at a maximum speed of 170 revolutions per minute. Suppose the flywheel is operating at its maximum speed for one minute and then the power supply to the toy is turned off. Each subsequent minute thereafter, the flywheel rotates two-fifths as many times as in the preceding minute. How many complete revolutions will the flywheel make before coming to a stop?

43. **Critical Thinking** Does $\lim\limits_{n\to\infty}\cos\frac{n\pi}{2}$ exist? Explain.

Pre-Cal Work Sheet 12-5A Name _____ Pd__

Write each expression in expanded form and then find the sum.

4. $\sum_{n=1}^{6} (n-3)$ 5. $\sum_{k=2}^{5} 4k$ 6. $\sum_{a=0}^{4} \frac{1}{2^a}$ 7. $\sum_{p=0}^{\infty} 5\left(\frac{3}{4}\right)^p$

Write each expression in expanded form and then find the sum.

14. $\sum_{n=1}^{4} (2n-7)$

17. $\sum_{k=2}^{6} (k+k^2)$

20. $\sum_{m=0}^{3} 3^{m-1}$

23. $\sum_{k=3}^{7} k!$

15. $\sum_{a=2}^{5} 5a$

18. $\sum_{n=5}^{8} \frac{n}{n-4}$

21. $\sum_{r=1}^{3} \left(\frac{1}{2}+4^r\right)$

24. $\sum_{p=0}^{\infty} 4(0.75)^p$

16. $\sum_{b=3}^{8} (6-4b)$

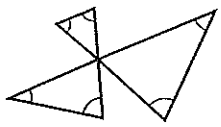
19. $\sum_{j=4}^{8} 2^j$

22. $\sum_{i=3}^{5} (0.5)^{-i}$

25. $\sum_{n=1}^{\infty} 4\left(\frac{2}{5}\right)^n$

26. Write $\sum_{n=2}^{5} n + i^n$ in expanded form. Then find the sum.

6. In the figure, what is the sum of the degree measures of the marked angles?

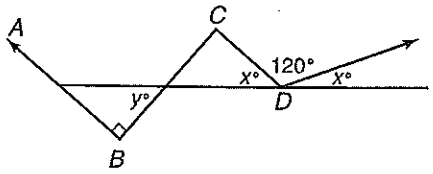


A 180 B 270 C 360 D 540
E It cannot be determined from the information given.

7. If $5x^2 + 6x = 70$ and $5x^2 - 6y = 10$, then what is the value of $10x + 10y$?

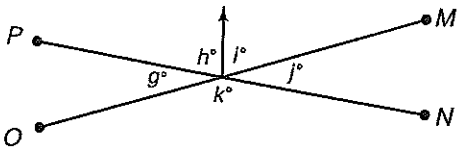
A 10 B 20 C 60 D 80 E 100

8. In the figure below, if $\overline{AB} \parallel \overline{CD}$, then what is the value of y? *Figure not drawn to scale.*



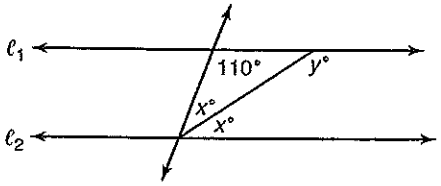
A 30 B 60 C 90 D 120 E 150

9. Which pair must be equal?



A h and i
B $(g + h)$ and $(i + j)$
C $(g + i)$ and $(h + j)$
D g and j
E $(g + j)$ and $(h + i)$

10. **Grid-In** If ℓ_1 is parallel to ℓ_2 in the figure below, what is the value of y?



Pre-Cal Worksheet 12-5B Name _____ Pd__

Express each series using sigma notation.

8. $5 + 10 + 15 + 20 + 25$

9. $2 + 4 + 10 + 28$

10. $2 - 4 - 10 - 16$

11. $\frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{32} + \cdots$

12. $-3 + 9 - 27 + \cdots$

Express each series using sigma notation.

27. $6 + 9 + 12 + 15$

29. $8 + 10 + 12 + \cdots + 24$

31. $10 + 50 + 250 + 1250$

33. $\frac{1}{9} + \frac{1}{14} + \frac{1}{19} + \cdots + \frac{1}{49}$

35. $4 - 9 + 16 - 25 + \cdots$

37. $-32 + 16 - 8 + 4 - \cdots$

28. $1 + 4 + 16 + \cdots + 256$

30. $-8 + 4 - 2 + 1$

32. $13 + 9 + 5 + 1$

34. $\frac{2}{3} + \frac{4}{5} + \frac{8}{7} + \frac{16}{9} + \cdots$

36. $5 + 5 + \frac{5}{2} + \frac{5}{6} + \frac{5}{24} + \cdots$

38. $2 + \frac{6}{2} + \frac{24}{3} + \frac{120}{4} + \cdots$

Simplify. Assume that n and m are positive integers, $a > b$, and $a > 2$.

42. $\frac{(a-2)!}{a!}$

43. $\frac{(a+1)!}{(a-2)!}$

44. $\frac{(a+b)!}{(a+b-1)!}$

13. **Aviation** Each October Albuquerque, New Mexico, hosts the Balloon Fiesta. In 1998, 873 hot air balloons participated in the opening day festivities. One of these balloons rose 389 feet after 1 minute. Because the air in the balloon was not reheated, each succeeding minute the balloon rose 63% as far as it did the previous minute.

   a. Use sigma notation to represent the height of the balloon above the ground after one hour. Then calculate the total height of the balloon after one hour to the nearest foot.

   b. What was the maximum height achieved by this balloon?

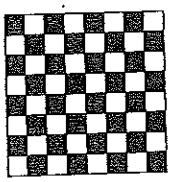
49. **Word Play** An *anagram* is a word or phrase that is made by rearranging the letters of another word or phrase. Consider the word "SILENT."

   a. How many different arrangements of the letters in this word are possible? Write this number as a factorial. (*Hint:* First solve a simpler problem to see a pattern, such as how many different arrangements are there of just 2 letters? 3 letters?)

   b. If a friend gives you a hint and tells you that an anagram of this word starts with "L," how many different arrangements still remain?

   c. Your friend gives you one more hint. The last letter in the anagram is "N." Determine how many possible arrangements remain and then determine the anagram your friend is suggesting.

50. **Chess** A standard chess board contains 64 small black or white squares. These squares make up many other larger squares of various sizes.



   a. How many 8×8 squares are there on a standard 8×8 chessboard? How many 7×7 squares?

   b. Continue this list until you have accounted for all 8 sizes of squares.

   c. Use sigma notation to represent the total number of squares found on an 8×8 chessboard. Then calculate this sum.

Pre-Cal Worksheet 12-1 A Name Key Pd__

Find the next four terms in each arithmetic sequence.

6. 6, 11, 16, ... 21, 26 D = 5

7. −15, −7, 1, ... 9, 17 D = 8

8. $a - 6, a - 2, a + 2, \ldots$ a+6, a+10 D = 4

18. −18, −7, 4, ...

21. $b, b + 4, b + 8, \ldots$

24. $5 + k, 5, 5 - k, \ldots$ 5 - 2k, 5 - 3k D = -k

19. 3, 4.5, 6, ...

22. $-x, 0, x, \ldots$

25. $2a - 5, 2a + 2, 2a + 9, \ldots$

$a_n = a_1 + (n-1)d$

9. Find the 17th term in the sequence for which $a_1 = 10$ and $d = -3$.

10. Find n for the sequence for which $a_n = 37$, $a_1 = -13$, and $d = 5$.

11. What is the first term in the sequence for which $d = -2$ and $a_7 = 3$? → $3 = a_1 + (7-1)(-2)$, $3 = a_1 + -12$, $15 = a_1$

12. Find d for the sequence for which $a_1 = 100$ and $a_{12} = 34$.

$34 = 100 + (12-1)d$

$34 = 100 + 11d$

$-66 = 11d$ $d = -6$

For Exercises 27-34, assume that each sequence or series is arithmetic.

27. Find the 25th term in the sequence for which $a_1 = 8$ and $d = 3$. $a_{25} = 8 + (25-1)(3) = 80$

28. Find the 18th term in the sequence for which $a_1 = 1.4$ and $d = 0.5$.

29. Find n for the sequence for which $a_n = -41$, $a_1 = 19$, and $d = -5$. $-41 = 19 + (n-1)(-5)$, $-60 = (n-1)(-5)$, $12 = n-1$, $13 = n$

30. Find n for the sequence for which $a_n = 138$, $a_1 = -2$, and $d = 7$.

31. What is the first term in the sequence for which $d = -3$, and $a_{15} = 38$?

$38 = a_1 + (15-1)(-3)$

$38 = a_1 + -42$

$80 = a_1$

For Exercises 35-49, assume that each sequence or series is arithmetic.

35. What is the eighth term in the sequence $-4 + \sqrt{5}, -1 + \sqrt{5}, 2 + \sqrt{5}, \ldots$?

36. What is the twelfth term in the sequence $5 - i, 6, 7 + i, \ldots$?

37. Find the 33rd term in the sequence 12.2, 10.5, 8.8, ...

38. Find the 79th term in the sequence $-7, -4, -1, \ldots$.

$D = 3$

(35) $a_8 = -4+\sqrt{5} + (8-1)(3)$

$a_8 = -4 + \sqrt{5} + 21$

$a_8 = 17 + \sqrt{5}$

(36) $D = 1+i$

$a_{12} = 5 - i + (12-1)(1+i)$

$a_{12} = 5 - i + 11 + 11i$

$a_{12} = 16 + 10i$

(37) $D = -1.7$

$a_{33} = 12.2 + (33-1)(-1.7)$

$a_{33} = -42.2$

(38) $D = 3$

$a_{79} = -7 + (79-1)(3)$

$a_{79} = 227$

69. **SAT/ACT Practice** If $a - 4b = 15$ and $4a - b = 15$, then $a - b = ?$

A 3 B 4 (C 6) D 15 E 30

$a - 4b = 15 \rightarrow a = 15 + 4b$

$4a - b = 15$

$4(15 + 4b) - b = 15$

$60 + 16b - b = 15$

$15b = -45$

$b = -3$

$a - 4(-3) = 15$

$-12 \quad -12$

$a = 3$

$3 - (-3) = 6$

Pre-Cal Worksheet 12-1 B Name Key

Pd_

13. Write a sequence that has two arithmetic means between 9 and 24.

14. What is the sum of the first 35 terms in the series $7 + 9 + 11 + \cdots$?

5. Find n for a series for which $a_1 = 30$, $d = -4$, and $S_n = -210$.

39. Write a sequence that has one arithmetic mean between 12 and 21.

40. Write a sequence that has two arithmetic means between -5 and 4.

41. Write a sequence that has two arithmetic means between $\sqrt{3}$ and 12.

42. Write a sequence that has three arithmetic means between 2 and 5.

43. Find the sum of the first 11 terms in the series $\frac{3}{2} + 1 + \frac{1}{2} + \ldots$.

44. Find the sum of the first 100 terms in the series $-5 - 4.8 - 4.6 - \ldots$.

$12 \underset{}{\overset{16.5}{\_\_}} 21$ $D = 4.5$

$-5 \underset{}{\overset{-2}{\_\_}} \underset{}{\overset{1}{\_\_}} 4$ $D = 3$

$\sqrt{3}\ \underset{}{\overset{4+\frac{2\sqrt{3}}{3}}{\_\_}}\ \underset{}{\overset{4+\frac{4\sqrt{3}}{3}}{\_\_}}\ 12$

$12 = \sqrt{3} + (4-1)D$

$12 - \sqrt{3} = 3D$

$4 - \frac{\sqrt{3}}{3} = D$

(42) $2 \underset{}{\overset{2.75}{\_\_}} \underset{}{\overset{3.5}{\_\_}} \underset{}{\overset{4.25}{\_\_}} 5$

$D = \frac{3}{4}$

(43) $a_{11} = \frac{3}{2} + (11-1)\left(-\frac{1}{2}\right)$

$a_{11} = 1.5 - 5$

$a_{11} = -3.5$

$S_{11} = \frac{11}{2}\left(\frac{3}{2} + \frac{-7}{2}\right)$

$= 5.5(-2) = -11$

(44) $a_{100} = -5 + (100-1)(.2)$

$= -5 + 19.8$

$= 14.8$

$S_{100} = \frac{100}{2}(-5 + 14.8)$

$= 490$

45. Find the sum of the first 26 terms in the series $-19 - 13 - 7 - \ldots$.

46. Find n for a series for which $a_1 = -7$, $d = 1.5$, and $S_n = -14$.

47. Find n for a series for which $a_1 = -3$, $d = 2.5$, and $S_n = 31.5$.

48. Write an expression for the nth term of the sequence 5, 7, 9, ...

49. Write an expression for the nth term of the sequence 6, -2, -10, ...

(46) ~~$a_n = -7 + (n-1)(1.5)$~~

~~$-14 = \frac{n}{2}[-7 + a_n]$~~

$-14 = \frac{n}{2}[-7 + -8.5 + 1.5n]$

$-14 = -7.75n + .75n^2$

$n = 2.\overline{3}$ & 8

(49) $a_n = 6 + (n-1)(-8)$

$a_n = -8n + 14$

$S_n = \frac{n}{2}(6 + -8n + 14)$

$= \frac{n}{2}(20 - 8n)$

$S_n = 10n - 4n^2$

Find the next 4

17. 5, −1, −7, ...

20. 5.6, 3.8, 2, ...

23. $5n, -n, -7n, \ldots$

26. Determine the common difference and find the next three terms of the arithmetic sequence $3 + \sqrt{7}, 5, 7 - \sqrt{7}, \ldots$.

$D = 5 - (3+\sqrt{7})$

$2 - \sqrt{7}$

32. What is the first term in the sequence for which $d = \frac{1}{3}$ and $a_7 = 10\frac{2}{3}$?

33. Find d for the sequence in which $a_1 = 6$ and $a_{14} = 58$.

34. Find d for the sequence in which $a_1 = 8$ and $a_{11} = 26$.

$10\frac{2}{3} = a_1 + (7-1)(\frac{1}{3})$

$10^{2/3} = a_1 + 2$

$8^{2/3} = a_1$

16. **Theater Design** The right side of the orchestra section of the Nederlander Theater in New York City has 19 rows, and the last row has 27 seats. The numbers of seats in each row increase by 1 as you move toward the back of the section. How many seats are in this section of the theater?

$S_9 = \frac{9}{2}(19 + 27)$

$= 4.5(46)$

$= 207$

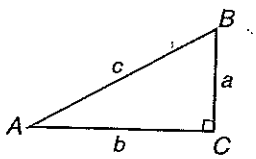
$27 = 19 + (n-1)(1)$

$8 = n - 1$

$9 = n$

50. **Keyboarding** Antonio has found that he can input statistical data into his computer at the rate of 2 data items faster each half hour he works. One Monday, he starts work at 9:00 A.M., inputting at a rate of 3 data items per minute. At what rate will Antonio be inputting data into the computer by lunchtime (noon)?

65. Solve triangle ABC if $B = 19°32'$ and $c = 4.5$. Round angle measures to the nearest minute and side measures to the nearest tenth. *(Lesson 5-5)*



Pre-Cal Worksheet 12-2 A Name Key Pd___

Determine the common ratio and find the next three terms of each geometric sequence.

7. $\frac{2}{3}, 4, 24, \ldots$

8. $2, 3, \frac{9}{2}, \ldots$

9. $1.8, -7.2, 28.8, \ldots$

Determine the common ratio and find the next three terms of each geometric sequence.

16. $10, 2, 0.4, \ldots$ $r = .2$ $.08, .016, .0032$

19. $\frac{3}{4}, \frac{3}{10}, \frac{3}{25}, \ldots$ $.048, .0192, .00768$ $r = .4$

22. $9, 3\sqrt{3}, 3, \ldots$ $\sqrt{3}, 1, \frac{\sqrt{3}}{3}$ $r = \frac{\sqrt{3}}{3}$

$r = \frac{3\sqrt{3}}{9} = \frac{\sqrt{3}}{3}$ $\frac{\sqrt{3}}{1} \cdot \frac{\sqrt{3}}{3} = \frac{3}{3} = 1$

17. $8, -20, 50, \ldots$

20. $-7, 3.5, -1.75, \ldots$

23. $i, -1, -i, \ldots$ $1 \cdot i, -1$

$r = \frac{-1}{i} = \frac{\cdot i^2}{i} = \cdot i$

For Exercises 10–14, assume that each sequence or series is geometric.

10. Find the seventh term of the sequence $7, 2.1, 0.63, \ldots$.

11. If $r = 2$ and $a_5 = 24$, find the first term of the sequence.

12. Find the first three terms of the sequence for which $a_4 = 2.5$ and $r = 2$.

For Exercises 26–40, assume that each sequence or series is geometric.

* 26. Find the fifth term of a sequence whose first term is 8 and common ratio is $\frac{3}{2}$. $a_5 = 8 \cdot \left(\frac{3}{2}\right)^4 = 40.5$

27. Find the sixth term of the sequence $\frac{1}{2}, -\frac{3}{8}, \frac{9}{32}, \ldots$.

28. Find the seventh term of the sequence $40, 0.4, 0.004, \ldots$.

* 29. Find the ninth term of the sequence $\sqrt{5}, \sqrt{10}, 2\sqrt{5}, \ldots$. $r = \frac{\sqrt{10}}{\sqrt{5}} = \sqrt{2}$ $a_9 = \sqrt{5}\left(\sqrt{2}\right)^8 = \sqrt{5} \cdot 16 = 16\sqrt{5}$

30. If $r = 4$ and $a_6 = 192$, what is the first term of the sequence?

31. If $r = -\sqrt{2}$ and $a_5 = 32\sqrt{2}$, what is the first term of the sequence?

32. Find the first three terms of the sequence for which $a_5 = -6$ and $r = -\frac{1}{3}$.

54. If $\csc \theta = 3$ and $0° \leq \theta \leq 90°$, find $\sin \theta$. *(Lesson 7-1)*

55. **Weather** The maximum normal daily temperatures in each season for Lincoln, Nebraska, are given below. Write a sinusoidal function that models the temperatures, using $t = 1$ to represent winter. *(Lesson 6-6)*

Normal Daily Temperatures for Lincoln, Nebraska

Winter	Spring	Summer	Fall
36°	61°	86°	65°

Source: Rand McNally & Company

56. Given $A = 43°$, $b = 20$, and $a = 11$, do these measurements determine one triangle, two triangles, or no triangle? *(Lesson 5-7)*

57. **SAT Practice** **Grid-In** If n and m are integers, and $-(n^2) \leq -\sqrt{49}$ and $m = n + 1$, what is the least possible value of mn?

$n = -7, 7$

$n^2 \leq \sqrt{49}$

54

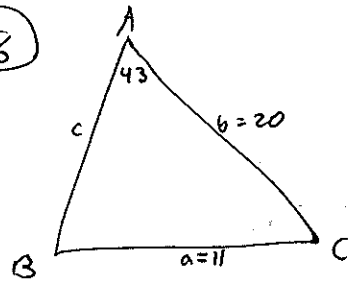
$\csc \theta = 3$

$\frac{1}{\sin \theta} = \frac{3}{1}$

$\sin \theta = \frac{1}{3}$

$\theta = \sin^{-1}\left(\frac{1}{3}\right) = 19.5$

56



$\frac{11}{\sin 43} = \frac{20}{\sin B}$

$\sin B = 1.2$

not possible

Pre-Cal Worksheet 12-2 B Name Key Pd_

13. Write a sequence that has two geometric means between 1 and 27.

14. Find the sum of the first nine terms of the series $0.5 - 1 + 2 - \cdots$.

$S_n = \frac{a_1 - a_1 r^n}{1 - r}$

34. Write a sequence that has three geometric means between 256 and 81.

35. Write a sequence that has two geometric means between -2 and 54.

36. Write a sequence that has one geometric mean between $\frac{4}{7}$ and 7.

37. What is the sum of the first five terms of the series $\frac{5}{3} + 5 + 15 + \cdots$?

38. What is the sum of the first six terms of the series $65 + 13 + 2.6 + \cdots$?

39. Find the sum of the first ten terms of the series $1 - \frac{3}{2} + \frac{9}{4} - \cdots$. $r = -\frac{3}{2}$ $n = 10$ $\frac{1 - 1\left(-\frac{3}{2}\right)^{10}}{1 - \left(-\frac{3}{2}\right)} = -22.6$

40. Find the sum of the first eight terms of the series $2 + 2\sqrt{3} + 6 + \cdots$.

18. $\frac{2}{9}, \frac{2}{3}, 2, \ldots$

21. $3\sqrt{2}, 6, 6\sqrt{2}, \ldots$

24. $t^8, t^5, t^2, \ldots$

25. The first term of a geometric sequence is $\frac{a}{b^2}$, and the common ratio is $\frac{b}{a^2}$. Find the next five terms of the geometric sequence.

33. Find the first three terms of the sequence for which $a_5 = 0.32$ and $r = 0.2$.

15. **Investment** Mika Rockwell invests in classic cars. He recently bought a 1978 convertible valued at $20,000. The value of the car is predicted to appreciate at a rate of 3.5% per year. Find the value of the car after 10, 20, and 40 years, assuming that the rate of appreciation remains constant.

41. **Biology** A cholera bacterium divides every half-hour to produce two complete cholera bacteria.
    a. If an initial colony contains a population of b_0 bacteria, write an equation that will determine the number of bacteria present after t hours.
    b. Suppose a petri dish contains 30 cholera bacteria. Use the equation from part **a** to determine the number of bacteria present 5 hours later.
    c. What assumptions are made in using the formula found in part **a**?

42. **Critical Thinking** Consider the geometric sequence with $a_4 = 4$ and $a_7 = 12$.
    a. Find the common ratio and the first term of the sequence.
    b. Find the 28th term of the sequence.

43. **Consumerism** High Tech Electronics advertises a weekly installment plan for the purchase of a popular brand of big screen TV. The buyer pays $5 at the end of the first week, $5.50 at the end of the second week, $6.05 at the end of the third week, and so on for one year.
    a. What will the payments be at the end of the 10th, 20th, and 40th weeks?
    b. Find the total cost of the TV.
    c. Why is the cost found in part **b** not entirely accurate?

44. **Statistics** A number x is said to be the *harmonic mean* of y and z if $\frac{1}{x}$ is the average of $\frac{1}{y}$ and $\frac{1}{z}$.
    a. Find the harmonic mean of 5 and 8.
    b. 8 is the harmonic mean of 20 and another number. What is the number?

Pre-Cal Worksheet 12-3 A Name Key Pd

Find each limit, or state that the limit does not exist and explain your reasoning.

5. $\lim_{n\to\infty} \frac{1}{5^n}$

6. $\lim_{n\to\infty} \frac{5-n^2}{2n}$

7. $\lim_{n\to\infty} \frac{3n-6}{7n}$

Find each limit, or state that the limit does not exist and explain your reasoning.

14. $\lim_{n\to\infty} \frac{7-2n}{5n}$ $-\frac{2}{5}$

15. $\lim_{n\to\infty} \frac{n^3-2}{n^2}$ ∞

16. $\lim_{n\to\infty} \frac{6n^2+5}{3n^2}$ 2

17. $\lim_{n\to\infty} \frac{9n^3+5n-2}{2n^3}$ $\frac{9}{2}$

18. $\lim_{n\to\infty} \frac{(3n+4)(1-n)}{n^2}$

$\frac{3n-3n^2+4-4n}{n^2}$

$\frac{-3n^2-n+4}{n^2}$

(-3)

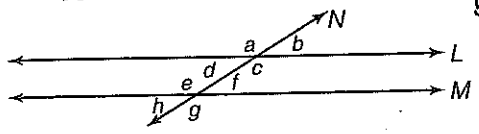
19. $\lim_{n\to\infty} \frac{8n^2+5n+2}{3+2n}$ ∞

After you work each problem, record your answer on the answer sheet provided or on a piece of paper.

Multiple Choice

1. In the figure below, line L is parallel to line M. Line N intersects both L and M, with angles a, b, c, d, e, f, g, and h as shown. Which of the following lists includes all of the angles that are supplementary to $\angle a$?

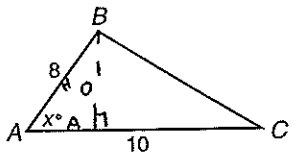
b d f h



(A) b, d, f, h B c, e, g C b, d, c

D e, f, g, h E d, c, h, g

2. In the figure below, what is the area of $\triangle ABC$ in terms of x?



$\sin x = \frac{opp.}{8}$

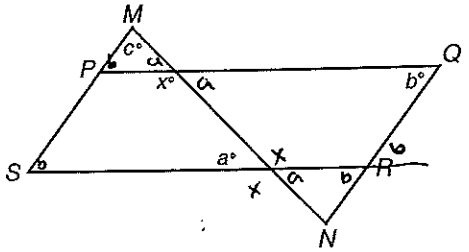
$8\sin x = opp.$

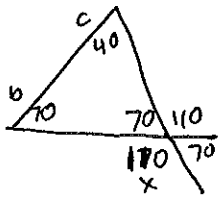
$Area = \frac{1}{2}(10)(8\sin x) = 40\sin x$

A $10 \sin x$ (B) $40 \sin x$ C $80 \sin x$

D $40 \cos x$ E $80 \cos x$

3. If $PQRS$ is a parallelogram and $\overline{MN}$ is a line segment, then x must equal



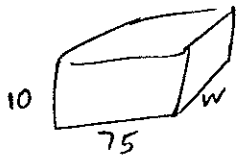


ext. angle thm.

A $180 - b$ B $180 - c$ C $a + b$

D $a + c$ (E) $b + c$

4. If a rectangular swimming pool has a volume of 16,500 cubic feet, a depth of 10 feet, and a length of 75 feet, what is the width of the pool, in feet?



(A) 22 B 26 C 32 D 110 E 1650

$\frac{750w}{750} = \frac{16500}{750}$

$w = 22$

5. $\frac{1}{10^{100}} - \frac{1}{10^{99}} =$

(A) $\frac{-9}{10^{100}}$ B $\frac{-1}{10^{100}}$ C $\frac{1}{10^{100}}$ D $\frac{1}{10}$ E $\frac{9}{10}$

$10^{-100} - 10^{-99}$

$10^{-100}(1 - 10^{1}) = 10^{-100}(-9) = \frac{1}{10^{100}} \cdot -9 = \frac{-9}{10^{100}}$

Pre-Cal Worksheet 12-3 B Name Key Pd

Find the sum of each infinite series, or state that the sum does not exist and explain your reasoning.

10. $-6 + 3 - \frac{3}{2} + \cdots$

11. $\frac{3}{4} + \frac{1}{4} + \frac{1}{12} + \cdots$

12. $\sqrt{3} + 3 + \sqrt{27} + \cdots$

Find the sum of each series, or state that the sum does not exist and explain your reasoning.

31. $16 + 12 + 9 + \cdots$ $R = .75$ $S = \frac{16}{1-.75} = \frac{16}{.25} = 64$

34. $6 + 5 + 4 + \cdots$ NOT GEOM

37. $\frac{6}{5} + \frac{4}{5} + \frac{8}{15} + \cdots$ $R = \frac{2}{3}$ $S = \frac{6/5}{1-2/3} = \frac{6}{5} \cdot \frac{3}{1} = \frac{18}{5} = 3.6$

32. $5 + 7.5 + 11.25 + \cdots$

35. $\frac{1}{8} + \frac{1}{4} + \frac{1}{2} + \cdots$

38. $\sqrt{5} + 1 + \frac{\sqrt{5}}{5} + \cdots$

33. $10 + 5 + 2.5 + \cdots$

36. $-\frac{2}{3} + \frac{1}{9} - \frac{1}{54} + \cdots$

39. $8 - 4\sqrt{3} + 6 - \cdots$

20. $\lim_{n\to\infty} \frac{4 - 3n + n^2}{2n^3 - 3n^2 + 5}$

21. $\lim_{n\to\infty} \frac{n}{3^n}$

22. $\lim_{n\to\infty} \frac{(-2)^n n}{4 + n}$

23. Find the limit of the sequence described by the general expression $\frac{5n + (-1)^n}{n^2}$, or state that the limit does not exist. Explain your reasoning.

13. **Entertainment** Pete's Pirate Ride operates like the bob of a pendulum. On its longest swing, the ship travels through an arc 75 meters long. Each successive swing is two-fifths the length of the preceding swing. If the ride is allowed to continue without intervention, what is the total distance the ship will travel before coming to rest?

40. **Physics** A basketball is dropped from a height of 35 meters and bounces $\frac{2}{5}$ of the distance after each fall.

a. Find the first five terms of the infinite series representing the vertical distance traveled by the ball.

b. What is the total vertical distance the ball travels before coming to rest? (*Hint:* Rewrite the series found in part a as the sum of two infinite geometric series.)

41. **Critical Thinking** Consider the sequence whose nth term is described by $\frac{n^2}{2n+1} - \frac{n^2}{2n-1}$.

a. Explain why $\lim_{n\to\infty}\left(\frac{n^2}{2n+1} - \frac{n^2}{2n-1}\right) \neq \lim_{n\to\infty}\frac{n^2}{2n+1} - \lim_{n\to\infty}\frac{n^2}{2n-1}$.

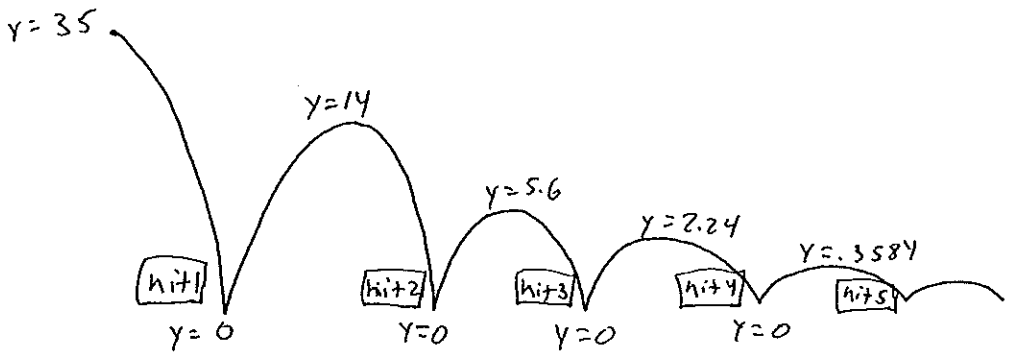
b. Find $\lim_{n\to\infty}\left(\frac{n^2}{2n+1} - \frac{n^2}{2n-1}\right)$.

42. **Engineering** Francisco designs a toy with a rotary flywheel that rotates at a maximum speed of 170 revolutions per minute. Suppose the flywheel is operating at its maximum speed for one minute and then the power supply to the toy is turned off. Each subsequent minute thereafter, the flywheel rotates two-fifths as many times as in the preceding minute. How many complete revolutions will the flywheel make before coming to a stop?

43. **Critical Thinking** Does $\lim_{n\to\infty}\cos\frac{n\pi}{2}$ exist? Explain.

Pre-Cal

12-3 #40



hit	1	2	3	4	5
new dist.	35	28	11.2	4.48	1.792

sum = 80.472

$\frac{28}{35} = .8$ $\frac{11.2}{28} = .4$ $\frac{4.48}{11.2} = .4$ $r = .4$ $(\frac{2}{5})$

or

$35 + \frac{28 - 28(.4)^4}{1 - .4}$

$35 + 45.472$

80.472

total distance

$\left[35 + \frac{28}{1-.4}\right] = 81.\overline{6}$

Pre-Cal Work Sheet 12-5A Name Key Pd_

Write each expression in expanded form and then find the sum.

4. $\sum_{n=1}^{6} (n-3)$

5. $\sum_{k=2}^{5} 4k$

6. $\sum_{a=0}^{4} \frac{1}{2^a}$

7. $\sum_{p=0}^{\infty} 5\left(\frac{3}{4}\right)^p$

Write each expression in expanded form and then find the sum.

14. $\sum_{n=1}^{4} (2n-7)$

17. $\sum_{k=2}^{6} (k+k^2)$

20. $\sum_{m=0}^{3} 3^{m-1}$

23. $\sum_{k=3}^{7} k!$

15. $\sum_{a=2}^{5} 5a$

18. $\sum_{n=5}^{8} \frac{n}{n-4}$

21. $\sum_{r=1}^{3} \left(\frac{1}{2}+4^r\right)$

24. $\sum_{p=0}^{\infty} 4(0.75)^p$

16. $\sum_{b=3}^{8} (6-4b)$

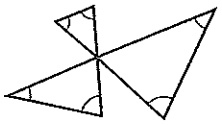
19. $\sum_{j=4}^{8} 2^j$

22. $\sum_{i=3}^{5} (0.5)^{-i}$

25. $\sum_{n=1}^{\infty} 4\left(\frac{2}{5}\right)^n$

26. Write $\sum_{n=2}^{5} n + i^n$ in expanded form. Then find the sum.

6. In the figure, what is the sum of the degree measures of the marked angles?

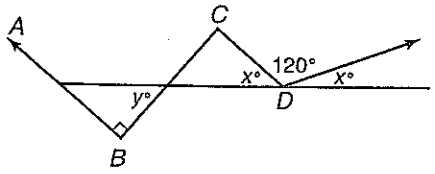


A 180 B 270 C 360 D 540
E It cannot be determined from the information given.

7. If $5x^2 + 6x = 70$ and $5x^2 - 6y = 10$, then what is the value of $10x + 10y$?

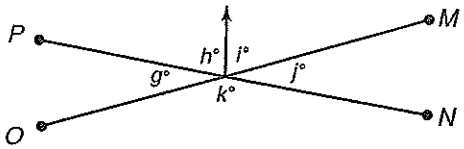
A 10 B 20 C 60 D 80 E 100

8. In the figure below, if $\overline{AB} \parallel \overline{CD}$, then what is the value of y? *Figure not drawn to scale.*



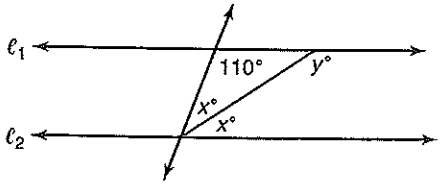
A 30 B 60 C 90 D 120 E 150

9. Which pair must be equal?



A h and i
B $(g + h)$ and $(i + j)$
C $(g + i)$ and $(h + j)$
D g and j
E $(g + j)$ and $(h + i)$

10. **Grid-In** If ℓ_1 is parallel to ℓ_2 in the figure below, what is the value of y?



Pre-Cal Worksheet 12-5B Name Key Pd

Express each series using sigma notation.

8. $5 + 10 + 15 + 20 + 25$

9. $2 + 4 + 10 + 28$

10. $2 - 4 - 10 - 16$

11. $\frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{32} + \cdots$

12. $-3 + 9 - 27 + \cdots$

Express each series using sigma notation.

27. $6 + 9 + 12 + 15$

29. $8 + 10 + 12 + \cdots + 24$

31. $10 + 50 + 250 + 1250$

33. $\frac{1}{9} + \frac{1}{14} + \frac{1}{19} + \cdots + \frac{1}{49}$

35. $4 - 9 + 16 - 25 + \cdots$

37. $-32 + 16 - 8 + 4 - \cdots$

28. $1 + 4 + 16 + \cdots + 256$

30. $-8 + 4 - 2 + 1$

32. $13 + 9 + 5 + 1$

34. $\frac{2}{3} + \frac{4}{5} + \frac{8}{7} + \frac{16}{9} + \cdots$

36. $5 + 5 + \frac{5}{2} + \frac{5}{6} + \frac{5}{24} + \cdots$

38. $2 + \frac{6}{2} + \frac{24}{3} + \frac{120}{4} + \cdots$

Simplify. Assume that n and m are positive integers, $a > b$, and $a > 2$.

42. $\frac{(a-2)!}{a!}$

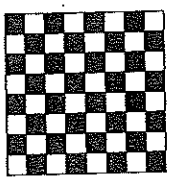
43. $\frac{(a+1)!}{(a-2)!}$

44. $\frac{(a+b)!}{(a+b-1)!}$

13. **Aviation** Each October Albuquerque, New Mexico, hosts the Balloon Fiesta. In 1998, 873 hot air balloons participated in the opening day festivities. One of these balloons rose 389 feet after 1 minute. Because the air in the balloon was not reheated, each succeeding minute the balloon rose 63% as far as it did the previous minute.
   a. Use sigma notation to represent the height of the balloon above the ground after one hour. Then calculate the total height of the balloon after one hour to the nearest foot.
   b. What was the maximum height achieved by this balloon?

49. **Word Play** An *anagram* is a word or phrase that is made by rearranging the letters of another word or phrase. Consider the word "SILENT."
   a. How many different arrangements of the letters in this word are possible? Write this number as a factorial. (*Hint:* First solve a simpler problem to see a pattern, such as how many different arrangements are there of just 2 letters? 3 letters?)
   b. If a friend gives you a hint and tells you that an anagram of this word starts with "L," how many different arrangements still remain?
   c. Your friend gives you one more hint. The last letter in the anagram is "N." Determine how many possible arrangements remain and then determine the anagram your friend is suggesting.

50. **Chess** A standard chess board contains 64 small black or white squares. These squares make up many other larger squares of various sizes.



   a. How many 8×8 squares are there on a standard 8×8 chessboard? How many 7×7 squares?
   b. Continue this list until you have accounted for all 8 sizes of squares.
   c. Use sigma notation to represent the total number of squares found on an 8×8 chessboard. Then calculate this sum.