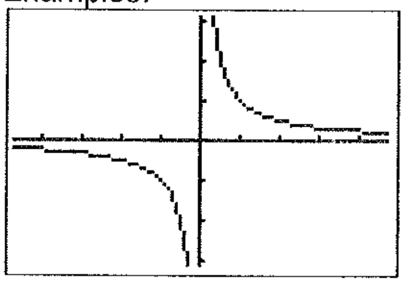
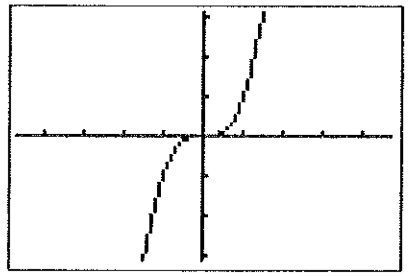
Pre-Calculus Chapter 3 Properties of Functions

Section 3.1 Symmetry Day 1

Odd Functions – Symmetry with respect to the Origin.

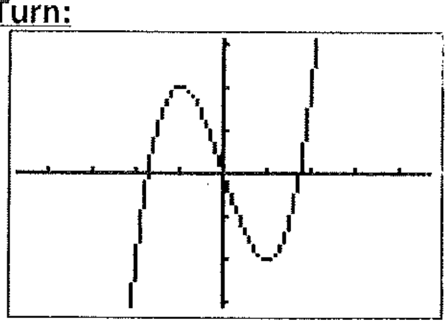
- Think of rotating 180 degrees about the origin.
- Think of flipping across the x-axis and the y-axis.
- Graphs that are symmetric about the origin are also called ODD Functions.
- To visualize an odd function, use $y = x^3$
- Examples:





Your Turn:

$f(x) = 3 / x^2$



Testing for symmetry about the origin. Complete the t-table for values on $y = x^3+x$

x	y
-3	
-2	
-1	
0	
1	
2	
3	

Patterns:

Summary:

Generalize

$f(-x) =$

Testing for symmetry about the origin. Complete the t-table for values on $y = x^5 - 2x^3 + x$

x	y
-3	
-2	
-1	
0	
1	
2	
3	

Patterns:

Summary:

Generalize

f(-x) =

To test for odd functions:

Concrete	Abstract
Find f(3) Find f(-3) Are these the same number with one positive and the other negative? If so, then it is odd. **This means that f(-3) = - f(3)**	Find f(x) Find f(-x) Are these the same equation with f(x) the opposite sign of f(-x)? If so, then it is odd. **This means that f(-x) = - f(x)**

Concrete Examples on Calculator
Number 7 on 3-1A

Your Turn
Number 17 on 3-1A

Abstract Examples by hand
Number 6 on 3-1A

Your Turn
Number 14 on 3-1A

Assignment:
Finish 3-1A

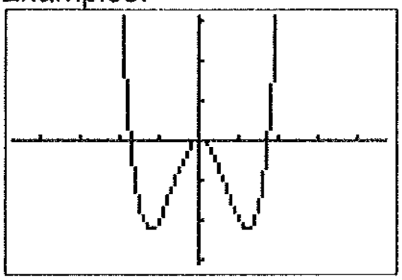
Concrete: 7, 17, 18, 19, 20

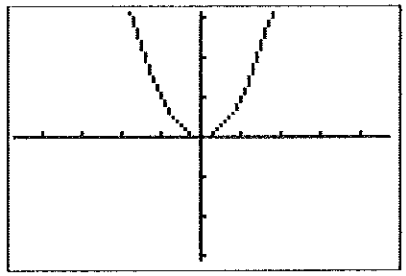
Abstract: 6, 14, 15, 16

Section 3.1 Symmetry   Day 2

Even Functions – Symmetry with respect to the Y-Axis.

- Think of flipping across the y-axis.
- These are also called EVEN functions.
- To visualize, think of $y = x^2$
- Examples:





Testing for symmetry about the origin. Complete the t-table for values on $y = x^2 - 2$

x	y
-3	
-2	
-1	
0	
1	
2	
3	

Patterns:

Summary:

Generalize

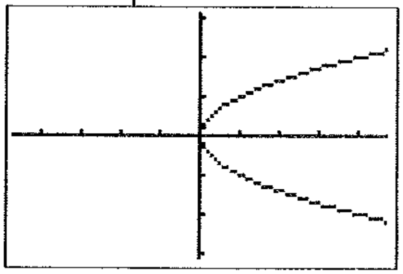
$f(-x) =$

To test for even functions:

Concrete	Abstract
Find f(3) Find f(-3) Are these the same number? If so, then it is even. **This means that f(-3) = f(3)**	Find f(x) Find f(-x) Are these the same equation? If so, then it is even. **This means that f(-x) = f(x)**

Symmetry with respect to the X-Axis.

- Think of flipping across the x-axis.
- To visualize, think of the bullet graph: $y = \pm\sqrt{x}$
- These do not pass the vertical line test.
- Examples:



Testing for symmetry about the x-axis. Complete the t-table for values on $y = \pm\sqrt{9 - x^2}$

x	y
-1	
0	
1	
2	
3	

Patterns:

Summary:

Generalize

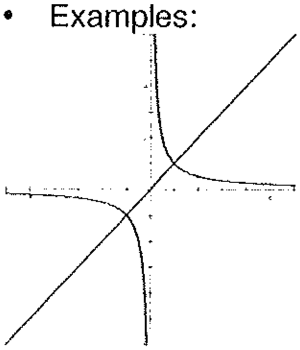
f(-x) =

To test for x-axis symmetry:

Concrete	Abstract
Find f(3) Does f(3) have 2 answers that are the same number with one positive and one negative? Then it is symmetric with respect to the x-axis. **This means that f(3) =** ± 1 **or another number.**	Find f(x) Does f(x) = c and f(x) = -c where c is a constant? Then it is symmetric with respect to the x-axis. **This means that f(x) =** $\pm c$

Symmetry with respect to the line y = x.

- Think of flipping across a 45° angle line (y = x).
- To visualize, think of the bullet graph: $y = \pm\sqrt{x}$
- These do not pass the vertical line test.
- Examples:



Testing for symmetry with respect to y=x. Complete the t-table for values on $y = \frac{2}{x}$

x	y
4	
3	
2	
1	
½	
2/3	

Patterns:

Summary:

Generalize

f(-x) =

To test for y=x symmetry:

Concrete	Abstract
Find f(3). Let's say that f(3) = 2 Now try the y-value as the x-value. In our example, try f(2). Does it equal the starting x-value (3 in our example)? Then it is symmetric about y=x. **This means that f(5) = -2 and f(-2) = 5 or To say it another way: (5, -2) and (-2, 5) are on the graph for all ordered pairs.**	Plug (a, b) into the equation. Plug (b, a) into the equation. Are these the same equation? If so, then it symmetric about the line y=x.

Concrete Examples on Calculator
Number 21 on 3-1B

Your Turn
Number 22 on 3-1B

Abstract Examples by hand
Number 23 on 3-1B

Your Turn
Number 24 on 3-1B

Assignment:
Finish 3-1B

Day 3
Example from 3-1C
Number 25

Assignment:
Finish 3-1C

Write the tests from memory

	Origin (Odd)	y-axis (even)	x-axis	y = x	y = -x
Test					
picture					

Write the tests from memory

	Origin (Odd)	y-axis (even)	x-axis	y = x	y = -x
Test					
picture					

Write the tests from memory

	Origin (Odd)	y-axis (even)	x-axis	y = x	y = -x
Test					
picture					

Write the tests from memory

	Origin (Odd)	y-axis (even)	x-axis	y = x	y = -x
Test					
picture					

Write the tests from memory

	Origin (Odd)	y-axis (even)	x-axis	y = x	y = -x
Test					
picture					

Write the tests from memory

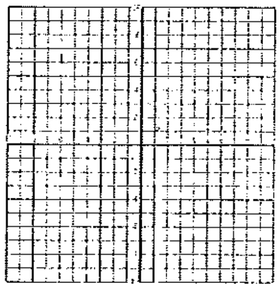
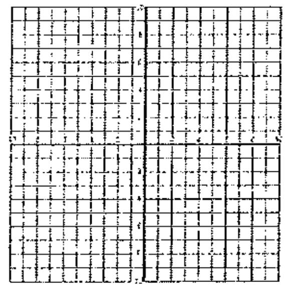
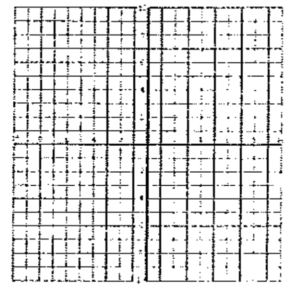
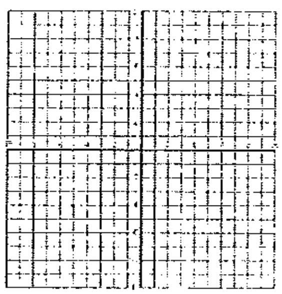
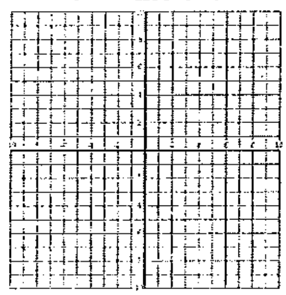
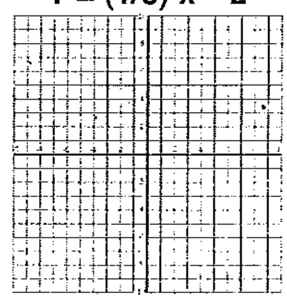
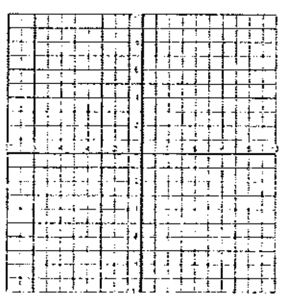
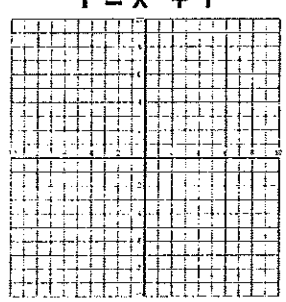
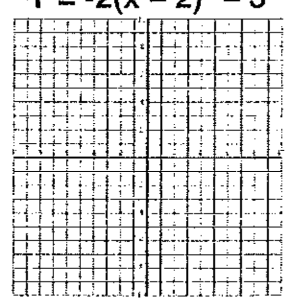
	Origin (Odd)	y-axis (even)	x-axis	y = x	y = -x
Test					
picture					

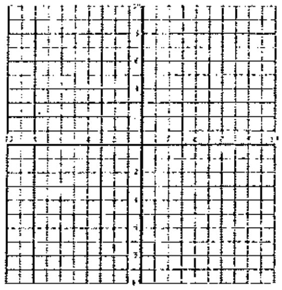
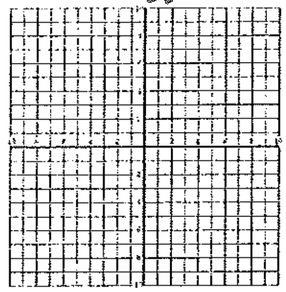
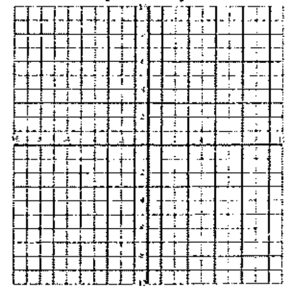
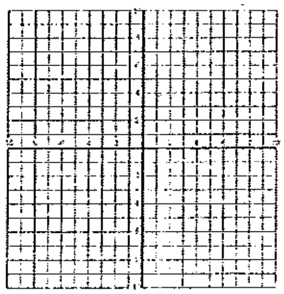
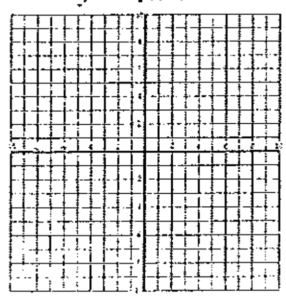
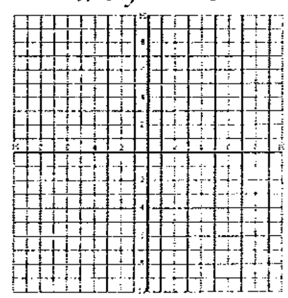
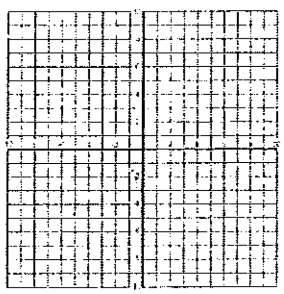
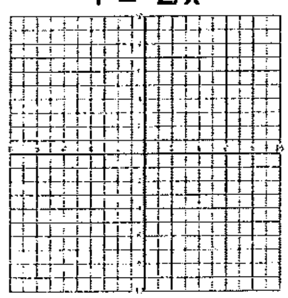
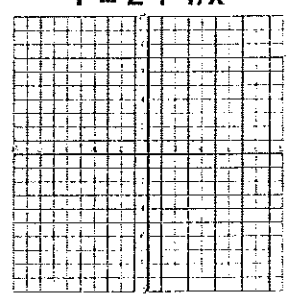
Section 3.2 Family of Functions

Day 1

Given $y = a * f(x + b) + c$

- a can flip and squeeze the graph f(x).
- b moves the graph left(+) and right(-).
- c moves the graph up(+) and down(-).

Name & Equation	Graph	Example 1	Example 2
Constant Function $Y = c$		$Y = 2$ 	$Y = -4$ 
Linear Equation $Y = mx + b$		$Y = -2x + 1$ 	$Y = (1/3)\ x - 2$ 
Quadratic (parabola) $Y = ax^2 + bx + c$ Or $Y = a(x + b)^2 + c$		$Y = x^2 + 1$ 	$Y = -2(x - 2)^2 - 3$ 

Name & Equation	Graph	Example 1	Example 2
Cubic $Y = ax^3 + bx^2 + cx + d$ **Or** $Y = a(x + b)^3 + c$		$Y = x^3 - 2$ 	$Y = (x - 1)^3 - 3$ 
Radical Function (Bullet) $y = \pm\sqrt{x}$		$y = \sqrt{x + 2}$ 	$x + y^2 = -1$ 
Rational Function $Y = 1 / x$		$Y = -2/x$ 	$Y = 2 + 1/x$ 

Discuss: Circle

Examples on 3-2A
Number 6

Your Turn on 3-2A
Number 7

Examples on 3-2A
Number 8

Your Turn on 3-2A
Number 9

Assignment:
Finish 3-2A

Section 3.3 Graphing Inequalities — Day 1

Inequalities	Line	Shade	Picture Example
y <	dashed	below	
y >	dashed	above	
y ≤	solid	below	
y ≥	solid	above	

Also consider testing a point (like the origin) by plugging it into the inequality.

1. First, graph the equation and find a point that is not on the graph.
2. Plug the point into the inequality.
3. If the point creates a true statement, like $4 < 8$, then that is the side to shade.
4. If the point creates a false statement, like $-2 \leq -7$, then that is NOT the side to shade.

Example from 3-3A
Number 13

Your Turn from 3-3A
Number 14

Example from 3-3A
Number 21

Your Turn from 3-3A
Number 23

Assignment
Finish 3-3A

Pre-Calculus Chapter 5 Basic Trigonometry

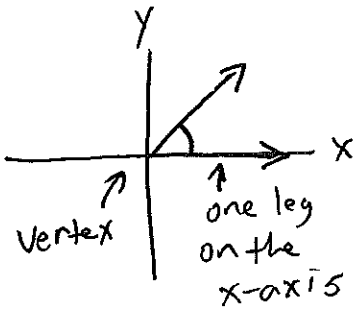
Section 5.1 Reference Angles Day 1

Vertex: corner of an angle

Note: The 2 sides of the angle are the initial side and the terminal side

STANDARD POSITION: An angle in which the vertex is at the origin and its initial side along the positive x-axis.

Picture:



Degrees, Minutes, and Seconds

1 full rotation = 360 degrees

Note:–Positive degrees are counterclockwise.

1 degree = 1 / 360th of a full rotation

1 minute = 1 / 60th of a degree (notation: 30')

1 second = 1 / 60th of a minute (notation: 10")

40° 20' 40''
deg min sec

Examples from 5-1A: 28, 30

Your Turn from 5-1A: 27, 32

Coterminal Angles

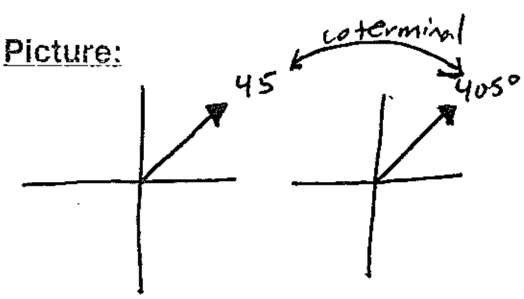
Since angles repeat every 360°, every angle has infinite angles that look identical. These are called Coterminal Angles.

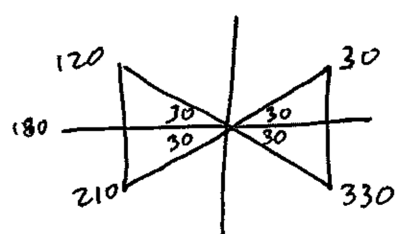
The angles coterminal to angle k are equal to k + 360n where n is an integer.

Reference Angles

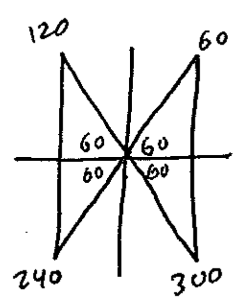
If an angle is in the standard position, the 3 angles formed by reflecting across the x-axis and y-axis. *Make the bow tie to illustrate.*

Picture:





30 is the ref. angle for 120, 180, 330



60 is the ref. angle for 120, 240, 300

Examples from 5-1A: 37, 45, 52

Your Turn from 5-1A: 38, 46, 53

Assignment: Finish 5-1A

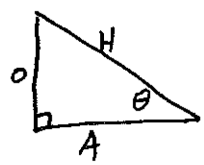
Section 5.2 Sine, Cosine, Tangent Day 1

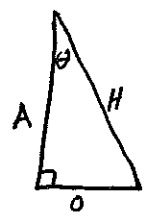
Note: Put your calculator in Degree Mode.

Right Triangle Trigonometry

H In a right triangle, the Hypotenuse H is the longest side.
O Given that one of the acute angles is theta, the side across from it is the Opposite, O
A The third side is the Adjacent side, A

Examples: label O, H, and A

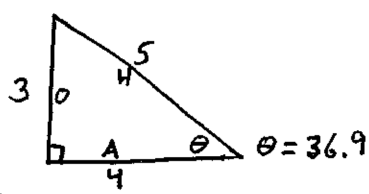




Sine, Cosine, and Tangent

Sin Θ = O/H Cos Θ = A/H Tan Θ = O/A Memory Trick: SohCahToa

Picture:



$\sin\theta = \frac{O}{H}$ $\cos\theta = \frac{A}{H}$ $\tan\theta = \frac{O}{A}$

$\sin 36.9 = \frac{3}{5}$ $\cos 36.9 = \frac{4}{5}$ $\tan 36.9 = \frac{3}{4}$

Examples from 5-2A: 5

Your Turn from 5-2A: 10

Solving Strategy: Make a proportion and always cross multiply & divide

Reciprocal Trigonometry

Cosecant	Reciprocal of Sine	sin Θ = 1 / csc Θ csc Θ = 1 / sin Θ	sin Θ = O/H csc Θ = H/O
Secant	Reciprocal of Cosine	cos Θ = 1 / sec Θ sec Θ = 1 / cos Θ	cos Θ = A/H sec Θ = H/A
Cotangent	Reciprocal of Tangent	tan Θ = 1 / cot Θ cot Θ = 1 / tan Θ	tan Θ = O/A cot Θ = A/O

Examples from 5-2A: 6, 7, 20

Your Turn from 5-2A: 14, 15, 21

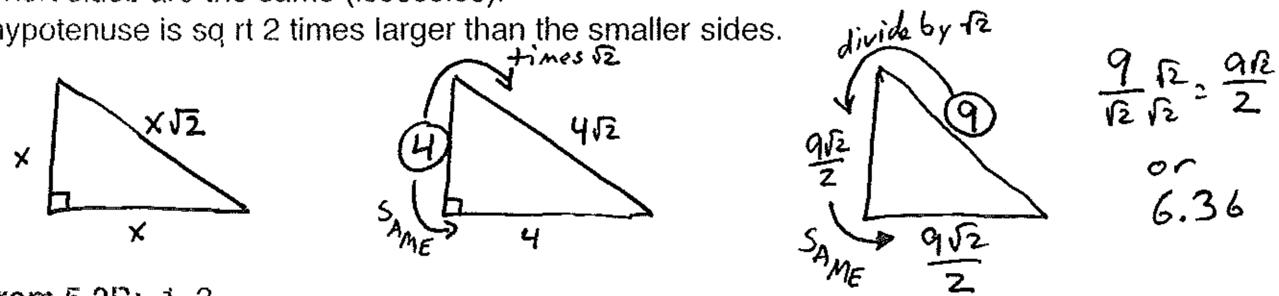
Assignment: Finish 5-2A

Section 5.2 Special Right Triangles Day 2

Special Right Triangle 1 = The 45-45-90 Triangle

- The short sides are the same (isosceles).
- The hypotenuse is sq rt 2 times larger than the smaller sides.

Picture:



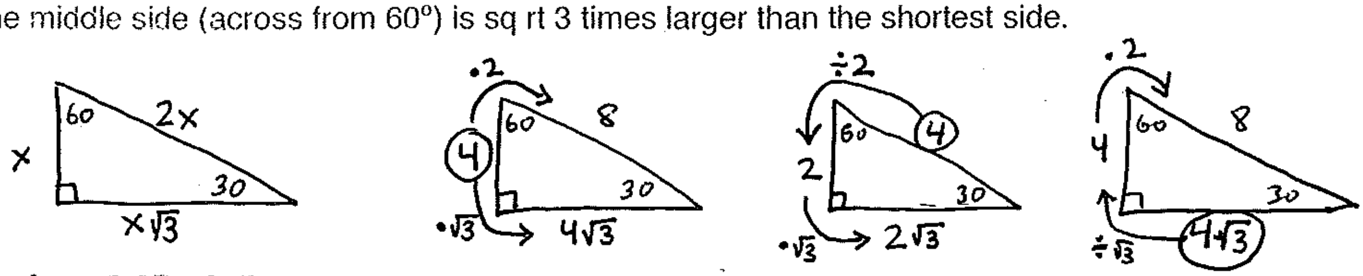
Examples from 5-2B: 1, 3

Your Turn from 5-2B: 4

Special Right Triangle 2 = The 30-60-90 Triangle

- The short side is across from the 30° angle.
- The hypotenuse is 2 times larger than the shortest side.
- The middle side (across from 60°) is sq rt 3 times larger than the shortest side.

Picture:



Examples from 5-2B: 2, 5

Your Turn from 5-2B: 6

Extra Practice:

45-45-90, shortest side is 3 30-60-90, hypotenuse is 8 45-45-90, hypotenuse is 4

Assignment: Finish 5-2B (Special Right Triangles)

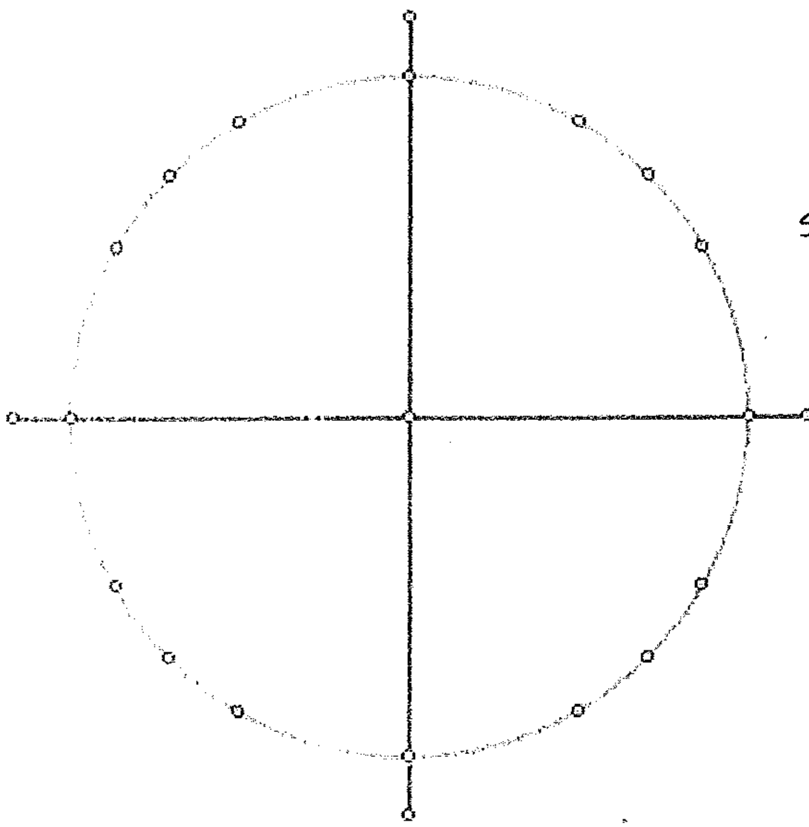
Section 5.3 The Unit Circle Day 1

Basics of the Unit Circle

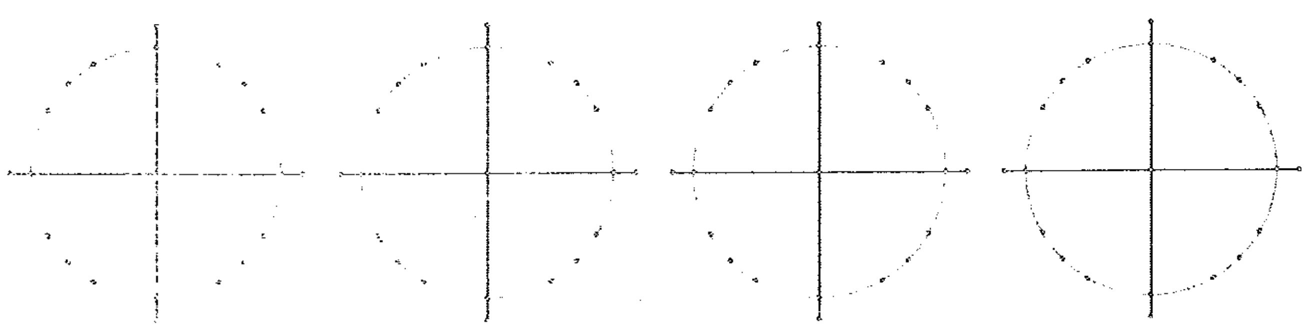
- Draw a circle centered at the origin with radius 1.
- State the ordered pairs where the x and y axes meet the circle
- State the ordered pairs every 30 degrees (this uses the 30-60-90 triangle)
- State the ordered pairs every 45 degrees (this uses the 45-45-90 triangle)

Note: There are only 3 fractions used in the unit circle. Memorize these: $\frac{1}{2}$ $\frac{\sqrt{3}}{2}$ $\frac{\sqrt{2}}{2}$

Picture:



See your unit circle Handout



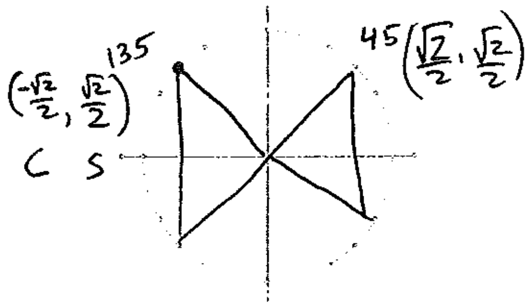
Section 5.3 The Unit Circle Day 2

Using the Unit Circle

Every point along the unit circle has an angle Θ and coordinates (x, y)

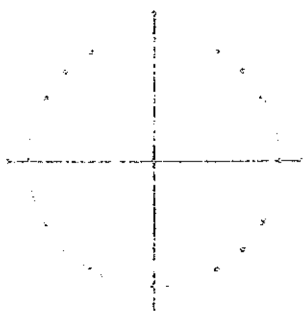
Sin Θ = the y value Cos Θ = the x value Tan Θ = y / x

Example: find the reference angle for 135 degrees and state the sin 135 and cos 135



sin 135 → $\frac{\sqrt{2}}{2}$ cos 135 → $-\frac{\sqrt{2}}{2}$

Examples from 5-3A: 5, 6, 7, 8



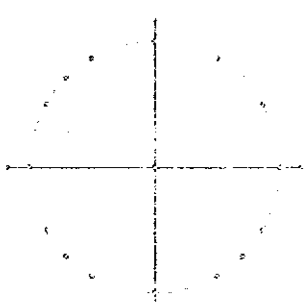
Note: one way to do this is to draw a triangle, identify O, H, & A and simply complete the 6 ratios:

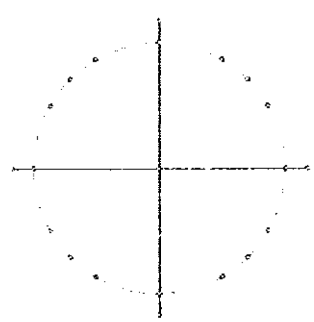
Sin Θ =	O/H	Csc Θ =	H/O
Cos Θ =	A/H	Sec Θ =	H/A
Tan Θ =	O/A	Cot Θ =	A/O

Your Turn from 5-3A: 14, 15, 17, 22, 23

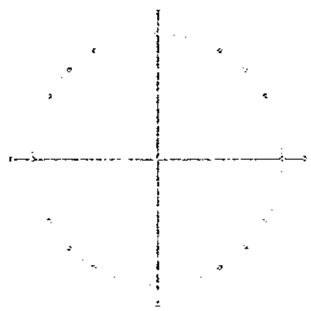
Extra Practice from 5-3A if necessary:

Assignment: Finish 5-3A (Special Right Triangles)





Section 5.3 The Unit Circle Day 3



Examples from 5-3B: 38, 44

Your Turn from 5-3B: 39

Examples from 5-3B: 46a

Your Turn from 5-3B: 46b

Assignment: finish worksheet 5-3B

Study for the Quiz!