Pre-Calculus Chapter 12 Sequences, Series, Sigma Notation, and Limits

Section 12.1 Arithmetic Sequences and Series Day 1

Opener: Practice quiz – family of functions

Notes: Arithmetic Sequence

- A sequence is a list of numbers.
- Arithmetic sequence means you add to the previous number to create new numbers.
- A is the starting value. New terms add D, the common difference.
- Terms: A, A + D, A + 2D, A + 3D, ...

Example: find the next 3 terms for

-5, -2, 1, ____, ____, ____

What is the value of D, the common difference?

Formula

The nth term of an arithmetic sequence (a_n) given the first term a_1 and the common difference D:

$$a_n = a_1 + (n - 1)D$$

Examples from 12-1A: 9-10

Your Turn from 12-1A: 11-12

Assignment: finish 8-1A

Section 12.1 Arithmetic Sequences and Series Day 2

Opener: Practice quiz – #1 #3 #5

Notes: Arithmetic Means

- Arithmetic means are numbers between 2 other numbers that we know.
- Put blanks where they go and use guess and check.

Example: find the 2 arithmetic means between 9 and 24.

Your Turn from 12-1B: 39, 40

Notes: Arithmetic Series

- A series is the sum of numbers from a sequence (list of numbers).

$$S_n = \frac{n}{2}\left(a_1 + a_n\right)$$

Examples from 12-1B: 14-15

Your Turn from 12-1B: 43-44

Assignment: finish 8-1B

Section 12.2 Geometric Sequences and Series   Day 1

Opener: Practice quiz – #2 #4 #6

Notes: Geometric Sequence

- A sequence is a list of numbers.
- Geometric sequence means you multiply to the previous number to create new numbers.
- A is the starting value. Each new term multiplies by r, the common ratio.
- Terms: $A, Ar, Ar^2, Ar^3, \ldots$

Examples from 12-1A: 7-9

Your Turn from 12-1A: 16, 19, 22

Formula

The nth term of an geometric sequence (a_n) given the first term a_1 and the common ratio, r:

$$a_n = a_1 r^{n-1}$$

Examples from 12-1A: 10-12

Your Turn from 12-1A: 26, 29

Assignment: finish 8-2A

Section 12.2 Geometric Sequences and Series Day 2

Opener: Practice quiz – #7 #9

Notes: Geometric Means

- Geometric means are numbers between 2 other numbers that we know.
- Put blanks where they go and use guess and check.

Example from 12-1B: 13, 34

Your Turn from 12-1B: 35, 36

Notes: Arithmetic Series

- A series is the sum of numbers from a sequence (list of numbers).

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$

Examples from 12-2B: 14, 37, 15

Your Turn from 12-2B: 38-39

Assignment: finish 8-2B

Section 12.3 Limits Day 1

Opener: Practice quiz – #8 #10 #11

Notes: Limits

- The answer is a y-value as x nears some value
- Limits are used when an x value is undefined (asymptote, hole) or unreachable (infinity)
- If the limit is undefined, look to the left and the right. If both sides approaches the same y-value, then the limit is that value.
- If the left and right do NOT approach the same value, then the limit is undefined.
- Notation:

$$\lim_{x \to c} f(x) = L$$

This says: what is the y value on the function at x value c.

Limits at Infinity On A Calculator

1. Put the function into y_1
2. Set the window from -1000 < x < 1000
3. Run ZoomFit
4. Trace to the right and watch the y values

Examples from 12-3A: 5 – 7

Your Turn from 12-3A: 14-16

Limits at Infinity By Hand

Method 1: For limits at infinity, plug in 10 to see what happens. For limits at negative infinity, plug in -10 to see what happens.

Method 2:

1. In a rational function, Find the degree of the numerator and the degree of the denominator:
2. If the higher degree is on top, the limit is infinity or negative infinity [top gets bigger faster]
3. If the high degree is on bottom, the limit is 0 [bottom gets bigger faster]
4. If the degree is equal, then the limit is the fraction made by the coefficients in front of the highest degree terms. [both get bigger together]

$$\lim_{x \to \infty} \frac{1}{2x} = \qquad \lim_{x \to \infty} \frac{x^3}{2x} = \qquad \lim_{x \to \infty} \frac{x^2}{2x^5} =$$

$$\lim_{x \to \infty} \frac{7x}{2x} = \qquad \lim_{x \to \infty} \frac{4^x}{2x} = \qquad \lim_{x \to \infty} \frac{9x - x^2}{2x^2 - 1} =$$

Your Turn from 12-3A: 17-19

Assignment: finish 12-3A

Section 12.3 Limits and the Infinite Geometric Series Day 2

Opener: Practice quiz – #12-14

Notes: Infinite Geometric Series

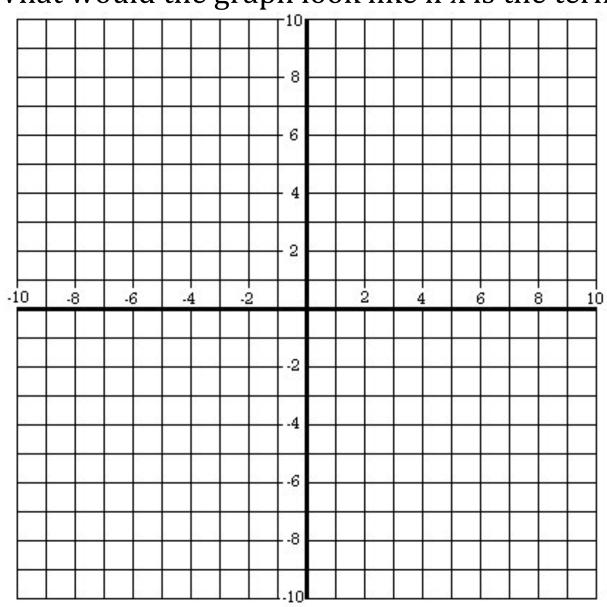
- Definition: the terms approach 0 as the number of terms increases. This means that the ratio must be between 0 and 1.
- Reminders: Sequence means a series of terms separated by commas. Geometric means you multiply from term to term. A series is the sum of all terms up through a certain number of terms.
- An infinite geometric series is the sum of terms from the first term through infinity.
- The value of the infinite sum can be found with the formula:

$$S = \frac{a_1}{1-r}$$

Examples: Are these Inf. Geom Seq?

4, 2, 1, .5, … 2, 6, 18, … 4, 4/5, 4/25, … 6, 4, 2, …

What would the graph look like if x is the term number and y is the value?



Your Turn from 12-3B: 10-12

Assignment: finish 12-3B

Section 12.5 Sigma Notation Day 1

Opener: Practice quiz – #15 – 20

Notes: Sigma Notation

For a sequence a_1, a_2, … the sum of the first k terms may be written where k is an integer value.

$$\sum_{n=1}^{k} a_n$$

This is read the summation from n = 1 to k of a_n

Examples from 12-5A: 4 – 5

Sequences on the Calculator:

- Use the sequence command to show the list of numbers
- The seq(command is found by pressing 2nd LIST and it is the 5th choice in the OPS menu.
- Example: State the first 5 terms for the sequence described by $a_i = i^2 - i$

seq(x^2 – x, x, 1, 5, 1)

Sums on the Calculator:

- Use the sum command to find the answer to the sum.
- The sum(command is found by pressing 2nd LIST and it is the 5th choice in the MATH menu.
- Example: State the sum of the first 6 terms of the sequence described by 6x – 2.

sum(seq(6x – 2, x, 1, 6, 1))

Examples from 12-5A: 14, 24

Your Turn from 12-5A: 17, 25

Assignment: finish 12-5A

Section 12.5 Sigma Notation Day 2

Opener: Practice quiz – #25-27

Examples from 12-5B: 8, 9, 12

Your Turn from 12-5A: 10, 11

Finish the practice Quiz!!

Assignment: finish 12-5B

Section 12.1 Arithmetic Sequences and Series Day 1

Opener: Practice quiz – family of functions

Notes: Arithmetic Sequence

- A sequence is a list of numbers.
- Arithmetic sequence means you add to the previous number to create new numbers.
- A is the starting value. New terms add D, the common difference.
- Terms: A, A + D, A + 2D, A + 3D, ...

Example: find the next 3 terms for

-5, -2, 1, ___, ___, ___

What is the value of D, the common difference?

Formula

The nth term of an arithmetic sequence (a_n) given the first term a_1 and the common difference D:

$$a_n = a_1 + (n - 1)D$$

Examples from 12-1A: 9-10

Your Turn from 12-1A: 11-12

Assignment: finish 8-1A

Section 12.1 Arithmetic Sequences and Series Day 2

Opener: Practice quiz – #1 #3 #5

Notes: Arithmetic Means

- Arithmetic means are numbers between 2 other numbers that we know.
- Put blanks where they go and use guess and check or use the a_n formula

Example: find the 2 arithmetic means between 9 and 24.

Your Turn from 12-1B: 39, 40

Notes: Arithmetic Series

- A series is the sum of numbers from a sequence (list of numbers).

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Examples from 12-1B: 14-15

Your Turn from 12-1B: 43-44

Assignment: finish 8-1B

Section 12.2 Geometric Sequences and Series   Day 1

Opener: Practice quiz – #2 #4 #6

Notes: Geometric Sequence

- A sequence is a list of numbers.
- Geometric sequence means you multiply to the previous number to create new numbers.
- A is the starting value. Each new term multiplies by r, the common ratio.
- Terms: $A, Ar, Ar^2, Ar^3, \ldots$

Examples from 12-1A: 7-9

Your Turn from 12-1A: 16, 19, 22

Formula

The nth term of an geometric sequence (a_n) given the first term a_1 and the common ratio, r:

$$a_n = a_1 r^{n-1}$$

Examples from 12-1A: 10-12

Your Turn from 12-1A: 26, 29

Assignment: finish 8-2A

Section 12.2 Geometric Sequences and Series Day 2

Opener: Practice quiz – #7 #9

Notes: Geometric Means

- Geometric means are numbers between 2 other numbers that we know.
- Put blanks where they go and use guess and check. or use the a_n formula

Example from 12-1B: **13, 34**

Your Turn from 12-1B: **35, 36**

Notes: Arithmetic Series

- A series is the sum of numbers from a sequence (list of numbers).

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$

Examples from 12-2B: **14, 37, 15**

Your Turn from 12-2B: **38-39**

Assignment: finish 8-2B

Section 12.3 Limits Day 1

Opener: Practice quiz – #8 #10 #11

Notes: Limits

- The answer is a y-value as x nears some value
- Limits are used when an x value is undefined (asymptote, hole) or unreachable (infinity)
- If the limit is undefined, look to the left and the right. If both sides approaches the same y-value, then the limit is that value.
- If the left and right do NOT approach the same value, then the limit is undefined.
- Notation:

$$\lim_{x \to c} f(x) = L$$

This says: what is the y value on the function at x value c.

Limits at Infinity On A Calculator

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1. Put the function into y_1
2. Set the window from -1000 < x < 1000
3. Run ZoomFit
4. Trace to the right and watch the y values

Examples from 12-3A: 5 – 7

Your Turn from 12-3A: 14-16

* Limits at Infinity NOT On A Calculator

plug in 10 → or -10

1. In a rational function, Find the degree of the numerator and the degree of the denominator:
2. If the higher degree is on top, the limit is infinity or negative infinity [top gets bigger faster]
3. If the high degree is on bottom, the limit is 0 [bottom gets bigger faster]
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Your Turn from 12-3A: 17-19

Assignment: finish 12-3A

Section 12.3 Limits and the Infinite Geometric Series — Day 2

Opener: Practice quiz – #12-14

if $0 < r < 1$ the sum is finite

Notes: Infinite Geometric Series

if $r > 0$ the sum is ∞ or $-\infty$

- Definition: the terms approach 0 as the number of terms increases
- Reminders: Sequence means a series of terms separated by commas. Geometric means you multiply from term to term.
- A series is the sum of all terms up through a certain number of terms.
- An infinite geometric series is the sum of terms from the first term through infinity.
- The value of the infinite sum can be found with the formula:

$$S = \frac{a_1}{1-r}$$

Examples: Are these Inf. Geom Seq?

4, 2, 1, .5, ... Yes — $\frac{2}{4} = .5$ $\frac{1}{2} = .5$ $\frac{.5}{1} = .5$

2, 6, 18, ... NO — $\frac{6}{2} = 3$ $\frac{18}{6} = 3$

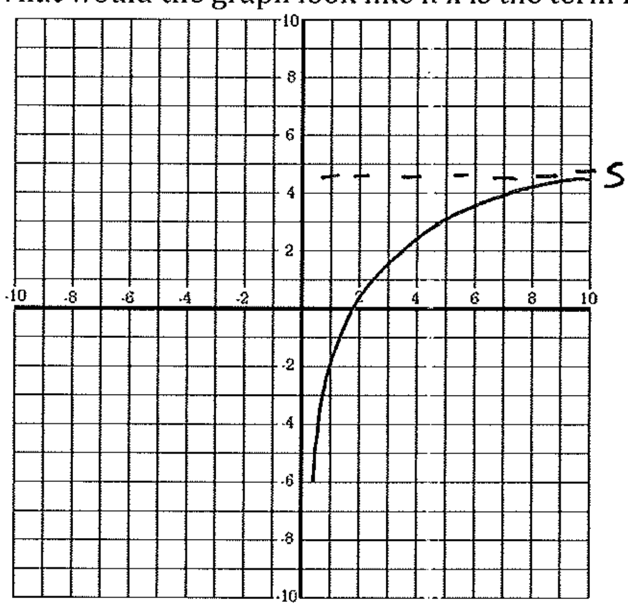
4, 4/5, 4/25, ... Yes — $\frac{\frac{4}{5}}{\frac{4}{1}} = \frac{1}{5}$ $\frac{\frac{4}{25}}{\frac{4}{5}} = \frac{1}{5}$

6, 4, 2, ... NO — $\frac{4}{6} = .\overline{6}$ $\frac{2}{4} = .5$

~~What can you say about the ratio in an infinite geometric Sequence?~~

if [illegible]

What would the graph look like if x is the term number and y is the value?



Your Turn from 12-3B: 10-12

Assignment: finish 12-3B

Section 12.5 Sigma Notation — Day 1

Opener: Practice quiz – #15 – 20

Notes: Sigma Notation

For a sequence $a_1, a_2, \ldots$ the sum of the first k terms may be written where k is an integer value.

$$\sum_{n=1 \leftarrow \text{start}}^{k \leftarrow \text{stop}} a_n \leftarrow \text{equation}$$

This is read the summation from n = 1 to k of a_n

Examples from 12-5A: 4 – 5

Sequences on the Calculator:

- Use the sequence command to show the list of numbers
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Sums on the Calculator:

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- Example: State the sum of the first 6 terms of the sequence described by 6x – 2.

sum(seq(6x – 2, x, 1, 6, 1))

Examples from 12-5A: 14, 24

Your Turn from 12-5A: 17, 25

Assignment: finish 12-5A

Section 12.5 Sigma Notation Day 2

Opener: Practice quiz – #25-27

Examples from 12-5B: 8, 9, 12

Your Turn from 12-5A: 10, 11

Finish the practice Quiz!!

Assignment: finish 12-5B