Calculus Test 4 Prep

Review for Quiz 11 (Sec 5-1 to 5-4)

Topic: Inverses (AP Book section 5-3, Sinclair Book section 6-1)

17. Assume that f is a one-to-one function.
(a) If $f(6) = 17$, what is $f^{-1}(17)$?
(b) If $f^{-1}(3) = 2$, what is $f(2)$?

39–42 Find $(f^{-1})'(a)$.

39. $f(x) = 3x^3 + 4x^2 + 6x + 5, \quad a = 5$

Use the following table to answer questions 1 and 2:

x	1.8	1.9	2	2.1	2.2
f(x)	8.2	7.8	8	8.1	7.8

1. Estimate f'(2)

2. Estimate $(f^{-1})'(8)$

43. Suppose f^{-1} is the inverse function of a differentiable function f and $f(4) = 5$, $f'(4) = \frac{2}{3}$. Find $(f^{-1})'(5)$.

Challenge:

45. If $f(x) = \int_3^x \sqrt{1 + t^3}\, dt$, find $(f^{-1})'(0)$.

Topic: Natural Log (AP Book section 5-1 and 5-2, Sinclair Book section 6-2*)

Differentiate the following:

19. $f(x) = \sin(\ln x)$

22. $y = \dfrac{1}{\ln x}$

Find the equation of the tangent line to the curve at the point:

48. $y = \ln(x^3 - 7),\ (2, 0)$

Integrate the following:

67. $\displaystyle\int_1^2 \frac{dt}{8 - 3t}$

69. $\displaystyle\int_1^e \frac{x^2 + x + 1}{x}\,dx$

72. $\displaystyle\int \frac{\cos x}{2 + \sin x}\,dx$

Topic: e^x (AP Book section 5-4, Sinclair Book section 6-3*)

Find the derivative:

47. $F(t) = e^{t \sin 2t}$

48. $y = e^x \cos(1 - e^{2x})$

56. Find an equation of the tangent line to the curve $xe^y + ye^x = 1$ at the point $(0, 1)$.

87. $\int e^x \sqrt{1 + e^x}\, dx$

91. $\int \frac{e^u}{(1 - e^u)^2}\, du$

Graphing calculator allowed:

1 What are all values of x for which the function f defined by $f(x) = (x^2 - 4)e^x$ is decreasing?

2 The slope of tangent line to the graph $3x - 3lny = -6$ at (-6, 1) is

3 Find the average value of the function $f(x) = 2^x$ on the interval [2, 4] ~~and express in perfect form (no decimal approximations)~~

4 If $y = x^2e^{2x}$ describe the interval on which y is decreasing.

5 A. particle moves along the curve $y = \frac{2}{x+1}$. As the particle passes the point (7, ¼), its x-coordinate is increasing at the rate of 8 units per minute.

a) How fast is the y-coordinate of the particle changing at this instant?

b) Write an accumulation function called A(n) that will find the area under the graph of y from 0 to n.

c) Find A(7)

d) Find the rate of change on A(n) as the particle moves through the point (7, ¼)

Calculus Test 4 Prep

5-1 to 5-4

Review for Quiz 11 (Sec ~~4.4 to 4.5~~)

Key

Topic: Inverses (AP Book section 5-3, Sinclair Book section 6-1)

17. Assume that f is a one-to-one function.
(a) If $f(6) = 17$, what is $f^{-1}(17)$? $= 6$
(b) If $f^{-1}(3) = 2$, what is $f(2)$? $= 3$

f	f^{-1}
$(6, 17)$	$(17, 6)$
$(2, 3)$	$(3, 2)$

39–42 Find $(f^{-1})'(a)$.

39. $f(x) = 3x^3 + 4x^2 + 6x + 5, \quad a = 5$

$f'(x) = 9x^2 + 8x + 6$
$f'(0) = 6$

f	f^{-1}
$(0, 5)$	$(5, 0)$
slope $= 6$	slope $= \frac{1}{6}$

$\frac{6}{1}$

Use the following table to answer questions 1 and 2:

x	1.8	1.9.	2	2.1	2.2
f(x)	8.2	7.8	8	8.1	7.8

1. Estimate f'(2) $\frac{7.8 - 8.1}{1.9 - 2.1} = \frac{-.3}{-.2} = 1.5$

2. Estimate $(f^{-1})'(8)$

f	f^{-1}
$(2, 8)$	$(8, 2)$
Slope $= 1.5$	Slope $= \frac{2}{3}$

$\frac{3}{2}$

43. Suppose f^{-1} is the inverse function of a differentiable function f and $f(4) = 5$, $f'(4) = \frac{2}{3}$. Find $(f^{-1})'(5)$.

$\frac{3}{2}$

Challenge:

45. If $f(x) = \int_3^x \sqrt{1 + t^3}\,dt$, find $(f^{-1})'(0)$.

f	f^{-1}
$(3, 0)$	$(0, 3)$

$f'(x) = \frac{d}{dx} \int_0^x \sqrt{1+t^3}\,dt$
$f'(x) = \sqrt{1+x^3}$
$f'(3) = \sqrt{1+3^3} = \sqrt{28}$

slope $= \frac{1}{\sqrt{28}}$

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Topic: Natural Log (AP Book section 5-1 and 5-2, Sinclair Book section 6-2*)

Differentiate the following:

19. $f(x) = \sin(\ln x)$

$f'(x) = \cos(\ln x)\left(\frac{1}{x}\right)$

22. $y = \frac{1}{\ln x}$

$(\ln x)^{-1}$

$y' = -1(\ln x)^{-2}\left(\frac{1}{x}\right)$

$y' = \frac{-1}{x(\ln x)^2}$

Find the equation of the tangent line to the curve at the point:

48. $y = \ln(x^3 - 7)$, $(2, 0)$

x y

$y - \underline{0} = \underline{12}(x - \underline{2})$

$y' = \frac{3x^2}{x^3 - 7}$

at $x = 2$ $\quad \frac{3(2)^2}{2^3 - 7} = \frac{12}{1} = 12$

Integrate the following:

* **67.** $\int_1^2 \frac{dt}{8 - 3t}$

$u = 8 - 3t$

$du = -3dt$

$-\frac{1}{3}du = dt$

$-\frac{1}{3}\int \frac{1}{u}du$

$-\frac{1}{3}\ln|u| \rightarrow -\frac{1}{3}\ln|8-3t| \Big|_1^2 = -.231 - -.536 = .305$

* **69.** $\int_1^e \frac{x^2 + x + 1}{x}\,dx = \int_1^e \left(x + 1 + \frac{1}{x}\right)dx$

$\frac{1}{2}x^2 + x + \ln|x| \Big|_1^e$

$7.413 - 1.5 = 5.9$

72. $\int \frac{\cos x}{2 + \sin x}\,dx$

$u = 2 + \sin x$

$du = \cos x\,dx$

$\int \frac{1}{u}du = \ln|u| = \ln|2 + \sin x| + C$

Topic: e^x (AP Book section 5-4, Sinclair Book section 6-3*)

Find the derivative:

47. $F(t) = e^{t\sin 2t}$

$f(t) = e^{t\sin 2t}(1\sin 2t + t\cos 2t \cdot 2)$

48. $y = e^x \cos(1 - e^{2x})$

$e^x\cos(1-e^{2x}) + -e^x\sin(1-e^{2x})(-e^{2x}(2))$

$+2e^x e^{2x}\sin(1-e^{2x})$

$e^x\cos(1-e^{2x}) + 2e^{3x}\sin(1-e^{2x})$

$e^x(\cos(1-e^{2x}) + 2e^{2x}(\sin(1-e^{2x})))$

56. Find an equation of the tangent line to the curve $xe^y + ye^x = 1$ at the point $(0, 1)$.

$y - 1 = (-1-e)(x-0)$

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$1e^y + xe^y y' + y'e^x + ye^x = 0$

$e^1 + 0 + y'e^0 + 1e^0 = 0$

$e + y' + 1 = 0$

$y' = -1 - e$

87. $\int e^x\sqrt{1 + e^x}\,dx$

$u = 1 + e^x$

$du = e^x dx$

$\int u^{1/2} du$

$\frac{2}{3}u^{3/2} \longrightarrow \frac{2}{3}(1+e^x)^{3/2} + C$

91. $\int \frac{e^u}{(1 - e^u)^2}\,du$

$v = 1 - e^u$

$dv = -e^u du$

$-dv = e^u du$

$-1\int v^{-2} dv$

$v^{-1} \longrightarrow \frac{1}{1-e^u} + C$

Graphing calculator allowed:

1 What are all values of x for which the function f defined by $f(x) = (x^2 - 4)e^x$ is decreasing?

deriv. is neg on

$-3.236 \le x \le 1.236$

so f is dec. on this interval.

2 The slope of tangent line to the graph $3x - 3\ln y = -6$ at $(-6, 1)$ is

$x \; y$

$3 - 3\left(\frac{y'}{y}\right) = 0$ at $y=1$

$3 - 3y' = 0$

$3 = 3y'$ Slope $= 1$

$1 = y'$

* 3 Find the average value of the function $f(x) = 2^x$ on the interval [2, 4] and express in perfect form (n~~o decimal approximations~~)

$\frac{1}{2}\int_2^4 2^x dx$

$\frac{1}{2}(17.312)$

$\boxed{8.656}$

* 4 If $y = x^2e^{2x}$ describe the interval on which y is decreasing. (no decimal approx.)

$y' = 2xe^{2x} + x^2e^{2x}(2)$

$y' = \underbrace{2e^{2x}}_{\text{always positive}}\underbrace{(x + x^2)}_{\text{determines } +/-}$

$x^2 + x$

$x(x+1) = 0$

$x = 0$ or -1

y inc dec inc

y' + − +

−1 0

y is dec. on $-1 \le x \le 0$

5 A. particle moves along the curve $y = \frac{2}{x+1}$. As the particle passes the point (7, ¼), its x-coordinate is increasing at the rate of 8 units per minute.

a) How fast is the y-coordinate of the particle changing at this instant?

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$\frac{dx}{dt} = 8$ units/min $\quad y = \frac{2}{x+1} \quad 2(x+1)^{-1}$

$\frac{dy}{dt} = \frac{-1}{4}$ units/min $\quad \frac{dy}{dt} = -2(x+1)^{-2}\frac{dx}{dt}$

$\frac{dy}{dt} = \frac{-2}{(x+1)^2}\frac{dx}{dt}$

$\frac{-2}{(7+1)^2}(8) = \frac{-1}{4}$

b) Write an accumulation function called A(n) that will find the area under the graph of y from 0 to n.

$A(n) = \int_0^n \frac{2}{x+1}\,dx$

c) Find A(7)

$\int_0^7 \frac{2}{x+1}\,dx$

$2\int \frac{1}{u}\,du = 2\ln|u|$

$= 2\ln|x+1|\Big|_0^7$

$u = x+1$

$du = dx$

$4.159 - 0 = 4.159$

d) Find the rate of change on A(n) as the particle moves through the point (7, ¼)

$\frac{d}{dn}\int_0^n \frac{2}{x+1}\,dx$

$\frac{2}{n+1} = \frac{2}{7+1} = \frac{2}{8} = \frac{1}{4}$ units²/min