Calculus Chapter 6 Area and Volume and Slope Fields

Section 6.0 Slope Fields Day 1 - 3

What is a slope field?

- Given a dy/dx function, the slope field will show the general shape of the differential equation y for all values of the constant C.

How to construct a slope field:

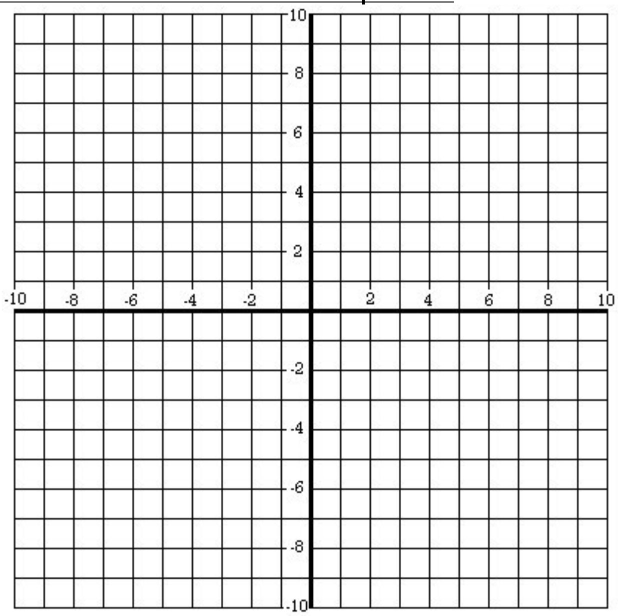
- Think of all integer points on the grid. (-5, -5), (-5, -4), (-5, -3), etc. Consider organizing the points into a table that looks like the grid.
- Plug each ordered pair into the dy/dx function. This will tell the slope at that location on y.
- For each slope generated, draw a short line segment at this location on the grid. For example, if the point (1, 2) has a slope of -2, then draw a segment through (1, 2) that displays a -2 slope.

Now complete the packet.

Section 6.1 Area between 2 curves Day 1

Consider: $y = x$ and $y = -x^2 + 2$

Sketch and find the intersection points:



Method 1: Find the area of each using separate Integrals and subtract the areas.

Method 2: Subtract inside a single integral.

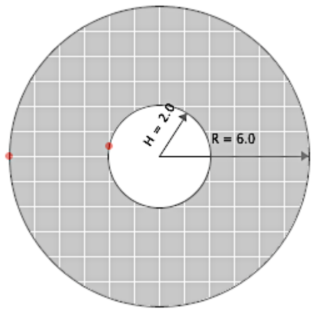
Section 6.1 Area between 2 curves Day 2

Opener: memorization practice

$$\lim_{x \to 0} \frac{\sin x}{x} = \qquad \lim_{x \to 0} \frac{1 - \cos x}{x} = \qquad \lim_{x \to 0} \frac{\tan x}{x} =$$

Section 6.2 Volumes of Revolution Day 1

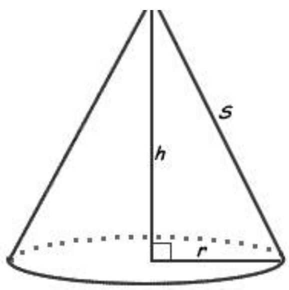
Opener: find the area of the shaded shape below:



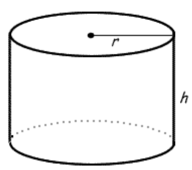
If this shape was the end of a roll of toilet paper of width 12 cm, then what is the volume?

Volumes of Revolutions:

Common Volume Formulas:



$V = \frac{1}{3}\pi r^2 h$



Volume $V = \pi r^2 h$

Notes:

- After a curve is revolved around an axis, a set of circles is created back-to-back. Think of pancakes stacked up. These pancakes do not have to be the same radius.
- The distance travelled from the first circle to the last circle is the height of the shape. Think of the height of the stack of pancakes.
- Think of the party favors!!

Examples: Revolve the following around the x-axis:

* y = 2 from x = 0 to x = 3 #1. y = -x+1 from x = 0 to 1

Setting up the Integral:

- Find the radius of the circle as a formula.
- Set up an integral with the limits of integration based on the height of the object and the area of a circle formula using your radius formula.
- Pull π and any other constants in front of the integral before integrating.

$$\boldsymbol{Volume = \int_a^b \pi(f(x))^2\, dx}$$

Examples where the shaded area is not tight to the axis of revolution:

Method 1: Find the volume of each using separate Integrals and subtract the volumes.

$$\boldsymbol{Volume = \int_a^b \pi(f(x))^2\, dx - \int_a^b \pi(g(x))^2 dx}$$

Method 2: Subtract inside a single integral.

$$\boldsymbol{Volume = \int_a^b \pi[(f(x))^2 - (g(x))^2]\, dx}$$

Examples where the shaded area is revolved around the y-axis

- The circles are created horizontally instead of vertically.
- The limits of integration are y-values.
- The radius is an x-value. This means we have to use a function for which x is solved (aka, find the inverse function).

Section 6.2 Volumes of Revolution Day 2

Opener: Memorization practice: draw a unit circle, quad 2 in radians

Examples where the shaded area is revolved around a "x = " line

- The circles are created horizontally.
- The limits of integration are y-values.
- The radius is an x-value. This means we have to use a function for which x is solved (aka, find the inverse function). After finding the x = function, you must add or subtract a constant based on the distance to the line of revolution.

Picture:

Examples where the shaded area is revolved around a "y = " line

- The circles are created vertically.
- The limits of integration are x-values.
- The radius is an y-value. You must add or subtract a constant based on the distance to the line of revolution.

Picture:

Section 6.2 Volumes of Revolution Day 3

Opener: Memorization practice: draw a unit circle, quad 2 in radians

Today is all AP style questions.