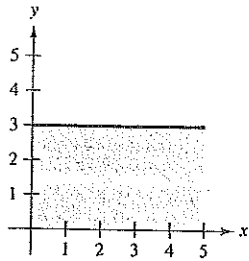
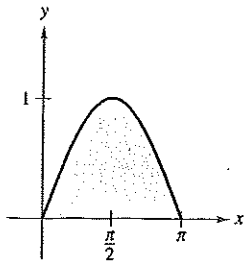


In Exercises 13–22, set up a definite integral that yields the area of the region. (Do not evaluate the integral.)

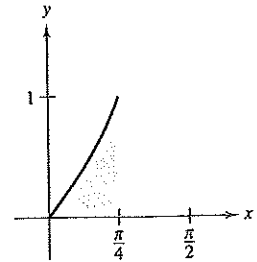
13. $f(x) = 3$



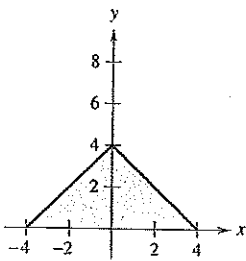
19. $f(x) = \sin x$



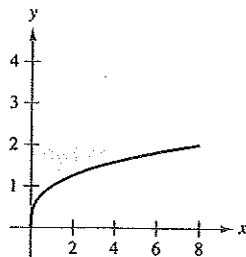
20. $f(x) = \tan x$



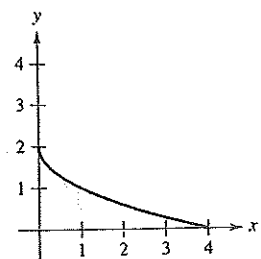
15. $f(x) = 4 - |x|$



21. $g(y) = y^3$



22. $f(y) = (y - 2)^2$



In Exercises 23–32, sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral ($a > 0, r > 0$).

23. $\int_0^3 4 \, dx$

24. $\int_{-a}^a 4 \, dx$

29. $\int_{-1}^1 (1 - |x|) \, dx$

25. $\int_0^4 x \, dx$

26. $\int_0^4 \frac{x}{2} \, dx$

31. $\int_{-3}^3 \sqrt{9 - x^2} \, dx$

In Exercises 33–40, evaluate the integral using the following values.

$\int_2^4 x^3 \, dx = 60, \quad \int_2^4 x \, dx = 6, \quad \int_2^4 dx = 2$

37. $\int_2^4 (x - 8) \, dx$

38. $\int_2^4 (x^3 + 4) \, dx$

33. $\int_4^2 x \, dx$

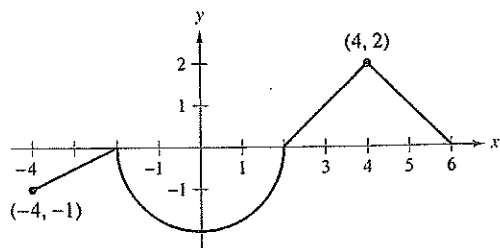
34. $\int_2^2 x^3 \, dx$

39. $\int_2^4 (\frac{1}{2}x^3 - 3x + 2) \, dx$

40. $\int_2^4 (6 + 2x - x^3) \, dx$

35. $\int_2^4 4x \, dx$

36. $\int_2^4 15 \, dx$



45. **Think About It** The graph of f consists of line segments and a semicircle, as shown in the figure. Evaluate each definite integral by using geometric formulas.

(a) $\int_0^2 f(x) \, dx$

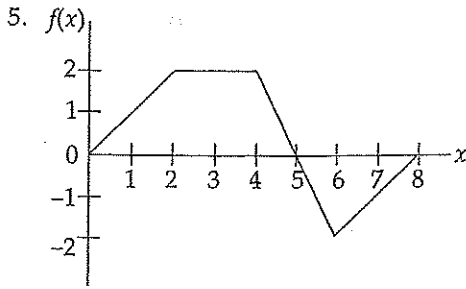
(b) $\int_2^6 f(x) \, dx$

(c) $\int_{-4}^2 f(x) \, dx$

(d) $\int_{-4}^6 f(x) \, dx$

(e) $\int_{-4}^6 |f(x)| \, dx$

(f) $\int_{-4}^6 [f(x) + 2] \, dx$



4.2

The graph of a piecewise linear function f , for $0 \leq x \leq 8$, is shown above. What is the value of $\int_0^8 f(x) dx$?

(A) 1 (B) 4 (C) 8 (D) 10 (E) 13

35. What is the trapezoidal approximation of $\int_0^3 e^x dx$ using $n = 4$ subintervals?

(A) 6.407 (B) 13.565 (C) 19.972 (D) 27.879 (E) 34.944

4.3

35. Approximate $\int_0^1 \sin^2 x dx$ using the Trapezoid Rule with $n = 4$, to three decimal places.

(A) 0.277
 (B) 0.273
 (C) 0.555
 (D) 1.109
 (E) 2.219

4.3

38. If the definite integral $\int_1^3 (x^2 + 1) dx$ is approximated by using the Trapezoid Rule with $n = 4$, the error is

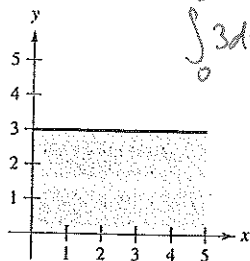
(A) 0 (B) $\frac{7}{3}$ (C) $\frac{1}{12}$ (D) $\frac{65}{6}$ (E) $\frac{97}{3}$

note: error = trap - actual

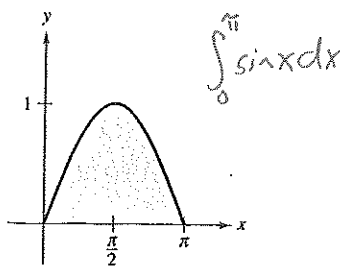
4.3

In Exercises 13–22, set up a definite integral that yields the area of the region. (Do not evaluate the integral.)

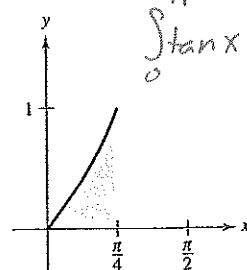
13. $f(x) = 3$



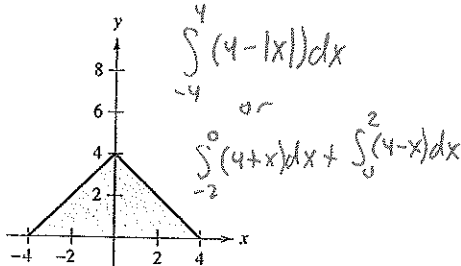
19. $f(x) = \sin x$



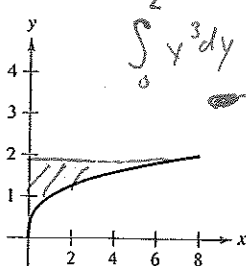
20. $f(x) = \tan x$



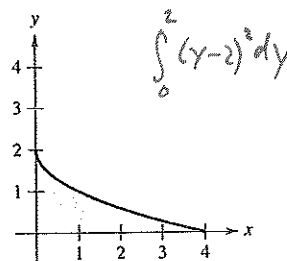
15. $f(x) = 4 - |x|$



21. $g(y) = y^3$

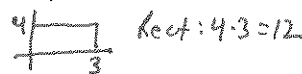


22. $f(y) = (y - 2)^2$

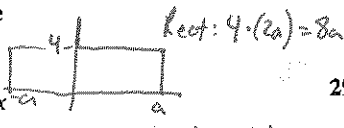


In Exercises 23–32, sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral ($a > 0, r > 0$).

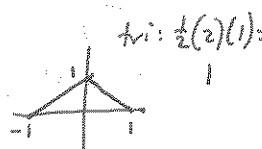
23. $\int_0^3 4 dx$



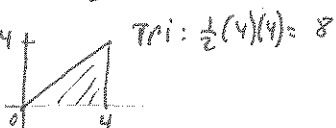
24. $\int_{-a}^a 4 dx$



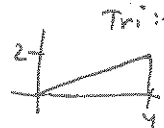
29. $\int_{-1}^1 (1 - |x|) dx$



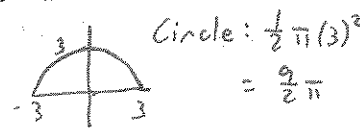
25. $\int_0^4 x dx$



26. $\int_0^4 \frac{x}{2} dx$



31. $\int_{-3}^3 \sqrt{9 - x^2} dx$



In Exercises 33–40, evaluate the integral using the following values.

$\int_2^4 x^3 dx = 60$, $\int_2^4 x dx = 6$, $\int_2^4 dx = 2$

33. $\int_4^2 x dx = -1$ $\int_2^4 x dx = 2$ $\int_2^4 x^2 dx = 12$ $\int_2^4 x^3 dx = 60$

35. $\int_2^4 4x dx = 4 \int_2^4 x dx = 4 * 6 = 24$ $\int_2^4 15 dx = 15 \int_2^4 dx = 15 * 2 = 30$



37. $\int_2^4 (x - 8) dx = \int_2^4 x dx - 8 \int_2^4 dx = 6 - 8 * 2 = -10$

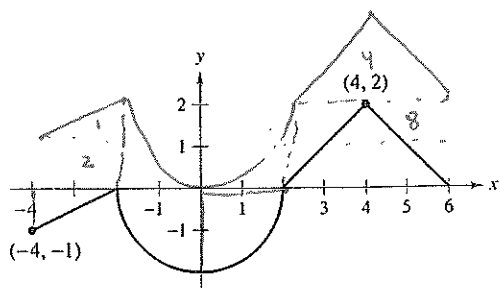
39. $\int_2^4 (\frac{1}{2}x^3 - 3x + 2) dx = \frac{1}{2} \int_2^4 x^3 dx - 3 \int_2^4 x dx + 2 \int_2^4 dx$

$\frac{1}{2} * 60 - 3 * 6 + 2 * 2 = 30 - 18 + 4 = 16$

38. $\int_2^4 (x^3 + 4) dx = \int_2^4 x^3 dx + 4 \int_2^4 dx = 60 + 4 * 2 = 68$

40. $\int_2^4 (6 + 2x - x^3) dx = 6 \int_2^4 dx + 2 \int_2^4 x dx - \int_2^4 x^3 dx$

$6 * 2 + 2 * 6 - 60 = 12 + 12 - 60 = -36$

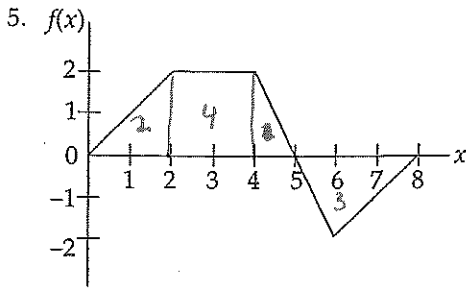


45. **Think About It** The graph of f consists of line segments and a semicircle, as shown in the figure. Evaluate each definite integral by using geometric formulas.

(a) $\int_0^2 f(x) dx = \frac{1}{4} \pi (2)^2 = \pi$ (b) $\int_2^4 f(x) dx = \frac{1}{2} * 4 * 2 = 4$

(c) $\int_{-4}^2 f(x) dx = \frac{1}{2} * 2 * 1 + \frac{1}{2} \pi (2)^2 = 1 + 2\pi$ (d) $\int_{-4}^6 f(x) dx = 1 + 2\pi + 4 = 5 + 2\pi$

(e) $\int_{-4}^6 |f(x)| dx = 5 + 2\pi$ (f) $\int_{-4}^6 [f(x) + 2] dx = 2 * 1 + 2\pi + 2 * 4 + 2 * 2 = 2 + 2\pi + 12 = 14 + 2\pi$



$$2+4+1+3$$

4.2

The graph of a piecewise linear function f , for $0 \leq x \leq 8$, is shown above. What is the value of $\int_0^8 f(x) dx$?

- (A) 1 (B) 4 (C) 8 (D) 10 (E) 13

35. What is the trapezoidal approximation of $\int_0^3 e^x dx$ using $n = 4$ subintervals?

4.3

- (A) 6.407 (B) 13.565 (C) 19.972 (D) 27.879 (E) 34.944

$$\frac{1+2.117}{2} \quad \frac{2.117+4.481}{2}$$

$$\frac{4.481+9.487}{2} \quad \frac{9.487+20.085}{2}$$

$$1.168 + 2.474 + 5.238 + 11.089$$

35. Approximate $\int_0^1 \sin^2 x dx$ using the Trapezoid Rule with $n = 4$, to three decimal places.

4.3

- (A) 0.277
(B) 0.273
(C) 0.555
(D) 1.109
(E) 2.219

$$y = (\sin(x))^2$$


calc.

38. If the definite integral $\int_1^3 (x^2 + 1) dx$ is approximated by using the Trapezoid Rule with $n = 4$, the error is

4.3

- (A) 0 (B) $\frac{7}{3}$ (C) $\frac{1}{12}$ (D) $\frac{65}{6}$ (E) $\frac{97}{3}$

$$\left. \begin{array}{l} \text{trap} = 10.750 \\ \text{actual} = 10.666 \end{array} \right\} \text{diff.} = .083$$


 **Graphical Reasoning** In Exercises 1–4, use a graphing utility to graph the integrand. Use the graph to determine whether the definite integral is positive, negative, or zero.

$$1. \int_0^{\pi} \frac{4}{x^2 + 1} dx$$

$$3. \int_{-2}^2 x\sqrt{x^2 + 1} dx$$

$$2. \int_0^{\pi} \cos x dx$$

$$4. \int_{-2}^2 x\sqrt{2-x} dx$$

 In Exercises 5–26, evaluate the definite integral of the algebraic function. Use a graphing utility to verify your result.

$$5. \int_0^1 2x dx$$

$$6. \int_2^7 3 dv$$

$$8. \int_2^5 (-3v + 4) dv$$

$$7. \int_{-1}^0 (x - 2) dx$$

$$9. \int_{-1}^1 (t^2 - 2) dt$$

$$11. \int_0^1 (2t - 1)^2 dt$$

$$13. \int_1^2 \left(\frac{3}{x^2} - 1\right) dx$$

$$15. \int_1^4 \frac{u - 2}{\sqrt{u}} du$$

$$17. \int_{-1}^1 (\sqrt[3]{t} - 2) dt$$

$$C \quad 19. \int_0^1 \frac{x - \sqrt{x}}{3} dx$$

$$C \quad 21. \int_{-1}^0 (t^{1/3} - t^{2/3}) dt$$

$$C \quad 18. \int_1^8 \sqrt{\frac{2}{x}} dx$$

$$C \quad 20. \int_0^2 (2 - t)\sqrt{t} dt$$

$$C \quad 22. \int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}} dx$$

t (hours)	0	2	5	7	8	10
$v(t)$ (miles per hour)	50	55	60	70	65	75

3. The table above gives the velocity $v(t)$ at selected times t of a car traveling along a straight road.
- (a) Use the values of the table to approximate the acceleration of the car at time $t = 6$. Show the work that leads to your answer and indicate units of measure. 2.2
- (b) Use a right Riemann sum with the subintervals given in the table to approximate $\int_0^{10} v(t) dt$. Indicate units of measure. What physical quantity does this integral represent? 4.3
- (c) The function $v(t)$ is twice differentiable on the interval $[0, 10]$. Show that there must be a moment of time when the acceleration of the car is equal to zero. 3.4

4. $\int_1^3 \frac{8}{x^3} dx =$ 4.4

- (A) $\frac{32}{9}$ (B) $\frac{40}{9}$ (C) 0 (D) $-\frac{40}{9}$ (E) $-\frac{32}{9}$


5. $\int_1^4 \frac{dx}{\sqrt{x}} =$ 4.4


- (A) $\frac{1}{2}$
 (B) 2
 (C) 4
 (D) $\frac{14}{3}$
 (E) $\frac{21}{2}$

15. $\int_{e^{-1}}^1 \frac{x^2 - x}{x^2} dx =$ 4.4

- (A) $-\frac{1}{e}$
 (B) $\frac{1}{e}$
 (C) $2 - \frac{1}{e}$
 (D) e
 (E) $e^2 - \frac{1}{e}$

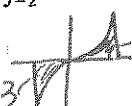
Graphical Reasoning In Exercises 1-4, use a graphing utility to graph the integrand. Use the graph to determine whether the definite integral is positive, negative, or zero.


1. $\int_0^{\pi} \frac{4}{x^2 + 1} dx$  5.050

2. $\int_0^{\pi} \cos x dx$  total: 0

3. $\int_{-2}^2 x\sqrt{x^2 + 1} dx$

4. $\int_{-2}^2 x\sqrt{2-x} dx$  1.508

 3.393
total: 0

 -3.641
total: -2.133

In Exercises 5-26, evaluate the definite integral of the algebraic function. Use a graphing utility to verify your result.

5. $\int_0^1 2x dx$ $[\frac{2}{2}x^2]_0^1 = 1^2 - 0^2 = 1$

6. $\int_2^7 3 dv$ $[3v]_2^7 = 21 - 6 = 15$

8. $\int_2^5 (-3v + 4) dv$ $[-\frac{3}{2}v^2 + 4v]_2^5 = -\frac{3}{2}(25) + 4(5) - (-\frac{3}{2}(4) + 4(2)) = -37.5 + 20 - (-6 + 8) = -19.5$

7. $\int_{-1}^0 (x - 2) dx$ $[\frac{1}{2}x^2 - 2x]_{-1}^0 = 0 - [\frac{1}{2}(-1)^2 - 2(-1)] = 0 - [\frac{1}{2} + 2] = -2.5$

9. $\int_{-1}^1 (t^2 - 2) dt$ $[\frac{1}{3}t^3 - 2t]_{-1}^1 = [\frac{1}{3} \cdot 1^3 - 2 \cdot 1] - [\frac{1}{3}(-1)^3 - 2(-1)] = -1.66 - 1.66 = -3.3$

11. $\int_0^1 \frac{4t^2 - 4t + 1}{(2t - 1)^2} dt$ $[\frac{4}{3}t^3 - \frac{4}{2}t^2 + t]_0^1 = [\frac{4}{3}(1)^3 - 2(1)^2 + 1] - [0] = \frac{1}{3}$

13. $\int_1^2 (\frac{3}{x^2} - 1) dx$ $[-\frac{3}{1}x^{-1} - x]_1^2 = [-3(\frac{1}{2}) - 2] - [-3(\frac{1}{1}) - (1)] = -3.5 + 4 = \frac{1}{2}$


$\frac{u}{4^{1/2}} - \frac{2}{4^{1/2}}$
15. $\int_1^4 \frac{u^{1/2} - 2u^{-1/2}}{\sqrt{u}} du$ $= [\frac{2}{3}u^{3/2} - 2(\frac{1}{2})u^{1/2}]_1^4 = [\frac{2}{3}(4)^{3/2} - 4(4)^{1/2}] - [\frac{2}{3}(1)^{3/2} - 2(1)^{1/2}] = [\frac{16}{3} - 8] - [\frac{2}{3} - 2] = \frac{2}{3}$

17. $\int_{-1}^1 (\frac{3}{4}t - 2) dt$ $[\frac{3}{4}t^2 - 2t]_{-1}^1 = [\frac{3}{4}(1)^2 - 2(1)] - [\frac{3}{4}(-1)^2 - 2(-1)] = -\frac{5}{4} - \frac{1}{4} = -1.5$

C 19. $\int_0^1 \frac{x - \sqrt{x}}{3} dx$  Area = -0.055

C 21. $\int_{-1}^0 (t^{1/3} - t^{2/3}) dt$  Area = -1.350

C 18. $\int_1^8 \sqrt{\frac{2}{x}} dx$  Area = 5.171

C 20. $\int_0^2 (2 - t)\sqrt{t} dt$  Area = 1.508

C 22. $\int_{-8}^{-1} \frac{x - x^2}{2\sqrt{x}} dx$  Area = 57.112

t (hours)	0	2	5	7	8	10
$v(t)$ (miles per hour)	50	55	60	70	65	75

3. The table above gives the velocity $v(t)$ at selected times t of a car traveling along a straight road.

(a) Use the values of the table to approximate the acceleration of the car at time $t = 6$. Show the work that leads to your answer and indicate units of measure.

$$\frac{70-60}{7-5} = \frac{10}{2} = 5 \text{ mph/h}$$

(b) Use a right Riemann sum with the subintervals given in the table to approximate $\int_0^{10} v(t) dt$. Indicate units of measure. What physical quantity does this integral represent?

t	2	3	7	8
v	55	60	70	65
Area	110	180	140	165

total: 645

(c) The function $v(t)$ is twice differentiable on the interval $[0, 10]$. Show that there must be a moment of time when the acceleration of the car is equal to zero.

accel. from $t=5$ to $t=7$ is 5 mph/h accel. from $t=7$ to $t=8$ is -5 mph/h . $a(t)$ is diff, so it is cont. so $a(t)=0$ for $5 < t < 8$ by IVT.

$$4. \int_1^3 \frac{8x^{-3}}{x^3} dx = \left[\frac{8}{-2} x^{-2} \right]_1^3 = \left[-4/x^2 \right]_1^3 = \frac{-4}{9} - \frac{-4}{1} = -\frac{4}{9} + \frac{36}{9} = \frac{32}{9}$$

4.4

- (A) $\frac{32}{9}$ (B) $\frac{40}{9}$ (C) 0 (D) $-\frac{40}{9}$ (E) $-\frac{32}{9}$

$$5. \int_1^4 \frac{dx}{\sqrt{x}} = \left[2x^{1/2} \right]_1^4 = 2(4)^{1/2} - 2(1)^{1/2} = 4 - 2 = 2$$

4.4

(A) $\frac{1}{2}$

(B) 2

(C) 4

(D) $\frac{14}{3}$

(E) $\frac{21}{2}$

$$15. \int_{e^{-1}}^1 \frac{x^2 - x}{x^2} dx = \left[x - \ln x \right]_{e^{-1}}^1 = \left[1 - \ln 1 \right] - \left[e^{-1} - \ln(e^{-1}) \right] = \left[1 - 0 \right] - \left[e^{-1} - (-1) \right] = 1 - \frac{1}{e} + 1$$

4.4

(A) $-\frac{1}{e}$

(B) $\frac{1}{e}$

(C) $2 - \frac{1}{e}$

(D) e

(E) $e^2 - \frac{1}{e}$

In Exercises 27–32, evaluate the definite integral of the trigonometric function. Use a graphing utility to verify your result.

27. $\int_0^{\pi} (1 + \sin x) dx$

C 29. $\int_{-\pi/6}^{\pi/6} \sec^2 x dx$

C 31. $\int_{-\pi/3}^{\pi/3} 4 \sec \theta \tan \theta d\theta$

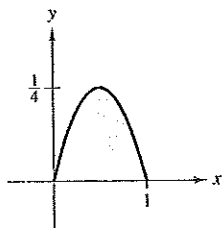
C 28. $\int_0^{\pi/4} \frac{1 - \sin^2 \theta}{\cos^2 \theta} d\theta$

C 30. $\int_{\pi/4}^{\pi/2} (2 - \csc^2 x) dx$

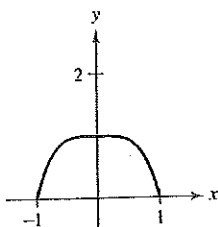
C 32. $\int_{-\pi/2}^{\pi/2} (2t + \cos t) dt$

In Exercises 35–40, determine the area of the indicated region.

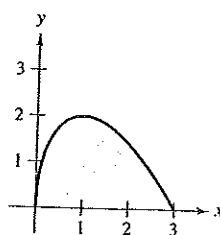
35. $y = x - x^2$



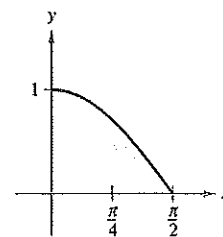
36. $y = 1 - x^4$



37. $y = (3 - x)\sqrt{x}$



39. $y = \cos x$



In Exercises 41–44, find the area of the region bounded by the graphs of the equations.

41. $y = 3x^2 + 1, \quad x = 0, \quad x = 2, \quad y = 0$

C 42. $y = 1 + \sqrt[3]{x}, \quad x = 0, \quad x = 8, \quad y = 0$

In Exercises 49–52, find the average value of the function over the interval and all values of x in the interval for which the function equals its average value.

<u>Function</u>	<u>Interval</u>
49. $f(x) = 4 - x^2$	$[-2, 2]$

51. $f(x) = \sin x$ $[0, \pi]$

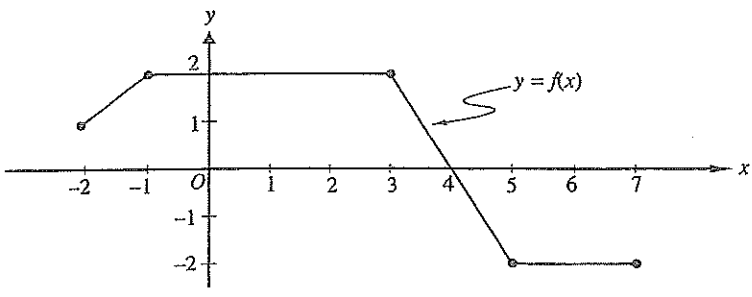
C 50. $f(x) = \frac{4(x^2 + 1)}{x^2}$ $[1, 3]$

C 52. $f(x) = \cos x$ $[0, \pi/2]$

4.4

C 9. $\int_2^{e+1} \left(\frac{4}{x-1}\right) dx =$

- (A) 4 (B) $4e$ (C) 0 (D) $-4e$ (E) -4



* Requires notes from Day 4

21. The graph of a piecewise-linear function $f(x)$, for $-2 \leq x \leq 7$, is shown in the figure above. If $F(x) = \int_2^x f(t) dt$, which of the following statements is true?

- (A) $F(2) > F(3) > F(7)$
 (B) $F(2) > F(7) > F(3)$
 (C) $F(3) > F(2) > F(7)$
 (D) $F(3) > F(7) > F(2)$
 (E) $F(7) > F(2) > F(3)$

4.4

C 79. What is the average value of the function f defined by $f(x) = \sin(x^2)$ on the closed interval $[1, 3]$?

- (A) 0.154
 (B) 0.232
 (C) 0.463
 (D) 0.696
 (E) 1.392

3.12
4.4

x	0	1	2	3
$f(x)$	2	5	4	3

4.3

C 91. The function f is continuous on the closed interval $[0, 3]$ and has values that are given in the table above. Using the subintervals $[0, 1]$, $[1, 2]$, and $[2, 3]$, what is the trapezoidal approximation to $\int_0^3 f(x) dx$?

- (A) 11
 (B) 11.5
 (C) 12
 (D) 12.5
 (E) 13

In Exercises 27–32, evaluate the definite integral of the trigonometric function. Use a graphing utility to verify your result.

NC 27. $\int_0^{\pi} (1 + \sin x) dx = [x - \cos x]_0^{\pi} = [\pi - \cos \pi] - [0 - \cos 0] = \pi - (-1) + 1 = 2 + \pi$

C 29. $\int_{-\pi/6}^{\pi/6} \sec^2 x dx = \int_{-\pi/6}^{\pi/6} (\frac{1}{\cos x})^2 = 1.154$

C 31. $\int_{-\pi/3}^{\pi/3} 4 \sec \theta \tan \theta d\theta = 4 \int_{-\pi/3}^{\pi/3} (1/\cos \theta) \tan \theta = 0$

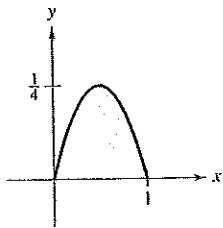
C 28. $\int_0^{\pi/4} \frac{1 - \sin^2 \theta}{\cos^2 \theta} d\theta = .785$

C 30. $\int_{\pi/4}^{\pi/2} (2 - \csc^2 x) dx = [2 - \int (\sin(x)^2)] = .570$

C 32. $\int_{-\pi/2}^{\pi/2} (2t + \cos t) dt = 2$

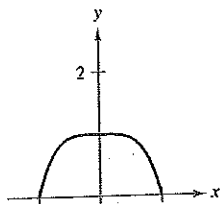
In Exercises 35–40, determine the area of the indicated region.

35. $y = x - x^2$



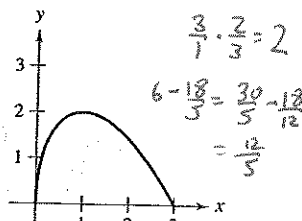
$\int_0^1 (x - x^2) dx = [\frac{1}{2}x^2 - \frac{1}{3}x^3]_0^1 = [\frac{1}{2} - \frac{1}{3}] - [0 - 0] = \frac{1}{6}$

36. $y = 1 - x^4$



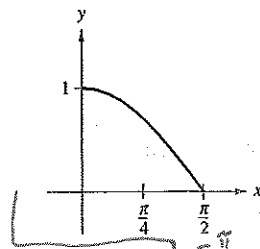
$\int_{-1}^1 (1 - x^4) dx = [x - \frac{1}{5}x^5]_{-1}^1 = [1 - \frac{1}{5}] - [-1 - \frac{1}{5}(-1)] = \frac{4}{5} - (-\frac{4}{5}) = \frac{8}{5}$

37. $y = (3 - x)\sqrt{x}$



$\int_0^3 (3x^{1/2} - x^{3/2}) dx = [\frac{3}{2}x^{3/2} - \frac{2}{5}x^{5/2}]_0^3 = [2(3)^{3/2} - \frac{2}{5}(3)^{5/2}] - [0 - 0] = 6\sqrt{3} - \frac{18}{5}\sqrt{3} = \frac{12}{5}\sqrt{3}$

39. $y = \cos x$



$\int_{\pi/4}^{\pi/2} \cos x dx = [\sin x]_{\pi/4}^{\pi/2} = \sin \frac{\pi}{2} - \sin \frac{\pi}{4} = 1 - \frac{1}{\sqrt{2}}$

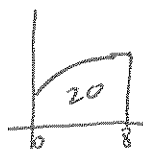
In Exercises 41–44, find the area of the region bounded by the graphs of the equations.

41. $y = 3x^2 + 1, x = 0, x = 2, y = 0$



$\int_0^2 (3x^2 + 1) dx = [\frac{3}{3}x^3 + x]_0^2 = [2^3 + 2] - [0^3 + 0] = 8 + 2 = 10$

C 42. $y = 1 + \sqrt[3]{x}, x = 0, x = 8, y = 0$



In Exercises 49–52, find the average value of the function over the interval and all values of x in the interval for which the function equals its average value.

49. $f(x) = 4 - x^2$

Interval $[-2, 2]$

$\frac{24}{3} - \frac{8}{3} = \frac{16}{3}$
 $-\frac{24}{3} + \frac{8}{3} = -\frac{16}{3}$

$\frac{1}{2 - (-2)} \int_{-2}^2 (4 - x^2) dx = \frac{1}{4} [4x - \frac{1}{3}x^3]_{-2}^2$

$\frac{1}{4} [4(2) - \frac{1}{3}(2)^3] - \frac{1}{4} [4(-2) - \frac{1}{3}(-2)^3] = \frac{1}{4} [\frac{16}{3}] - \frac{1}{4} [-\frac{16}{3}] = \frac{4}{3} + \frac{4}{3} = \frac{8}{3}$

C 50. $f(x) = \frac{4(x^2 + 1)}{x^2}$ [1, 3]

$\frac{1}{3-1} \int_1^3 \frac{4}{x} dx = \frac{1}{2} [10.6] = 5.3$

51. $f(x) = \sin x$

[0, π]

$\frac{1}{\pi - 0} \int_0^{\pi} \sin x dx = \frac{1}{\pi} [-\cos x]_0^{\pi}$

$\frac{1}{\pi} [-\cos \pi] - \frac{1}{\pi} [-\cos 0] = \frac{1}{\pi} (1) - \frac{1}{\pi} (-1) = \frac{1}{\pi} + \frac{1}{\pi} = \frac{2}{\pi}$

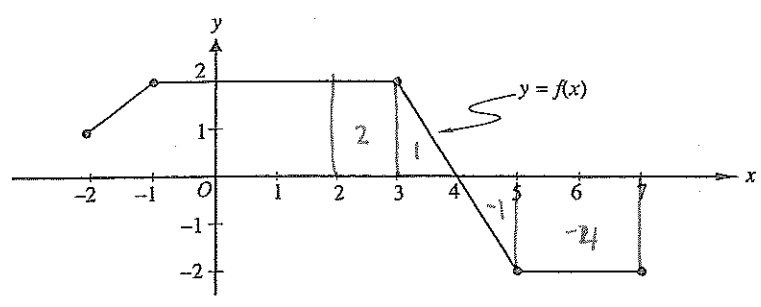
C 52. $f(x) = \cos x$

[0, $\pi/2$]

$\frac{1}{\pi/2 - 0} \int_0^{\pi/2} \cos x dx = \frac{1}{\pi/2} [1] = \frac{2}{\pi}$

9. $\int_2^{e+1} \left(\frac{4}{x-1} \right) dx =$
 (A) 4

- (B) $4e$ (C) 0 (D) $-4e$ (E) -4



21. The graph of a piecewise-linear function $f(x)$, for $-2 \leq x \leq 7$, is shown in the figure above. If $F(x) = \int_2^x f(t) dt$, which of the following statements is true?

- (A) $F(2) > F(3) > F(7)$
 (B) $F(2) > F(7) > F(3)$
 (C) $F(3) > F(2) > F(7)$
 (D) $F(3) > F(7) > F(2)$
 (E) $F(7) > F(2) > F(3)$

Area from 2 to x
 $F(2) = \int_2^2 f(t) dt = 0$
 $F(3) = \int_2^3 f(t) dt = 2$
 $F(7) = \int_2^7 f(t) dt = 3 + (-5) = -2$

4.4

79. What is the average value of the function f defined by $f(x) = \sin(x^2)$ on the closed interval $[1, 3]$?

- (A) 0.154
 (B) 0.232
 (C) 0.463
 (D) 0.696
 (E) 1.392

$\frac{1}{3-1} \int_1^3 \sin(x^2) dx$
 $\frac{1}{2} [0.463] = 0.232$

3.2
 4.4

x	0	1	2	3
$f(x)$	2	5	4	3

4.3

91. The function f is continuous on the closed interval $[0, 3]$ and has values that are given in the table above. Using the subintervals $[0, 1]$, $[1, 2]$, and $[2, 3]$, what is the trapezoidal approximation to $\int_0^3 f(x) dx$?

- (A) 11
 (B) 11.5
 (C) 12
 (D) 12.5
 (E) 13

	avg H	w	Area
0 to 1	3.5	1	3.5
1 to 2	4.5	1	4.5
2 to 3	3.5	1	3.5
			11.5

In Exercises 69–74, find F as a function of x and evaluate it at $x = 2$, $x = 5$, and $x = 8$.

69. $F(x) = \int_0^x (t - 5) dt$

70. $F(x) = \int_2^x (t^3 + 2t - 2) dt$

71. $F(x) = \int_1^x \frac{10}{v^2} dv$

72. $F(x) = \int_2^x -\frac{2}{t^3} dt$

In Exercises 75–80, (a) integrate to find F as a function of x and (b) demonstrate the Second Fundamental Theorem of Calculus by differentiating the result in part (a).

75. $F(x) = \int_0^x (t + 2) dt$

76. $F(x) = \int_0^x t(t^2 + 1) dt$

77. $F(x) = \int_8^x \sqrt[3]{t} dt$

~~78.~~ $F(x) = \int_4^x \sqrt{t} dt$

In Exercises 81–86, use the Second Fundamental Theorem of Calculus to find $F'(x)$.

81. $F(x) = \int_{-2}^x (t^2 - 2t) dt$

82. $F(x) = \int_1^x \frac{t^2}{t^2 + 1} dt$

85. $F(x) = \int_0^x t \cos t dt$

86. $F(x) = \int_0^x \sec^3 t dt$

In Exercises 87–92, find $F'(x)$.

87. $F(x) = \int_x^{x+2} (4t + 1) dt$

88. $F(x) = \int_{-x}^x t^3 dt$

~~91.~~ $F(x) = \int_0^{x^3} \sin t^2 dt$

~~92.~~ $F(x) = \int_0^{x^2} \sin \theta^2 d\theta$

x	0	1	3	7	10
$f(x)$	1	-1	4	2	3

4.3

19. The function f is continuous on the closed interval $[0, 10]$ and has values given in the table above. Using the subintervals $[0, 1]$, $[1, 3]$, $[3, 7]$, and $[7, 10]$, what is the left Riemann sum estimate for $\int_0^{10} f(x) dx$?

- (A) 15
- (B) 17.5
- (C) 20
- (D) 21
- (E) 22.5

11. The area of the region in the first quadrant bounded by the graph of $y = x\sqrt{9+x^2}$, the x -axis, and the line $x = 4$ is

- (A) $9 \sec^3 4 - 9$
- (B) $\frac{16}{3}$
- (C) $\frac{13}{3}\sqrt{13} - 9$
- (D) $\frac{64}{3}$
- (E) $\frac{98}{3}$

4.4

53. State the Fundamental Theorem of Calculus.

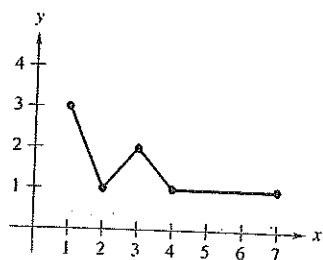


Figure for 54

54. The graph of f is given in the figure.

- (a) Evaluate $\int_1^7 f(x) dx$.
- (b) Determine the average value of f on the interval $[1, 7]$.
- (c) Determine the answers to parts (a) and (b) if the graph is translated two units upward.

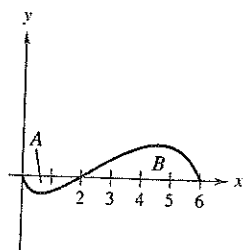


Figure for 55-60

In Exercises 55-60, use the graph of f shown in the figure. The shaded region A has an area of 1.5, and $\int_0^6 f(x) dx = 3.5$. Use this information to fill in the blanks.

59. $\int_0^6 [2 + f(x)] dx =$

60. The average value of f over the interval $[0, 6]$ is

In Exercises 69-74, find F as a function of x and evaluate it at $x = 2$, $x = 5$, and $x = 8$.

69. $F(x) = \int_0^x (t-5) dt$
 $F(x) = \left[\frac{1}{2}t^2 - 5t \right]_0^x$
 $F(x) = \frac{1}{2}x^2 - 5x$
 $F(2) = \frac{1}{2}(2)^2 - 5(2) = -8$
 $F(5) = \frac{1}{2}(5)^2 - 5(5) = 2.5$
 $F(8) = \frac{1}{2}(8)^2 - 5(8) = -8$

70. $F(x) = \int_2^x (t^3 + 2t - 2) dt$
 $F(x) = \left[\frac{1}{4}t^4 + t^2 - 2t \right]_2^x$
 $F(x) = \frac{1}{4}x^4 + x^2 - 2x - [8 + 4 - 2]$
 $F(x) = \frac{1}{4}x^4 + x^2 - 2x - 10$
 $F(2) = 0$
 $F(5) = \frac{1}{4}(625) + 25 - 10 - 10 = 161.25$
 $F(8) = \frac{1}{4}(4096) + 64 - 16 - 10 = 1062$

71. $F(x) = \int_1^x \frac{10}{v^2} dv$
 $F(x) = \left[-10v^{-1} \right]_1^x$
 $F(x) = \frac{-10}{x} + 10$
 $F(2) = \frac{-10}{2} + 10 = 5$
 $F(5) = \frac{-10}{5} + 10 = 8$
 $F(8) = \frac{-10}{8} + 10 = -1.25 + 10 = 8.75$

72. $F(x) = \int_2^x -\frac{2}{t^3} dt$
 $F(x) = \left[\frac{-4}{-2}t^{-2} \right]_2^x$
 $F(x) = 2/x^2 - 2/4 = \frac{2}{x^2} - \frac{1}{2}$
 $F(2) = \frac{2}{4} - \frac{1}{2} = 0$
 $F(5) = \frac{2}{25} - \frac{1}{2} = -.42$
 $F(8) = \frac{2}{64} - \frac{1}{2} = -.468$

In Exercises 75-80, (a) integrate to find F as a function of x and (b) demonstrate the Second Fundamental Theorem of Calculus by differentiating the result in part (a).

75. $F(x) = \int_0^x (t+2) dt$
 $F(x) = \left[\frac{1}{2}t^2 + 2t \right]_0^x$
 $F(x) = \frac{1}{2}x^2 + 2x - 0$
 $F'(x) = x + 2$

76. $F(x) = \int_0^x t(t^2+1) dt$
 $F(x) = \left[\frac{1}{4}t^4 + t \right]_0^x$
 $F(x) = \frac{1}{4}x^4 + x - 0$
 $F'(x) = x^3 + 1$
 $= x(x^2+1)$

77. $F(x) = \int_8^x \sqrt[3]{t} dt$
 $F(x) = \left[\frac{3}{4}t^{4/3} \right]_8^x$
 $F(x) = \frac{3}{4}x^{4/3} - \frac{3}{4}(8)^{4/3}$
 $F'(x) = x^{1/3} - 0$
 $= \sqrt[3]{x}$

78. $F(x) = \int_4^x \sqrt{t} dt$

In Exercises 81-86, use the Second Fundamental Theorem of Calculus to find $F'(x)$.

81. $F(x) = \int_{-2}^x (t^2 - 2t) dt$
 $F'(x) = x^2 - 2x$

82. $F(x) = \int_1^x \frac{t^2}{t^2+1} dt$
 $F'(x) = \frac{x^2}{x^2+1}$

85. $F(x) = \int_0^x t \cos t dt$
 $F'(x) = x \cos x$

86. $F(x) = \int_0^x \sec^3 t dt$
 $F'(x) = \sec^3 x$

In Exercises 87-92, find $F'(x)$.

87. $F(x) = \int_x^{x+2} (4t+1) dt$
 $F(x) = \left[2t^2 + t \right]_x^{x+2}$
 $= 2(x+2)^2 + (x+2) - 2x^2 - x$
 $F'(x) = 4(x+2) \cdot 1 + 1 - 4x - 1$
 $4x + 8 - 4x$
 $F'(x) = 8$

88. $F(x) = \int_{-x}^x t^3 dt$
 $F(x) = \left[\frac{1}{4}t^4 \right]_{-x}^x = \frac{1}{4}x^4 - \frac{1}{4}(-x)^4$
 $F(x) = 0$
 $F'(x) = 0$

91. $F(x) = \int_0^x \sin t^2 dt$
 $F(x) = 0$

92. $F(x) = \int_0^{x^2} \sin \theta^2 d\theta$
 $2x \sin x^4$

x	0	1	3	7	10
$f(x)$	1	-1	4	2	3

4.3

19. The function f is continuous on the closed interval $[0, 10]$ and has values given in the table above. Using the subintervals $[0, 1]$, $[1, 3]$, $[3, 7]$, and $[7, 10]$, what is the left Riemann sum estimate for $\int_0^{10} f(x) dx$?

- (A) 15
- (B) 17.5
- (C) 20
- (D) 21**
- (E) 22.5

	L Height	w	Area
0 to 1	1	1	1
1 to 3	-1	2	-2
3 to 7	4	4	16
7 to 10	2	3	6

Area: 21

11. The area of the region in the first quadrant bounded by the graph of $y = x\sqrt{9+x^2}$, the x -axis, and the line $x = 4$ is

- (A) $9 \sec^3 4 - 9$
- (B) $\frac{16}{3}$
- (C) $\frac{13}{3} \sqrt{13} - 9$
- (D) $\frac{64}{3}$
- (E) $\frac{98}{3}$**

J

32.6

4.4

53. State the Fundamental Theorem of Calculus.

a) $2 + 1.5 + 1.5 + 3 = 8$

b) $\frac{1}{7-1} \int_1^7 f(x) dx = \frac{1}{6} \cdot 8 = \frac{8}{6}$

c) $\int_1^7 2 dx = 12$

$8 + 12 = 20$ area

$\frac{20}{6} = \text{ave. val.}$

54. The graph of f is given in the figure.

- (a) Evaluate $\int_1^7 f(x) dx$.
- (b) Determine the average value of f on the interval $[1, 7]$.
- (c) Determine the answers to parts (a) and (b) if the graph is translated two units upward.

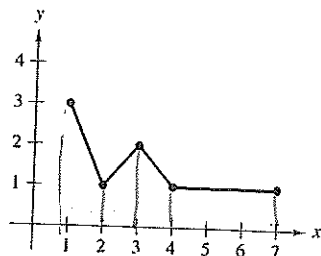


Figure for 54

In Exercises 55–60, use the graph of f shown in the figure. The shaded region A has an area of 1.5, and $\int_0^6 f(x) dx = 3.5$. Use this information to fill in the blanks.

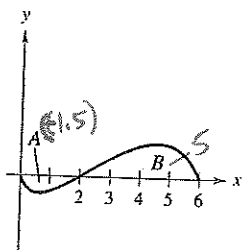


Figure for 55–60

59. $\int_0^6 [2 + f(x)] dx = \int_0^6 2 dx + \int_0^6 f(x) dx = 12 + 3.5 = 15.5$

60. The average value of f over the interval $[0, 6]$ is $\frac{1}{6-0} \int_0^6 f(x) dx = \frac{1}{6} (3.5) = .58\bar{3}$