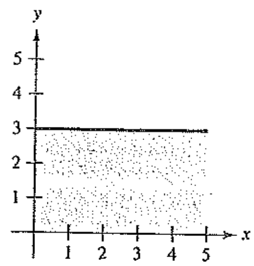
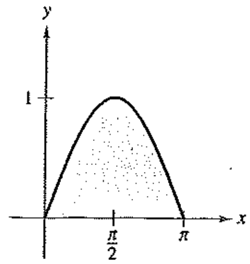
Calculus Worksheet 4-3A Name______________________________ Pd______

In Exercises 13–22, set up a definite integral that yields the area of the region. (Do not evaluate the integral.)

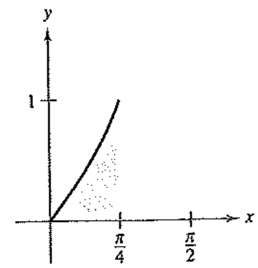
13. $f(x) = 3$



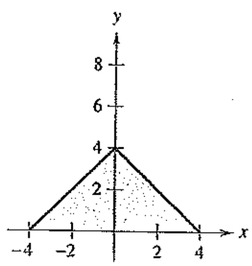
19. $f(x) = \sin x$



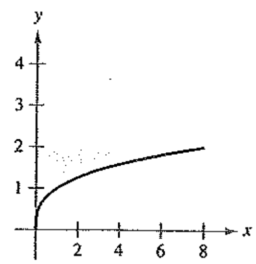
20. $f(x) = \tan x$



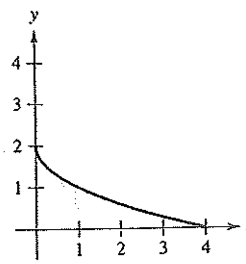
15. $f(x) = 4 - |x|$



21. $g(y) = y^3$



22. $f(y) = (y - 2)^2$



In Exercises 23–32, sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral ($a > 0, r > 0$).

23. $\displaystyle\int_0^3 4\,dx$

24. $\displaystyle\int_{-a}^{a} 4\,dx$

25. $\displaystyle\int_0^4 x\,dx$

26. $\displaystyle\int_0^4 \frac{x}{2}\,dx$

29. $\displaystyle\int_{-1}^{1} (1 - |x|)\,dx$

31. $\displaystyle\int_{-3}^{3} \sqrt{9 - x^2}\,dx$

In Exercises 33–40, evaluate the integral using the following values.

$$\int_2^4 x^3\,dx = 60, \qquad \int_2^4 x\,dx = 6, \qquad \int_2^4 dx = 2$$

33. $\displaystyle\int_4^2 x\,dx$

34. $\displaystyle\int_2^2 x^3\,dx$

35. $\displaystyle\int_2^4 4x\,dx$

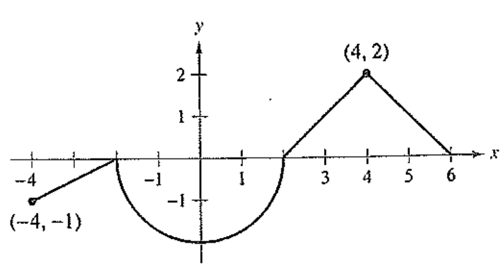
36. $\displaystyle\int_2^4 15\,dx$

37. $\displaystyle\int_2^4 (x - 8)\,dx$

38. $\displaystyle\int_2^4 (x^3 + 4)\,dx$

39. $\displaystyle\int_2^4 \left(\tfrac{1}{2}x^3 - 3x + 2\right) dx$

40. $\displaystyle\int_2^4 (6 + 2x - x^3)\,dx$



45. ***Think About It*** The graph of f consists of line segments and a semicircle, as shown in the figure. Evaluate each definite integral by using geometric formulas.

(a) $\displaystyle\int_0^2 f(x)\,dx$

(b) $\displaystyle\int_2^6 f(x)\,dx$

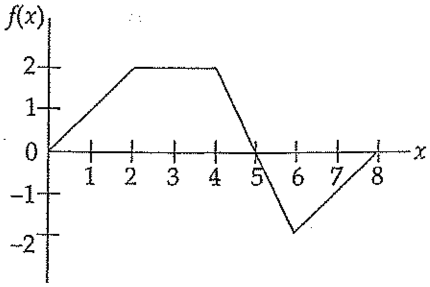
(c) $\displaystyle\int_{-4}^{2} f(x)\,dx$

(d) $\displaystyle\int_{-4}^{6} f(x)\,dx$

(e) $\displaystyle\int_{-4}^{6} |f(x)|\,dx$

(f) $\displaystyle\int_{-4}^{6} [f(x) + 2]\,dx$

5.



4.2

The graph of a piecewise linear function f, for $0 \le x \le 8$, is shown above. What is the value of $\int_0^8 f(x)\,dx$?

(A) 1 (B) 4 (C) 8 (D) 10 (E) 13

C

4.2

35. What is the trapezoidal approximation of $\int_0^3 e^x dx$ using $n = 4$ subintervals?

(A) 6.407 (B) 13.565 (C) 19.972 (D) 27.879 (E) 34.944

C

4.3

35. Approximate $\int_0^1 \sin^2 x\,dx$ using the Trapezoid Rule with $n = 4$, to three decimal places.

(A) 0.277
(B) 0.273
(C) 0.555
(D) 1.109
(E) 2.219

4.3

C

38. If the definite integral $\int_1^3 (x^2+1)dx$ is approximated by using the Trapezoid Rule with $n = 4$, the error is

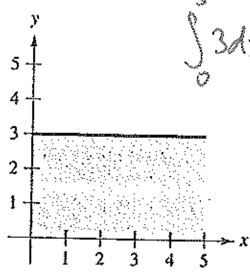
note: error = trap − actual

(A) 0 (B) $\frac{7}{3}$ (C) $\frac{1}{12}$ (D) $\frac{65}{6}$ (E) $\frac{97}{3}$

Calculus Worksheet 4-3A Name Key Pd______

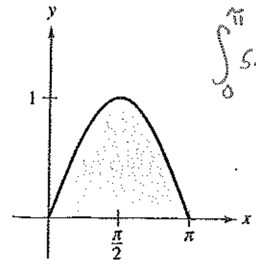
In Exercises 13–22, set up a definite integral that yields the area of the region. (Do not evaluate the integral.)

13. $f(x) = 3$



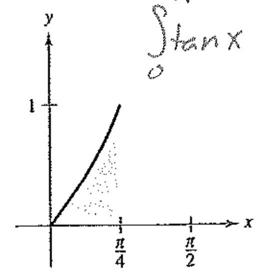
$\int_0^5 3dx$

19. $f(x) = \sin x$



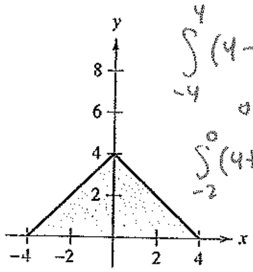
$\int_0^{\pi} \sin x dx$

20. $f(x) = \tan x$



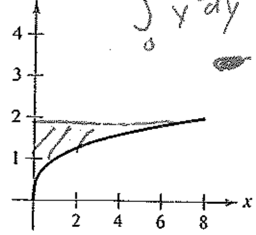
$\int_0^{\pi/4} \tan x$

15. $f(x) = 4 - |x|$



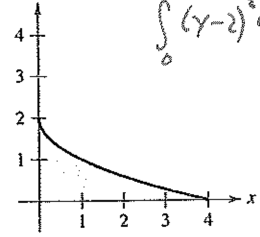
$\int_{-4}^{4} (4-|x|)dx$ or $\int_{-2}^{0}(4+x)dx + \int_0^2 (4-x)dx$

21. $g(y) = y^3$



$\int_0^2 y^3 dy$

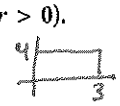
22. $f(y) = (y-2)^2$



$\int_0^2 (y-2)^2 dy$

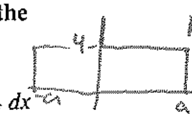
In Exercises 23–32, sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral ($a > 0, r > 0$).

23. $\int_0^3 4\,dx$



Rect: $4 \cdot 3 = 12$

24. $\int_{-a}^{a} 4\,dx$

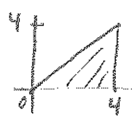


Rect: $4 \cdot (2a) = 8a$

29. $\int_{-1}^{1} (1 - |x|)\,dx$

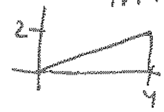
tri: $\frac{1}{2}(2)(1) = 1$

25. $\int_0^4 x\,dx$



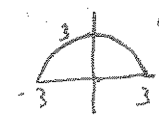
Tri: $\frac{1}{2}(4)(4) = 8$

26. $\int_0^4 \frac{x}{2}\,dx$



Tri: $\frac{1}{2}(2)(4) = 4$

31. $\int_{-3}^{3} \sqrt{9 - x^2}\,dx$



Circle: $\frac{1}{2}\pi(3)^2 = \frac{9}{2}\pi$

In Exercises 33–40, evaluate the integral using the following values.

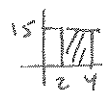
$$\int_2^4 x^3\,dx = 60, \qquad \int_2^4 x\,dx = 6, \qquad \int_2^4 dx = 2$$

33. $\int_4^2 x\,dx$ $= -1\int_2^4 x dx = -2 \cdot 6 = -12$

34. $\int_2^2 x^3\,dx$ $= 0$

35. $\int_2^4 4x\,dx$ $4\int_2^4 x dx = 4 \cdot 6 = 24$

36. $\int_2^4 15\,dx$ $= 15\int_2^4 dx = 15 \cdot 2 = 30$



$A = 2 \cdot 15 = 30$

37. $\int_2^4 (x - 8)\,dx$

$\int_2^4 x dx - 8\int_2^4 dx$

$6 - 8 \cdot 2 = -10$

38. $\int_2^4 (x^3 + 4)\,dx$

$\int_2^4 x^3 dx + 4\int_2^4 dx = 60 + 4 \cdot 2$

68

39. $\int_2^4 \left(\frac{1}{2}x^3 - 3x + 2\right)dx$

$\frac{1}{2}\int_2^4 x^3 dx - 3\int_2^4 x dx + 2\int_2^4 dx$

$\frac{1}{2} \cdot 60 - 3 \cdot 6 + 2 \cdot 2$

$30 - 18 + 4$

16

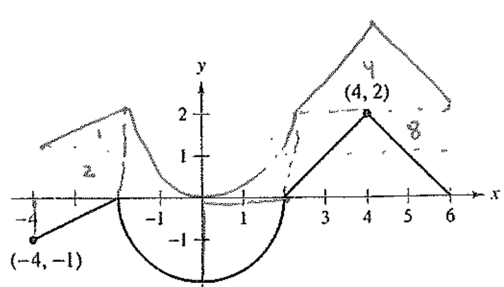
40. $\int_2^4 (6 + 2x - x^3)\,dx$

$6\int_2^4 dx + 2\int_2^4 x dx - \int_2^4 x^3 dx$

$6 \cdot 2 + 2 \cdot 6 - 60$

$12 + 12 - 60$

-36



45. ***Think About It*** The graph of f consists of line segments and a semicircle, as shown in the figure. Evaluate each definite integral by using geometric formulas.

(a) $\int_0^2 f(x)\,dx$ $\frac{1}{4}\pi(2)^2 = \pi$

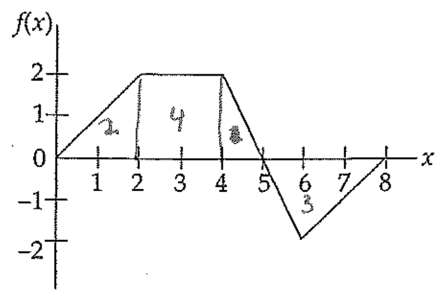
(b) $\int_2^6 f(x)\,dx$ $\frac{1}{2} \cdot 4 \cdot 2 = 4$

(c) $\int_{-4}^{2} f(x)\,dx$ $\frac{1}{2} \cdot 2 \cdot 1 + \frac{1}{2}\pi 2^2$ $1 + 2\pi$

(d) $\int_{-4}^{6} f(x)\,dx$ $1 + 2\pi + 4 = 5 + 2\pi$

(e) $\int_{-4}^{6} |f(x)|\,dx$ $5 + 2\pi$

(f) $\int_{-4}^{6} [f(x) + 2]\,dx$ $2 + 1 + 8 - 2\pi + 12$ $23 - 2\pi$

5. 

$2+4+1+3$

The graph of a piecewise linear function f, for $0 \le x \le 8$, is shown above. What is the value of $\int_0^8 f(x)\,dx$?

(A) 1 (B) 4 (C) 8 (D) 10 (E) 13

4.2

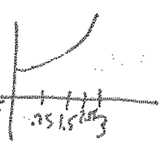
C

4.2

35. What is the trapezoidal approximation of $\int_0^3 e^x dx$ using $n = 4$ subintervals?

(A) 6.407 (B) 13.565 (C) 19.972 (D) 27.879 (E) 34.944

$\frac{1+2.117}{2}$ $\frac{2.117+4.481}{2}$ $\frac{4.481+9.487}{2}$ $\frac{9.487+20.085}{}$



$1.168+2.474+5.238+11.089$

C

4.3

35. Approximate $\int_0^1 \sin^2 x\,dx$ using the Trapezoid Rule with $n = 4$, to three decimal places.

$Y_1 = (\sin(x))^2$

(A) 0.277
(B) 0.273
(C) 0.555
(D) 1.109
(E) 2.219

Calc.

4.3

C

38. If the definite integral $\int_1^3 (x^2+1)dx$ is approximated by using the Trapezoid Rule with $n = 4$, the error is

(A) 0 (B) $\frac{7}{3}$ (C) $\frac{1}{12}$ (D) $\frac{65}{6}$ (E) $\frac{97}{3}$

trap = 10.750
actual = 10.666

diff. = $.08\overline{3}$

Calculus Worksheet 4-4A Name______________________________ Pd______

Graphical Reasoning **In Exercises 1–4, use a graphing utility to graph the integrand. Use the graph to determine whether the definite integral is positive, negative, or zero.**

1. $\displaystyle\int_0^{\pi} \frac{4}{x^2+1}\,dx$

2. $\displaystyle\int_0^{\pi} \cos x\,dx$

3. $\displaystyle\int_{-2}^{2} x\sqrt{x^2+1}\,dx$

4. $\displaystyle\int_{-2}^{2} x\sqrt{2-x}\,dx$

In Exercises 5–26, evaluate the definite integral of the algebraic function. Use a graphing utility to verify your result.

5. $\displaystyle\int_0^{1} 2x\,dx$

6. $\displaystyle\int_2^{7} 3\,dv$

8. $\displaystyle\int_2^{5} (-3v+4)\,dv$

7. $\displaystyle\int_{-1}^{0} (x-2)\,dx$

9. $\displaystyle\int_{-1}^{1} (t^2-2)\,dt$

11. $\displaystyle\int_0^{1} (2t-1)^2\,dt$

13. $\displaystyle\int_1^{2} \left(\frac{3}{x^2}-1\right)dx$

15. $\displaystyle\int_1^{4} \frac{u-2}{\sqrt{u}}\,du$

17. $\displaystyle\int_{-1}^{1} (\sqrt[3]{t}-2)\,dt$

C 19. $\displaystyle\int_0^{1} \frac{x-\sqrt{x}}{3}\,dx$

C 21. $\displaystyle\int_{-1}^{0} (t^{1/3}-t^{2/3})\,dt$

C 18. $\displaystyle\int_1^{8} \sqrt{\frac{2}{x}}\,dx$

C 20. $\displaystyle\int_0^{2} (2-t)\sqrt{t}\,dt$

C 22. $\displaystyle\int_{-8}^{-1} \frac{x-x^2}{2\sqrt[3]{x}}\,dx$

t (hours)	0	2	5	7	8	10
$v(t)$ (miles per hour)	50	55	60	70	65	75

C

3. The table above gives the velocity $v(t)$ at selected times t of a car traveling along a straight road. 2.2

(a) Use the values of the table to approximate the acceleration of the car at time $t = 6$. Show the work that leads to your answer and indicate units of measure. 4.3

(b) Use a right Riemann sum with the subintervals given in the table to approximate $\int_0^{10} v(t)\,dt$. Indicate units of measure. What physical quantity does this integral represent? 3.4

(c) The function $v(t)$ is twice differentiable on the interval $[0, 10]$. Show that there must be a moment of time when the acceleration of the car is equal to zero.

4.4

4. $\int_1^3 \frac{8}{x^3}\,dx =$

(A) $\frac{32}{9}$ (B) $\frac{40}{9}$ (C) 0 (D) $-\frac{40}{9}$ (E) $-\frac{32}{9}$

5. $\int_1^4 \frac{dx}{\sqrt{x}} =$

4.4

(A) $\frac{1}{2}$

(B) 2

(C) 4

(D) $\frac{14}{3}$

(E) $\frac{21}{2}$

15. $\int_{e^{-1}}^{1} \frac{x^2 - x}{x^2}\,dx =$

(A) $-\frac{1}{e}$

4.4

(B) $\frac{1}{e}$

(C) $2 - \frac{1}{e}$

(D) e

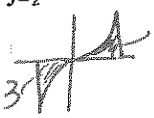
(E) $e^2 - \frac{1}{e}$

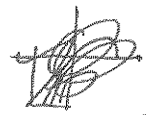
Calculus Worksheet 4-4A Name Key Pd____

Graphical Reasoning In Exercises 1–4, use a graphing utility to graph the integrand. Use the graph to determine whether the definite integral is positive, negative, or zero.

1. $\int_0^{\pi} \frac{4}{x^2+1}\,dx$  5.050

2. $\int_0^{\pi} \cos x\,dx$  total: 0

3. $\int_{-2}^{2} x\sqrt{x^2+1}\,dx$  3.393, −3.393 total: 0

4. $\int_{-2}^{2} x\sqrt{2-x}\,dx$   1.508, −3.641 total: −2.133

In Exercises 5–26, evaluate the definite integral of the algebraic function. Use a graphing utility to verify your result.

5. $\int_0^1 2x\,dx$ $\left[\frac{2}{2}x^2\right]_0^1 = 1^2 - 0^2 = 1$

6. $\int_2^7 3\,dv$ $\left[3v\right]_2^7 = 21 - 6 = 15$

8. $\int_2^5 (-3v+4)\,dv$ $\left[-\frac{3}{2}v^2+4v\right]_2^5 = -\frac{3}{2}(25)+4(5) - \left(-\frac{3}{2}(4)+4(2)\right) = -37.5+20-(-6+8) = -19.5$

7. $\int_{-1}^{0} (x-2)\,dx$ $\left[\frac{1}{2}x^2-2x\right]_{-1}^{0} = 0 - \left[\frac{1}{2}(-1)^2-2(-1)\right] = 0-\left(\frac{1}{2}+2\right) = -2.5$

9. $\int_{-1}^{1} (t^2-2)\,dt$ $\left[\frac{1}{3}t^3-2t\right]_{-1}^{1} = \left[\frac{1}{3}\cdot1^3-2\cdot1\right]-\left[\frac{1}{3}(-1)^3-2(-1)\right] = -1.\overline{6}-1.\overline{6} = -3.\overline{3}$

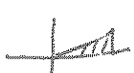
11. $\int_0^1 (2t-1)^2\,dt$ $4t^2-4t+1$ $\left[\frac{4}{3}t^3-\frac{4}{2}t^2+t\right]_0^1 = \left[\frac{4}{3}(1)^3-2(1)^2+1\right]-[0] = \frac{1}{3}$

13. $\int_1^2 \left(\frac{3}{x^2}-1\right)dx$ $3x^{-2}-1$ $= \left[\frac{3}{-1}x^{-1}-x\right]_1^2 = \left[-3\left(\frac{1}{2}\right)-2\right]-\left[-3\left(\frac{1}{1}\right)-(1)\right] = -3.5+4 = \frac{1}{2}$

$\frac{u}{u^{1/2}}-\frac{2}{u^{1/2}}$

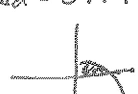
15. $\int_1^4 \frac{u-2}{\sqrt{u}}\,du$ $u^{1/2}-2u^{-1/2}$ $= \left[\frac{2}{3}u^{\frac{3}{2}}-2\left(\frac{2}{1}\right)u^{1/2}\right]_1^4 = \left[\frac{2}{3}(4)^{3/2}-4(4)^{1/2}\right]-\left[\frac{2}{3}(1)^{3/2}-4(1)^{1/2}\right] = \left[\frac{16}{3}-8\right]-\left[\frac{2}{3}-4\right] = 2/3$

17. $\int_{-1}^{1} (\sqrt[3]{t}-2)\,dt$ $t^{1/3}-2$ $= \left[\frac{3}{4}t^{4/3}-2t\right]_{-1}^{1} = \left[\frac{3}{4}(1)^{4/3}-2(1)\right]-\left[\frac{3}{4}(-1)^{4/3}-2(-1)\right] = \frac{-5}{4}-\frac{11}{4} = -4$

C 19. $\int_0^1 \frac{x-\sqrt{x}}{3}\,dx$  Area = −.0556

C 21. $\int_{-1}^{0} (t^{1/3}-t^{2/3})\,dt$  Area = −1.350

C 18. $\int_1^8 \sqrt{\frac{2}{x}}\,dx$  Area = 5.171

C 20. $\int_0^2 (2-t)\sqrt{t}\,dt$  Area = 1.508

C 22. $\int_{-8}^{-1} \frac{x-x^2}{2\sqrt[3]{x}}\,dx$  Area = 57.112

t (hours)	0	2	5	7	8	10
$v(t)$ (miles per hour)	50	55	60	70	65	75

C

3. The table above gives the velocity $v(t)$ at selected times t of a car traveling along a straight road. 2.2

(a) Use the values of the table to approximate the acceleration of the car at time $t = 6$. Show the work that leads to your answer and indicate units of measure. $\frac{70-60}{7-5} = \frac{10}{2} = 5$ mph/h 4.3

(b) Use a right Riemann sum with the subintervals given in the table to approximate $\int_0^{10} v(t)\,dt$. Indicate units of measure. What physical quantity does this integral represent?

B	2	3	2	1	2
H	55	60	70	65	75
Area	110	180	140	65	150

total: 645 3.4

(c) The function $v(t)$ is twice differentiable on the interval [0, 10]. Show that there must be a moment of time when the acceleration of the car is equal to zero. accel. from $t=5$ to $t=7$ is 5 mph/h accel. from $t=7$ to $t=8$ is -5 mph/h. $a(t)$ is diff, so it is cont. so $a(t)=0$ for $5<t<8$ by IVT.

---

4. $\int_1^3 \frac{8}{x^3}\,dx =$ $8x^{-3}$ $\left[\frac{8}{-2}x^{-2}\right]_1^3 = \left[-4/x^2\right]_1^3 = \frac{-4}{9} - \frac{-4}{1}$ 4.4

$-\frac{4}{9} + \frac{36}{9} = \frac{32}{9}$

(A) $\frac{32}{9}$ (B) $\frac{40}{9}$ (C) 0 (D) $-\frac{40}{9}$ (E) $-\frac{32}{9}$

5. $\int_1^4 \frac{dx}{\sqrt{x}} =$ $x^{-1/2}$ $\left[2x^{1/2}\right]_1^4 = 2(4)^{1/2} - 2(1)^{1/2} = 4 - 2 = 2$

4.4

(A) $\frac{1}{2}$

(B) 2

(C) 4

(D) $\frac{14}{3}$

(E) $\frac{21}{2}$

15. $\int_{e^{-1}}^{1} \frac{x^2 - x}{x^2}\,dx =$ $1 - x^{-1}$ $\left[x - \ln x\right]_{e^{-1}}^{1} = \left(1 - \ln 1\right) - \left[e^{-1} - \ln(e^{-1})\right]$

$[1 - 0] - e^{-1} - (-1)$

$1 - \frac{1}{e} + 1$

4.4

(A) $-\frac{1}{e}$

(B) $\frac{1}{e}$

(C) $2 - \frac{1}{e}$

(D) e

(E) $e^2 - \frac{1}{e}$

In Exercises 27–32, evaluate the definite integral of the trigonometric function. Use a graphing utility to verify your result.

27. $\displaystyle\int_0^{\pi} (1 + \sin x)\, dx$

C 28. $\displaystyle\int_0^{\pi/4} \frac{1 - \sin^2\theta}{\cos^2\theta}\, d\theta$

C 29. $\displaystyle\int_{-\pi/6}^{\pi/6} \sec^2 x\, dx$

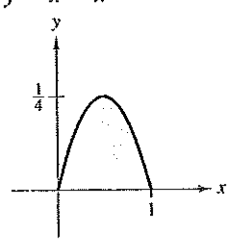
C 30. $\displaystyle\int_{\pi/4}^{\pi/2} (2 - \csc^2 x)\, dx$

C 31. $\displaystyle\int_{-\pi/3}^{\pi/3} 4 \sec\theta \tan\theta\, d\theta$

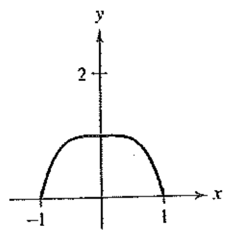
C 32. $\displaystyle\int_{-\pi/2}^{\pi/2} (2t + \cos t)\, dt$

In Exercises 35–40, determine the area of the indicated region.

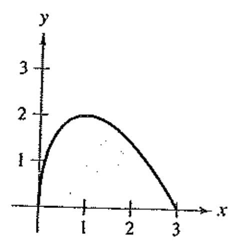
35. $y = x - x^2$



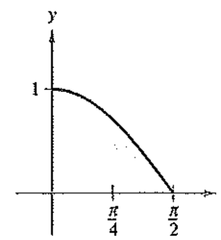
36. $y = 1 - x^4$



37. $y = (3 - x)\sqrt{x}$



39. $y = \cos x$



In Exercises 41–44, find the area of the region bounded by the graphs of the equations.

41. $y = 3x^2 + 1, \quad x = 0, \quad x = 2, \quad y = 0$

C 42. $y = 1 + \sqrt[3]{x} \quad x = 0, \quad x = 8, \quad y = 0$

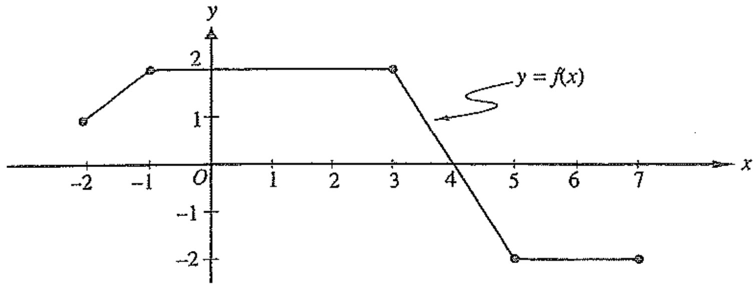
In Exercises 49–52, find the average value of the function over the interval and all values of x in the interval for which the function equals its average value.

Function	*Interval*
49. $f(x) = 4 - x^2$	$[-2, 2]$
C 50. $f(x) = \dfrac{4(x^2+1)}{x^2}$	$[1, 3]$
51. $f(x) = \sin x$	$[0, \pi]$
C 52. $f(x) = \cos x$	$[0, \pi/2]$

4.4

C 9. $\int_2^{e+1} \left(\frac{4}{x-1}\right) dx =$

(A) 4 (B) $4e$ (C) 0 (D) $-4e$ (E) -4



* Requires Notes from Day 4

21. The graph of a piecewise-linear function $f(x)$, for $-2 \le x \le 7$, is shown in the figure above. If $F(x) = \int_2^x f(t)\,dt$, which of the following statements is true?

(A) $F(2) > F(3) > F(7)$

(B) $F(2) > F(7) > F(3)$

(C) $F(3) > F(2) > F(7)$

(D) $F(3) > F(7) > F(2)$ 4.4

(E) $F(7) > F(2) > F(3)$

c

79. What is the average value of the function f defined by $f(x) = \sin(x^2)$ on the closed interval $[1, 3]$?

(A) 0.154

(B) 0.232 ~~3.12~~

(C) 0.463 4.4

(D) 0.696

(E) 1.392

x	0	1	2	3
$f(x)$	2	5	4	3

4.3

C

91. The function f is continuous on the closed interval $[0, 3]$ and has values that are given in the table above. Using the subintervals $[0, 1]$, $[1, 2]$, and $[2, 3]$, what is the trapezoidal approximation to $\int_0^3 f(x)\,dx$?

(A) 11

(B) 11.5

(C) 12

(D) 12.5

(E) 13

Calculus Worksheet 4-4B Name Key Pd____

In Exercises 27–32, evaluate the definite integral of the trigonometric function. Use a graphing utility to verify your result.

N.C. **27.** $\int_0^{\pi} (1 + \sin x)\,dx$ $\left[x - \cos x\right]_0^{\pi} = \left[\pi - \cos\pi\right] - \left[0 - \cos 0\right]$ $\pi - (-1) + 1 = 2 + \pi$

C **28.** $\int_0^{\pi/4} \frac{1 - \sin^2\theta}{\cos^2\theta}\,d\theta$ $.785$

C **29.** $\int_{-\pi/6}^{\pi/6} \sec^2 x\,dx$ $= \int \left(\frac{1}{\cos x}\right)^2 = 1.154$

C **30.** $\int_{\pi/4}^{\pi/2} (2 - \csc^2 x)\,dx$ $\int 2 - 1/(\sin(x)^2) = .570$

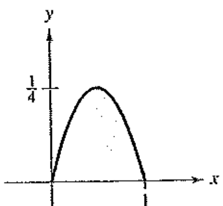
C **31.** $\int_{-\pi/3}^{\pi/3} 4\sec\theta\tan\theta\,d\theta$

$4(1/\cos\theta)\tan\theta = 0$

C **32.** $\int_{-\pi/2}^{\pi/2} (2t + \cos t)\,dt$ $= 2$

In Exercises 35–40, determine the area of the indicated region.

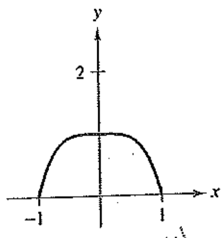
35. $y = x - x^2$



$\int_0^1 (x - x^2)\,dx = \left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1$

$\left[\frac{1}{2} - \frac{1}{3}\right] - [0 - 0] = \frac{1}{6}$

36. $y = 1 - x^4$

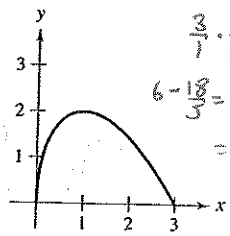


$\int_{-1}^{1} (1 - x^4) = \left[x - \frac{1}{5}x^5\right]_{-1}^{1}$

$\left[1 - \frac{1}{5}\right] - \left[-1 - \frac{1}{5}(-1)\right]$

$\frac{4}{5} - \left(-\frac{4}{5}\right) = \frac{8}{5}$

37. $y = (3 - x)\sqrt{x}$



$\frac{3}{1} \cdot \frac{2}{3} = 2$

$6 - \frac{18}{5} = \frac{30}{5} - \frac{18}{5}$

$= \frac{12}{5}$

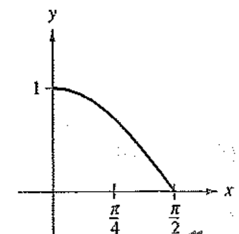
$\int_0^3 (3x^{1/2} - x^{3/2})\,dx = \left[\frac{3}{\frac{3}{2}}x^{3/2} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^3$

$\left[2(3)^{3/2} - \frac{2}{5}(3)^{\frac{5}{2}}\right] - [0 - 0]$

$2 \cdot 3\sqrt{3} - \frac{2}{5} \cdot 9\sqrt{3}$

$6\sqrt{3} - \frac{18}{5}\sqrt{3} = \frac{12}{5}\sqrt{3}$

39. $y = \cos x$



$\int_0^{\frac{\pi}{2}} \cos x\,dx$

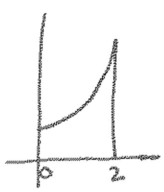
$\left[\sin x\right]_0^{\pi/2}$

$\sin\frac{\pi}{2} - \sin 0$

$1 - 0 = 1$

In Exercises 41–44, find the area of the region bounded by the graphs of the equations.

41. $y = 3x^2 + 1,\quad x = 0,\quad x = 2,\quad y = 0$

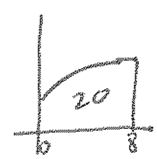


$\int_0^2 (3x^2 + 1)\,dx = \left[\frac{3}{3}x^3 + x\right]_0^2$

$\left[2^3 + 2\right] - \left[0^3 + 0\right]$

$8 + 2 = 10$

C **42.** $y = 1 + \sqrt[3]{x}\quad x = 0,\quad x = 8,\quad y = 0$



In Exercises 49–52, find the average value of the function over the interval and all values of x in the interval for which the function equals its average value.

Function	*Interval*
49. $f(x) = 4 - x^2$	$[-2, 2]$

$\frac{24}{3} - \frac{8}{3} = \frac{16}{3}$

$\frac{-24}{3} + \frac{8}{3} = \frac{-16}{3}$

$\frac{1}{2 - -2}\int_{-2}^{2} (4 - x^2)\,dx = \frac{1}{4}\left[4x - \frac{1}{3}x^3\right]_{-2}^{2}$

$\frac{1}{4}\left[4(2) - \frac{1}{3}(2)^3\right] - \frac{1}{4}\left[4(-2) - \frac{1}{3}(-2)^3\right] = \frac{1}{4}\left[\frac{16}{3}\right] - \frac{1}{4}\left[\frac{-16}{3}\right] = \frac{4}{3} + \frac{4}{3} = \frac{8}{3}$

C **50.** $f(x) = \frac{4(x^2 + 1)}{x^2}$ $[1, 3]$

$\frac{1}{3 - 1}\int_1^3 f(x)\,dx = \frac{1}{2}\left[10.\overline{6}\right] = 5.\overline{3}$

51. $f(x) = \sin x$ $[0, \pi]$

$\frac{1}{\pi - 0}\int_0^{\pi} \sin x\,dx = \frac{1}{\pi}\left[-\cos x\right]_0^{\pi}$

$\frac{1}{\pi}\left[-\cos\pi\right] - \frac{1}{\pi}\left[-\cos 0\right] = \frac{1}{\pi}(1) - \frac{1}{\pi}(-1)$

$\frac{1}{\pi} + \frac{1}{\pi} = \frac{2}{\pi}$

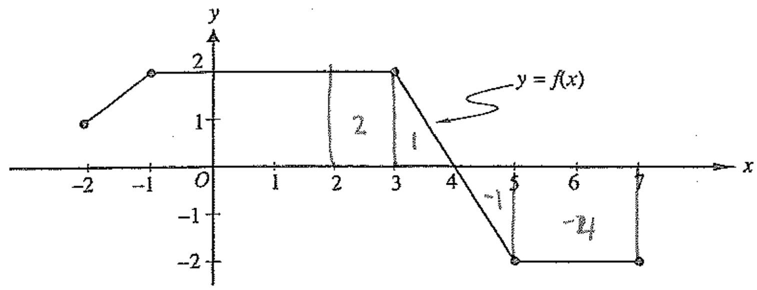
C **52.** $f(x) = \cos x$ $[0, \pi/2]$

$\frac{1}{\frac{\pi}{2} - 0}\int_0^{\frac{\pi}{2}} \cos x\,dx = \frac{1}{\frac{\pi}{2}}[1] = \frac{2}{\pi}$

4.4

C 9. $\int_{2}^{e+1} \left(\frac{4}{x-1}\right) dx =$

(A) 4 (B) $4e$ (C) 0 (D) $-4e$ (E) -4



21. The graph of a piecewise-linear function $f(x)$, for $-2 \le x \le 7$, is shown in the figure above. If $F(x) = \int_2^x f(t)\,dt$, which of the following statements is true?

Area from 2 to x

(A) $F(2) > F(3) > F(7)$

(B) $F(2) > F(7) > F(3)$

(C) $F(3) > F(2) > F(7)$

(D) $F(3) > F(7) > F(2)$

(E) $F(7) > F(2) > F(3)$

$F(2) = \int_2^2 f(t)dt = 0$
$F(3) = \int_2^3 f(t)dt = 2$
$F(7) = \int_2^7 f(t)dt = 3 + -5 = -2$

4.4

C

79. What is the average value of the function f defined by $f(x) = \sin(x^2)$ on the closed interval $[1, 3]$?

(A) 0.154

(B) 0.232

(C) 0.463

(D) 0.696

(E) 1.392

$\frac{1}{3-1}\int_1^3 \sin(x^2)dx$

$\frac{1}{2}[.463) = .232$

4.4

x	0	1	2	3
$f(x)$	2	5	4	3

4.3

C

91. The function f is continuous on the closed interval $[0, 3]$ and has values that are given in the table above. Using the subintervals $[0, 1]$, $[1, 2]$, and $[2, 3]$, what is the trapezoidal approximation to $\int_0^3 f(x)\,dx$?

(A) 11

(B) 11.5

(C) 12

(D) 12.5

(E) 13

	ave H	W	Area
0 to 1	3.5	1	3.5
1 to 2	4.5	1	4.5
2 to 3	3.5	1	3.5
			11.5

Calculus Worksheet 4-4C Name______________________________ Pd______

In Exercises 69–74, find F as a function of x and evaluate it at $x = 2, x = 5,$ and $x = 8$.

69. $F(x) = \int_0^x (t - 5)\,dt$

C

70. $F(x) = \int_2^x (t^3 + 2t - 2)\,dt$

71. $F(x) = \int_1^x \frac{10}{v^2}\,dv$

C

72. $F(x) = \int_2^x -\frac{2}{t^3}\,dt$

In Exercises 75–80, (a) integrate to find F as a function of x and (b) demonstrate the Second Fundamental Theorem of Calculus by differentiating the result in part (a).

75. $F(x) = \int_0^x (t + 2)\,dt$

76. $F(x) = \int_0^x t(t^2 + 1)\,dt$

77. $F(x) = \int_8^x \sqrt[3]{t}\,dt$

~~**78.**~~ $F(x) = \int_4^x \sqrt{t}\,dt$

In Exercises 81–86, use the Second Fundamental Theorem of Calculus to find $F'(x)$.

81. $F(x) = \int_{-2}^x (t^2 - 2t)\,dt$

82. $F(x) = \int_1^x \frac{t^2}{t^2 + 1}\,dt$

85. $F(x) = \int_0^x t \cos t\,dt$

86. $F(x) = \int_0^x \sec^3 t\,dt$

In Exercises 87–92, find $F'(x)$.

87. $F(x) = \int_x^{x+2} (4t + 1)\,dt$

88. $F(x) = \int_{-x}^x t^3\,dt$

~~**91.**~~ $F(x) = \int_0^{x^3} \sin t^2\,dt$

~~**92.**~~ $F(x) = \int_0^{x^2} \sin \theta^2\,d\theta$

x	0	1	3	7	10
$f(x)$	1	-1	4	2	3

4.3

19. The function f is continuous on the closed interval $[0, 10]$ and has values given in the table above. Using the subintervals $[0, 1]$, $[1, 3]$, $[3, 7]$, and $[7, 10]$, what is the left Riemann sum estimate for $\int_0^{10} f(x)\,dx$?

(A) 15

(B) 17.5

(C) 20

(D) 21

(E) 22.5

C

11. The area of the region in the first quadrant bounded by the graph of $y = x\sqrt{9+x^2}$, the x-axis, and the line $x = 4$ is

(A) $9\sec^3 4 - 9$

(B) $\frac{16}{3}$

(C) $\frac{13}{3}\sqrt{13} - 9$

(D) $\frac{64}{3}$

(E) $\frac{98}{3}$

4.4

53. State the Fundamental Theorem of Calculus.

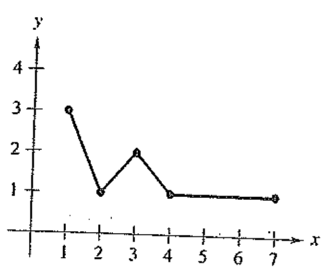


Figure for 54

54. The graph of f is given in the figure.

(a) Evaluate $\int_1^7 f(x)\,dx$.

(b) Determine the average value of f on the interval $[1, 7]$.

(c) Determine the answers to parts (a) and (b) if the graph is translated two units upward.

In Exercises 55–60, use the graph of f shown in the figure. The shaded region A has an area of 1.5, and $\int_0^6 f(x)\,dx = 3.5$. Use this information to fill in the blanks.

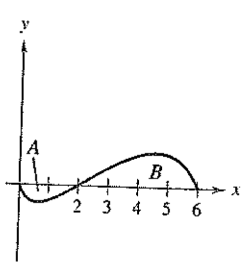


Figure for 55–60

59. $\int_0^6 [2 + f(x)]\,dx =$

60. The average value of f over the interval $[0, 6]$ is .

Calculus Worksheet 4-4C Name Key Pd____

In Exercises 69–74, find F as a function of x and evaluate it at $x = 2$, $x = 5$, and $x = 8$.

69. $F(x) = \int_0^x (t-5)\,dt$

$F(x) = \left[\frac{1}{2}t^2 - 5t\right]_0^x$

$F(x) = \frac{1}{2}x^2 - 5x$

$F(2) = \frac{1}{2}(2)^2 - 5(2) = -8$

$F(5) = \frac{1}{2}(5)^2 - 5(5) = 2.5$

$F(8) = \frac{1}{2}(8)^2 - 5(8) = -8$

C

70. $F(x) = \int_2^x (t^3 + 2t - 2)\,dt$

$F(x) = \left[\frac{1}{4}t^4 + t^2 - 2t\right]_2^x$

$F(x) = \frac{1}{4}x^4 + x^2 - 2x - [8 + 4 - 2]$

$F(x) = \frac{1}{4}x^4 + x^2 - 2x - 10$

$F(2) = 0$

$F(5) = \frac{1}{4}(625) + 25 - 10 - 10 = 161.25$

$F(8) = \frac{1}{4}(4096) + 64 - 16 - 10 = 1062$

$10v^{-2}$

71. $F(x) = \int_1^x \frac{10}{v^2}\,dv$

$F(x) = \left[-10v^{-1}\right]_1^x$

$F(x) = \frac{-10}{x} + 10$

$F(2) = \frac{-10}{2} + 10 = 5$

$F(5) = \frac{-10}{5} + 10 = 8$

$F(8) = \frac{-10}{8} + 10 = -1.25 + 10 = 8.75$

$-2t^{-3}$

C

72. $F(x) = \int_2^x -\frac{2}{t^3}\,dt$

$F(x) = \left[\frac{-4}{-2}t^{-2}\right]_2^x$

$F(x) = 2/x^2 - 2/4 = \frac{2}{x^2} - \frac{1}{2}$

$F(2) = \frac{2}{4} - \frac{1}{2} = 0$

$F(5) = \frac{2}{25} - \frac{1}{2} = -.42$

$F(8) = \frac{2}{64} - \frac{1}{2} = -.468$

In Exercises 75–80, (a) integrate to find F as a function of x and (b) demonstrate the Second Fundamental Theorem of Calculus by differentiating the result in part (a).

75. $F(x) = \int_0^x (t+2)\,dt$

$F(x) = \left[\frac{1}{2}t^2 + 2t\right]_0^x$

$F(x) = \frac{1}{2}x^2 + 2x - 0$

$F'(x) = x + 2$

$t^3 + t$

76. $F(x) = \int_0^x t(t^2+1)\,dt$

$F(x) = \left[\frac{1}{4}t^4 + t\right]_0^x$

$F(x) = \frac{1}{4}x^4 + \frac{1}{2}x^2 - 0$

$F'(x) = x^3 + x$

$= x(x^2+1)$

$t^{1/3}$

77. $F(x) = \int_8^x \sqrt[3]{t}\,dt$

$F(x) = \left[\frac{3}{4}t^{4/3}\right]_8^x$

$F(x) = \frac{3}{4}x^{4/3} - \frac{3}{4}(8)^{4/3}$

$F'(x) = x^{1/3} - 0$

$= \sqrt[3]{x}$

78. $F(x) = \int_4^x \sqrt{t}\,dt$

$t^{1/2}$

In Exercises 81–86, use the Second Fundamental Theorem of Calculus to find $F'(x)$.

81. $F(x) = \int_{-2}^x (t^2 - 2t)\,dt$

$F'(x) = x^2 - 2x$

82. $F(x) = \int_1^x \frac{t^2}{t^2+1}\,dt$

$F'(x) = \frac{x^2}{x^2+1}$

85. $F(x) = \int_0^x t\cos t\,dt$

$F'(x) = x\cos x$

86. $F(x) = \int_0^x \sec^3 t\,dt$

$F'(x) = \sec^3 x$

In Exercises 87–92, find $F'(x)$.

87. $F(x) = \int_x^{x+2} (4t+1)\,dt$

$F(x) = \left[2t^2 + t\right]_x^{x+2}$

$= 2(x+2)^2 + (x+2) - 2x^2 - x$

$F'(x) = 4(x+2)^1 \cdot 1 + 1 - 4x - 1$

$4x + 8 - 4x$

$F'(x) = 8$

88. $F(x) = \int_{-x}^x t^3\,dt$

$F(x) = \left[\frac{1}{4}t^4\right]_{-x}^x = \frac{1}{4}x^4 - \frac{1}{4}x^4$

$F(x) = 0$

$F'(x) = 0$

91. $F(x) = \int_0^{x^3} \sin t^2\,dt$

$F(x) =$

92. $F(x) = \int_0^{x^2} \sin\theta^2\,d\theta$

$2x \sin x^4$

x	0	1	3	7	10
$f(x)$	1	-1	4	2	3

4.3

19. The function f is continuous on the closed interval $[0, 10]$ and has values given in the table above. Using the subintervals $[0, 1]$, $[1, 3]$, $[3, 7]$, and $[7, 10]$, what is the left Riemann sum estimate for $\int_0^{10} f(x)\,dx$?

(A) 15

(B) 17.5

(C) 20

(D) 21

(E) 22.5

	L Hght	W	Area
0 to 1	1	1	1
1 to 3	-1	2	-2
3 to 7	4	4	16
7 to 10	2	3	6

Area: 21

C

11. The area of the region in the first quadrant bounded by the graph of $y = x\sqrt{9 + x^2}$, the x-axis, and the line $x = 4$ is

(A) $9 \sec^3 4 - 9$

(B) $\frac{16}{3}$

(C) $\frac{13}{3}\sqrt{13} - 9$

(D) $\frac{64}{3}$

(E) $\frac{98}{3}$

$\int$

$32.\overline{6}$

4.4

53. State the Fundamental Theorem of Calculus.

a) $2+1.5+1.5+3 = 8$

b) $\frac{1}{7-1}\int_1^7 f(x)dx = \frac{1}{6}\cdot 8 = \frac{8}{6}$

c) $8+12 = 20$ area

$\frac{20}{6}$ = ave. val.

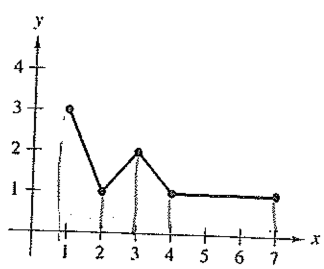


Figure for 54

54. The graph of f is given in the figure.

(a) Evaluate $\int_1^7 f(x)\,dx$.

(b) Determine the average value of f on the interval $[1, 7]$.

(c) Determine the answers to parts (a) and (b) if the graph is translated two units upward.

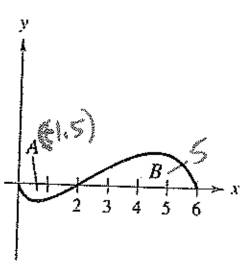


Figure for 55–60

In Exercises 55–60, use the graph of f shown in the figure. The shaded region A has an area of 1.5, and $\int_0^6 f(x)\,dx = 3.5$. Use this information to fill in the blanks.

59. $\int_0^6 [2 + f(x)]\,dx =$ ____ $\int_0^6 2dx + \int_0^6 f(x)dx = 12 + 3.5 = 15.5$

60. The average value of f over the interval $[0, 6]$ is ____ $\frac{1}{6-0}\int_0^6 f(x)dx = \frac{1}{6}(3.5) = .58\overline{3}$