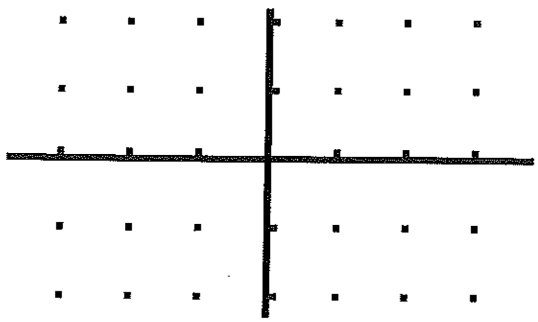
Name

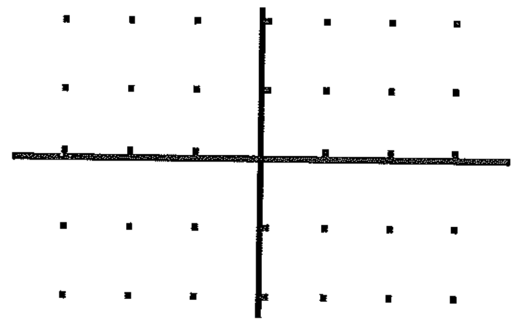
SLOPE FIELDS

Draw a slope field for each of the following differential equations.

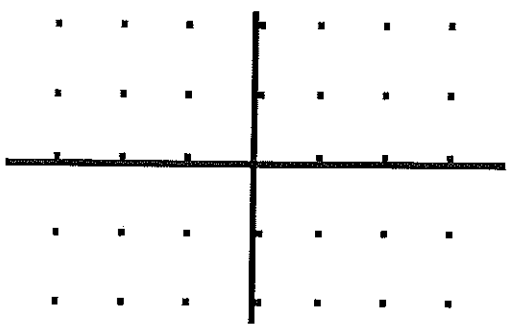
1. $\dfrac{dy}{dx} = x+1$



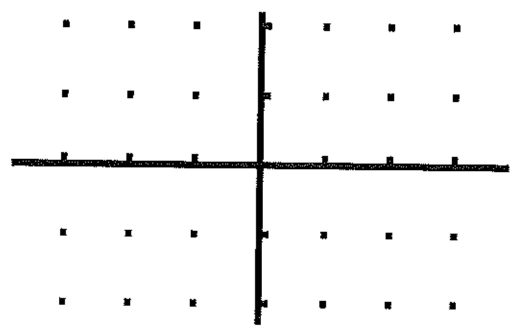
2. $\dfrac{dy}{dx} = 2y$



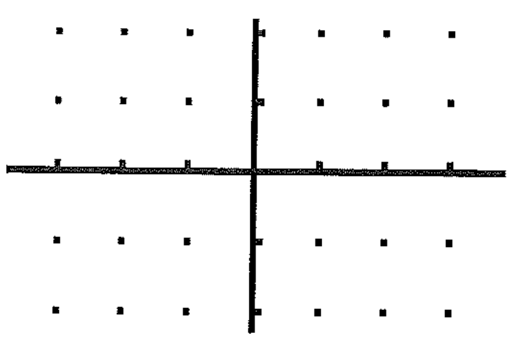
3. $\dfrac{dy}{dx} = x+y$



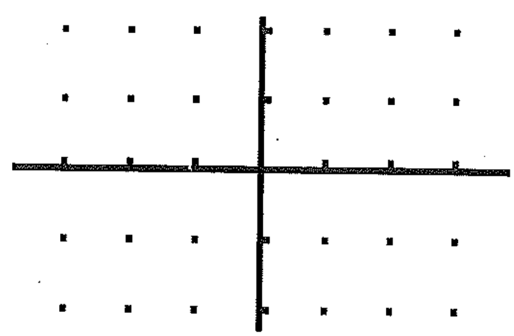
4. $\dfrac{dy}{dx} = 2x$



5. $\dfrac{dy}{dx} = y-1$

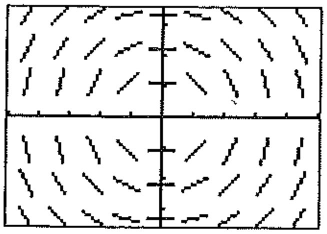


6. $\dfrac{dy}{dx} = -\dfrac{y}{x}$

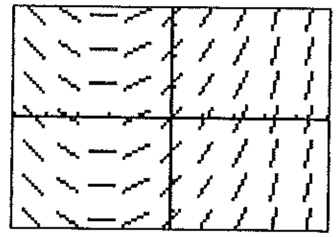


Match the slope fields with their differential equations.

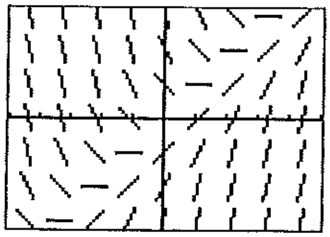
(A)



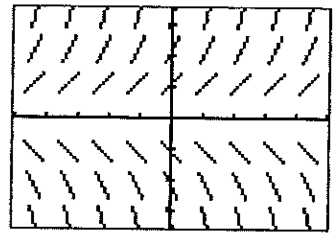
(B)



(C)



(D)



15. $\dfrac{dy}{dx} = \dfrac{1}{2}x + 1$

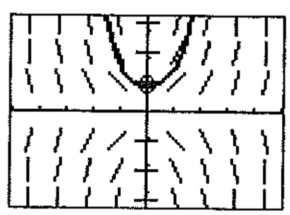
16. $\dfrac{dy}{dx} = y$

17. $\dfrac{dy}{dx} = x - y$

18. $\dfrac{dy}{dx} = -\dfrac{x}{y}$

---

19. The calculator drawn slope field for the differential equation $\dfrac{dy}{dx} = xy$ is shown in the figure below. The solution curve passing through the point (0, 1) is also shown.
    (a) Sketch the solution curve through the point (0, 2).
    (b) Sketch the solution curve through the point $(0, -1)$.



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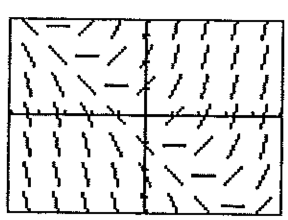
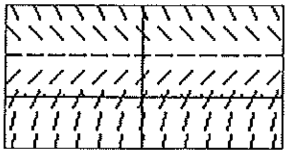
20. The calculator drawn slope field for the differential equation $\dfrac{dy}{dx} = x + y$ is shown in the figure below.
    (a) Sketch the solution curve through the point (0, 1).
    (b) Sketch the solution curve through the point $(-3, 0)$.

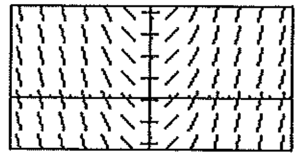


Match the slope fields with their differential equations. Name ________

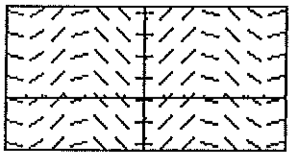
(A)



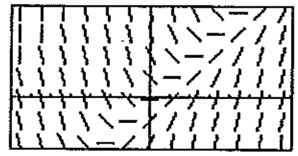
(B)



(C)



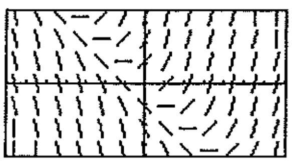
(D)



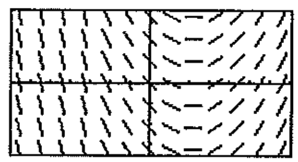
7. $\dfrac{dy}{dx} = \sin x$ 8. $\dfrac{dy}{dx} = x - y$ 9. $\dfrac{dy}{dx} = 2 - y$ 10. $\dfrac{dy}{dx} = x$

Match the slope fields with their differential equations.

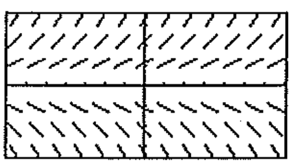
(A)



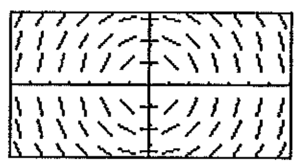
(B)



(C)



(D)



11. $\dfrac{dy}{dx} = .5x - 1$ 12. $\dfrac{dy}{dx} = .5y$ 13. $\dfrac{dy}{dx} = -\dfrac{x}{y}$ 14. $\dfrac{dy}{dx} = x + y$

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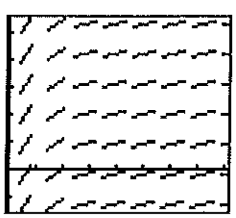
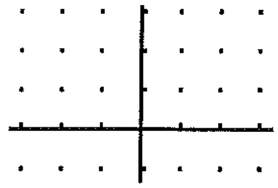
15. (From the AP Calculus Course Description)



The slope field from a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$ (B) $y = e^x$ (C) $y = e^{-x}$ (D) $y = \cos x$ (E) $y = \ln x$

19. Consider the differential equation given by $\frac{dy}{dx}=\frac{2x}{x^2+1}$.



(a) On the axes provided, sketch a slope field for the given differential equation.
(b) Sketch a solution curve that passes through the point (0, 1) on your slope field.
(c) Find $\frac{d^2y}{dx^2}$. For what values of x is the graph of the solution $y=f(x)$ concave up? Concave down?

---

20. Consider the logistic differential equation $\frac{dy}{dt}=\frac{1}{2}y(2-y)$;
(a) On the axes provided, sketch a slope field for the given differential equation.

(b) Sketch a solution curve that passes through the point (4, 1) on your slope field.
(c) Show that $y=\frac{2}{1+2e^{-t}}$ satisfies the given differential equation.
(d) Find $\lim_{t\to\infty} y$ by using the solution curve given in part (c).
(e) Find $\frac{d^2y}{dt^2}$. For what values of y, $0<y<2$, does the graph of $y=f(t)$ have an inflection point?

---

21. (a) On the slope field for $\frac{dP}{dt}=3P-3P^2$, sketch three solution curves showing different types of behavior for the population P.
(b) Is there a stable value of the population? If so, what is it?
(c) Describe the meaning of the shape of the solution curves for the population: Where is P increasing? Decreasing? What happens in the long run? Are there any inflection points? Where? What do they mean for the population?
(d) Sketch a graph of $\frac{dP}{dt}$ against P. Where is $\frac{dP}{dt}$ positive? Negative? Zero? Maximum? How do your observations about $\frac{dP}{dt}$ explain the shapes of your solution curves?

(Problem 21 is from Calculus (Third Edition) by Hughes-Hallett, Gleason, et al)

The solution curves are parabolas.

C10

The solution curves are hyperbolas.

C3

$\lim_{x \to \infty} y = 2$

C8

If $y > 0$ and $x \neq 0$, the solution curve is concave up. If $y < 0$ and $x \neq 0$, the solution curve is concave down.

C9

The solution curve that passes through the point $(0, -1)$ is the line $y = x - 1$.

C2

The solution curve that passes through the point (1, 1) has a local maximum at (1, 1).

C5

The solution curves have a horizontal asymptote only at $y = 0$.

C6

The solution curve that passes through the point $(-1, 0)$ is the line $y = -x - 1$.

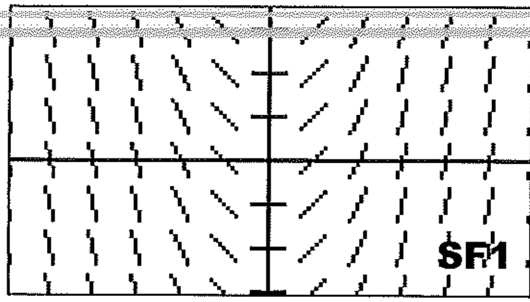
C4

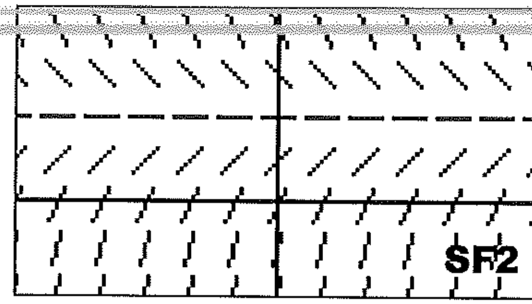
The solution curves are circles.

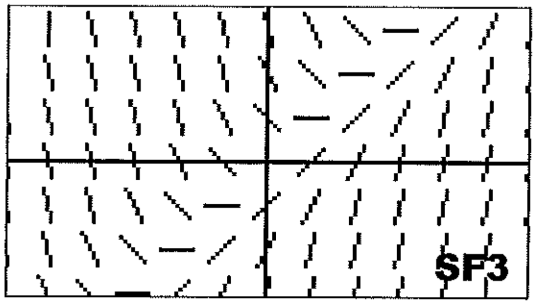
C1

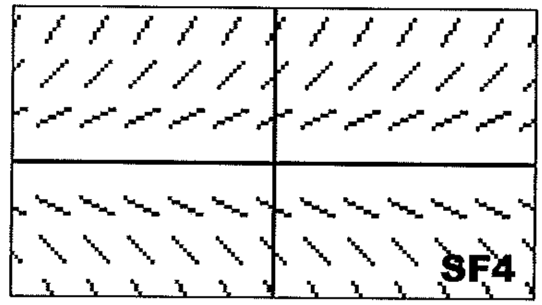
For $y > 0$, the solution curve is logistic and has two horizontal asymptotes.

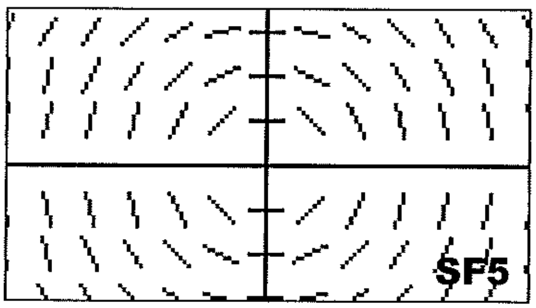
C7

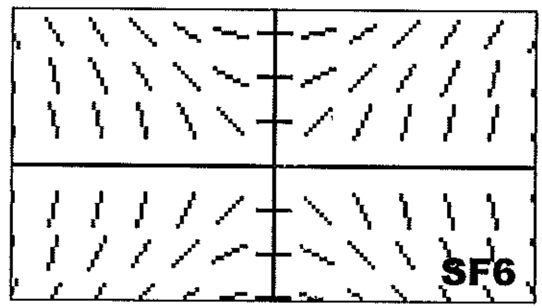

SF1

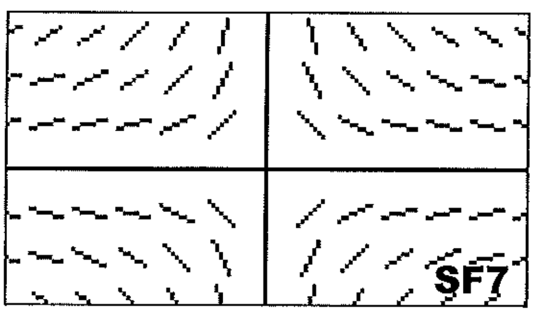

SF2

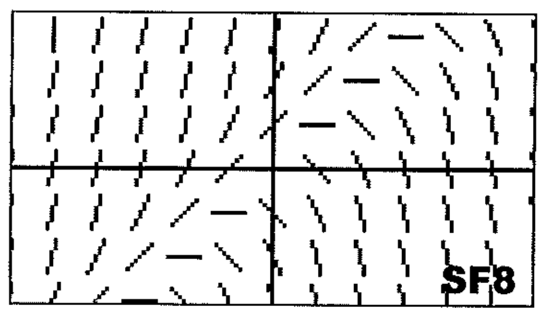

SF3

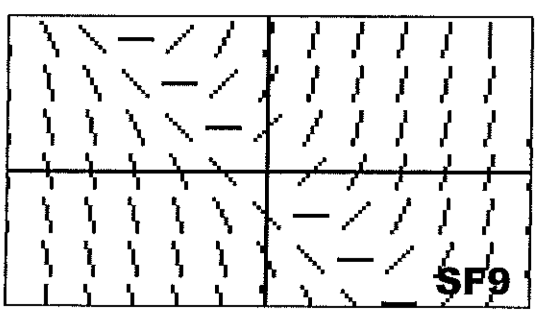

SF4

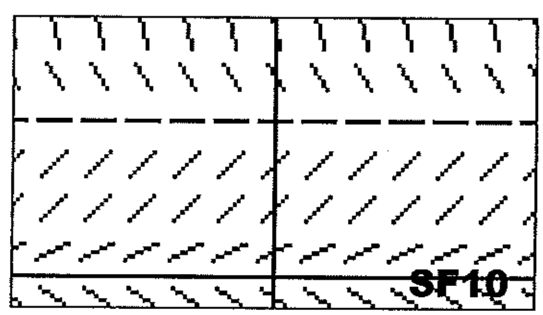

SF5


SF6


SF7


SF8


SF9


SF10

SLOPE FIELD CARD MATCH

Slope Fields	Differential Equations	Conclusions
SF 1		
SF 2		
SF 3		
SF 4		
SF 5		
SF 6		
SF 7		
SF 8		
SF 9		
SF 10		

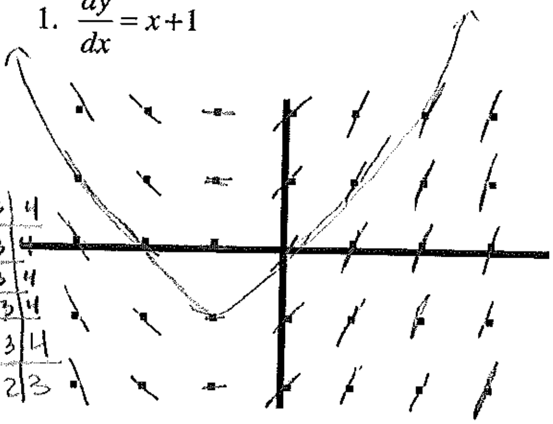
30 27 Key

SLOPE FIELDS

Draw a slope field for each of the following differential equations.

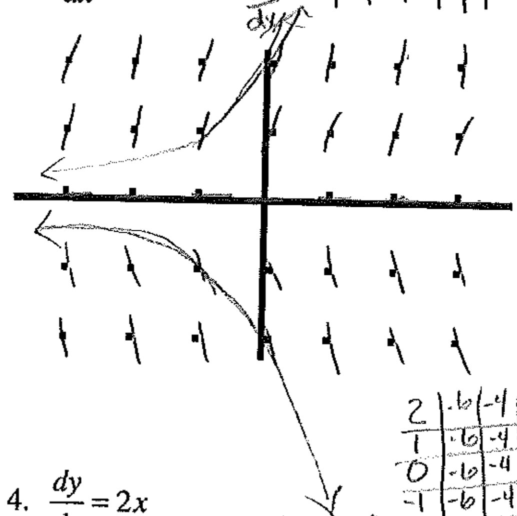
1. $\dfrac{dy}{dx} = x+1$

2	-2	-1	0	1	2	3	4
1	-2	-1	0	1	2	3	4
0	-2	-1	0	1	2	3	4
-1	-2	-1	0	1	2	3	4
-2	-2	-1	0	1	2	3	4
dy/dx	-3	-2	-1	0	1	2	3



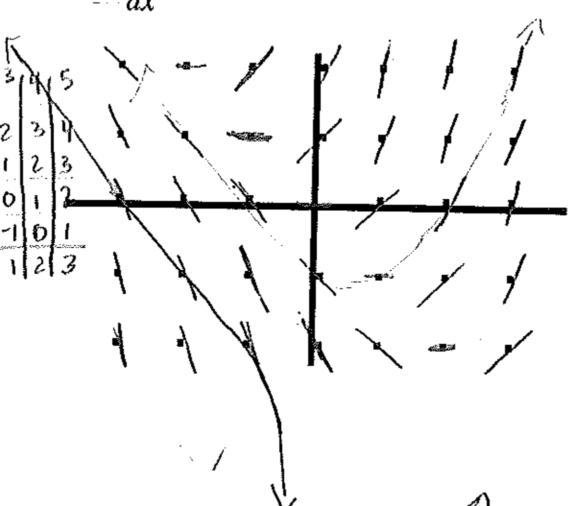
2. $\dfrac{dy}{dx} = 2y$

2	4	4	4	4	4	4	4
1	2	2	2	2	2	2	2
0	0	0	0	0	0	0	0
-1	-2	-2	-2	-2	-2	-2	-2
-2	-4	-4	-4	-4	-4	-4	-4
dy/dx	-3	-2	-1	0	1	2	3



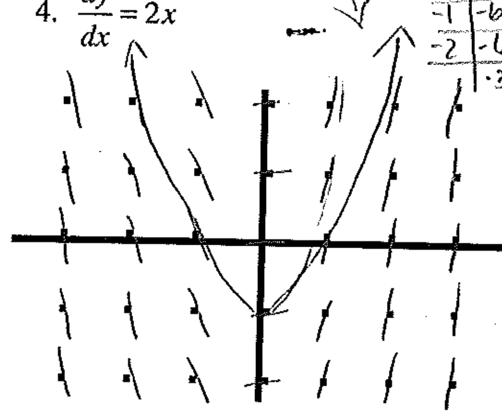
3. $\dfrac{dy}{dx} = x+y$

[illegible]	-1	0	1	2	3	4	5
[illegible]	-2	-1	0	1	2	3	4
0	[illegible]	-2	-1	0	1	2	3
-1	[illegible]	-3	-2	-1	0	1	2
-2	-5	-4	-3	-2	-1	0	1
y/x	-3	-2	-1	0	1	2	3



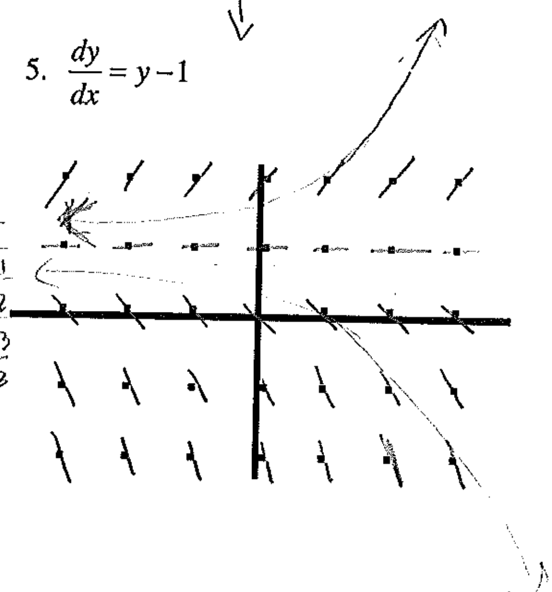
4. $\dfrac{dy}{dx} = 2x$

2	-6	-4	-2	0	2	4	6
1	-6	-4	-2	0	2	4	6
0	-6	-4	-2	0	2	4	6
-1	-6	-4	-2	0	2	4	6
-2	-6	-4	-2	0	2	4	6
	-3	-2	-1	0	1	2	3



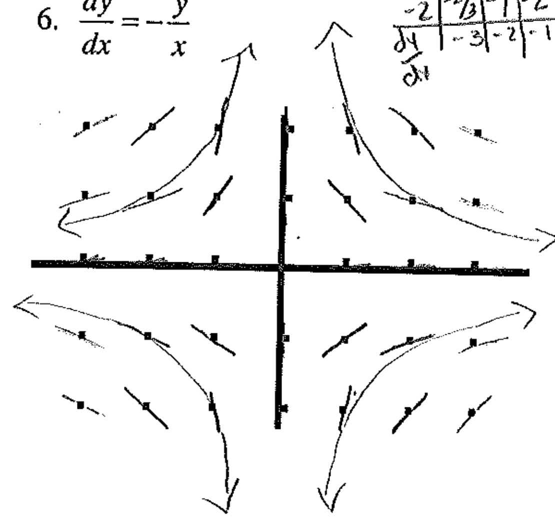
5. $\dfrac{dy}{dx} = y-1$

1	1	1	1	1	1	1
0	0	0	0	0	0	0
-1	-1	-1	-1	-1	-1	-1
-2	-2	-2	-2	-2	-2	-2
-3	-3	-3	-3	-3	-3	-3
-3	-2	-1	0	1	2	3



6. $\dfrac{dy}{dx} = -\dfrac{y}{x}$

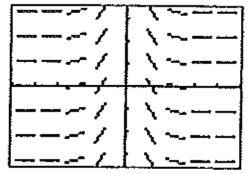
2	2/3	1	2	[illegible]	-2	-1	-2/3
1	1/3	1/2	1	[illegible]	-1	-1/2	-1/3
0	0	0	0	[illegible]	0	0	0
-1	-1/3	-1/2	-1	[illegible]	1	1/2	1/3
-2	-2/3	-1	-2	[illegible]	2	1	2/3
dy/dx	-3	-2	-1	0	1	2	3



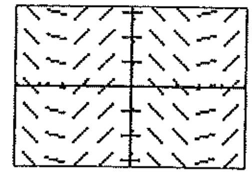
42 6

Match each slope field with the equation that the slope field could represent.

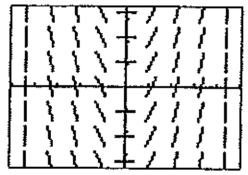
(A)



(B)



(C)



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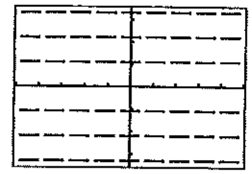
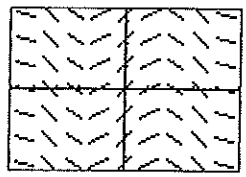
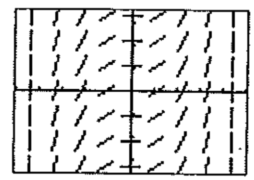


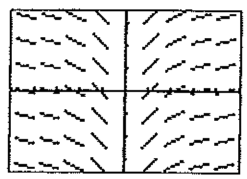
(E)



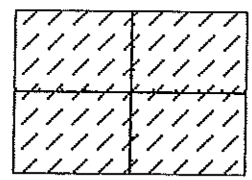
(F)



(G)



(H)



7. $y = 1$ D

8. $y = x$ H

9. $y = x^2$ C

10. $y = \frac{1}{6}x^3$ F

11. $y = \frac{1}{x^2}$ A

12. $y = \sin x$ E

13. $y = \cos x$ B

14. $y = \ln|x|$ G

$\frac{1}{y}dy = x\,dx$

$\overline{8}$

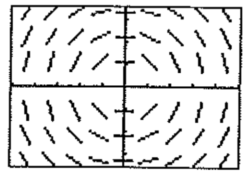
$\ln y = \frac{1}{2}x^2 + C$

$\cancel{y = C \cdot \frac{1}{2}x^2}$

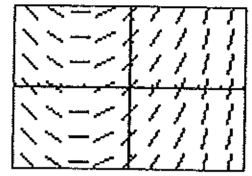
$y = C \cdot e^{\frac{1}{2}x^2}$

Match the slope fields with their differential equations.

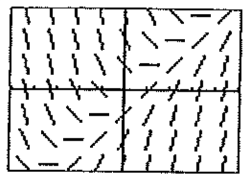
(A)



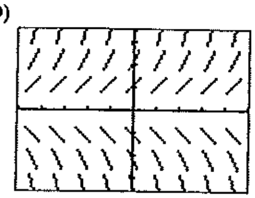
(B)



(C)



(D)



4

15. $\frac{dy}{dx} = \frac{1}{2}x + 1$ B

16. $\frac{dy}{dx} = y$ D

17. $\frac{dy}{dx} = x - y$ C

18. $\frac{dy}{dx} = -\frac{x}{y}$ A

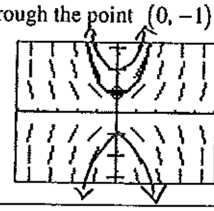
---

19. The calculator drawn slope field for the differential equation $\frac{dy}{dx} = xy$ is shown in the figure below. The solution curve passing through the point (0, 1) is also shown.

2

(a) Sketch the solution curve through the point (0, 2).

(b) Sketch the solution curve through the point $(0, -1)$.



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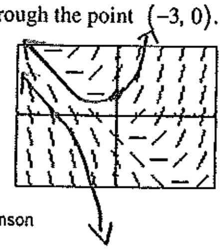
20. The calculator drawn slope field for the differential equation $\frac{dy}{dx} = x + y$ is shown in the figure below.

2

(a) Sketch the solution curve through the point (0, 1).

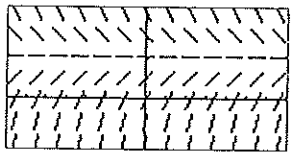
(b) Sketch the solution curve through the point $(-3, 0)$.



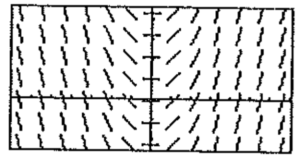
8

Match the slope fields with their differential equations. Key

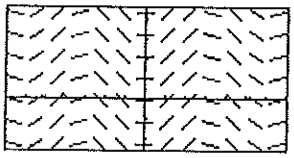
(A)



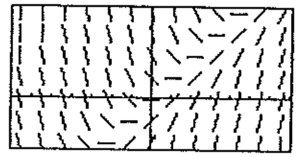
(B)



(C)



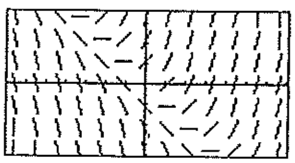
(D)



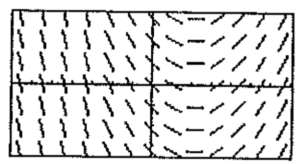
7. $\frac{dy}{dx} = \sin x$ C
8. $\frac{dy}{dx} = x - y$ D
9. $\frac{dy}{dx} = 2 - y$ A
10. $\frac{dy}{dx} = x$ B

Match the slope fields with their differential equations.

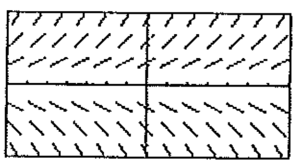
(A)



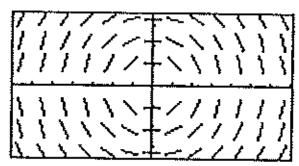
(B)



(C)

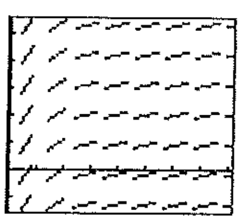


(D)



11. $\frac{dy}{dx} = .5x - 1$ B
12. $\frac{dy}{dx} = .5y$ C
13. $\frac{dy}{dx} = -\frac{x}{y}$ D
14. $\frac{dy}{dx} = x + y$ A

15. (From the AP Calculus Course Description)



The slope field from a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$ (B) $y = e^x$ (C) $y = e^{-x}$ (D) $y = \cos x$ (E) $y = \ln x$

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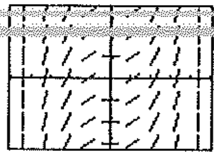
16.



The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(1)

(A) $y = \sin x$ (B) $y = \cos x$ (C) $y = x^2$ (D) $y = \frac{1}{6}x^3$ (E) $y = \ln x$

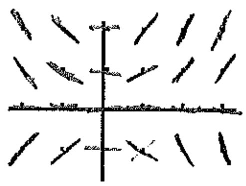
---

17. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

$\ln|y| = \frac{1}{4}x^2 - \frac{1}{4}$

(a) On the axes provided, sketch a slope field for the given differential equation.

$|y| = e^{\frac{1}{4}x^2 - \frac{1}{4}}$



c.) $\frac{1}{y}\,dy = \frac{1}{2}x\,dx$

$\ln|y| = \frac{1}{4}x^2 + C$

$\ln|1| = \frac{1}{4} + C$

$C = -\frac{1}{4}$

$y = \pm e^{-1/4} \cdot e^{\frac{1}{4}x^2}$

$f(1.2) = 1.416$

(b) Let f be the function that satisfies the given differential equation. Write an equation for the tangent line to the curve $y = f(x)$ through the point (1, 1). Then use your tangent line equation to estimate the value of $f(1.2)$

$y - 1 = \frac{1}{2}(x - 1)$

$y - 1 = \frac{1}{2}(1.2 - 1)$ $f(1.2) = 1.1$

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$. Use your solution to find $f(1.2)$.

(d) Compare your estimate of $f(1.2)$ found in part (b) to the actual value of $f(1.2)$ found in part (c). Was your estimate from part (b) an underestimate or an overestimate? Use your slope field to explain why.

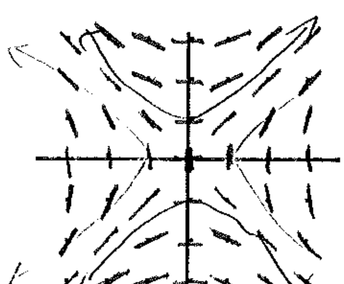
f(1.2) is an underestimate at x=1.2 f(x) is concave up ∴ The tangent line would have a smaller y-value at x=1.2.

---

18. Consider the differential equation given by $\frac{dy}{dx} = \frac{x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation.

(1)



c.) $y\,dy = x\,dx$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$

$C = \frac{1}{2}$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + \frac{1}{2}$

$y^2 = x^2 + 1$

(1) (b) Sketch a solution curve that passes through the point (0, 1) on your slope field.

(2) (c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = 1$.

(1) (d) Sketch a solution curve that passes through the point $(0, -1)$ on your slope field.

(2) (e) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = -1$.

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$

$\frac{1}{2} = C$

$y^2 = x^2 + 1$

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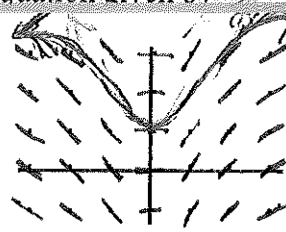
19. Consider the differential equation given by $\frac{dy}{dx} = \frac{2x}{x^2+1}$.

$-2x^2+2=0$
$-2(x^2-1)=0$
$x=\pm 1$



$f(x)$ is conc. up on $(-1,1)$ because $f''(x)>0$. $f(x)$ is conc. down on $(-\infty,-1)\cup(1,\infty)$ because $f''(x)<0$.

1 (a) On the axes provided, sketch a slope field for the given differential equation.
1 (b) Sketch a solution curve that passes through the point (0, 1) on your slope field.

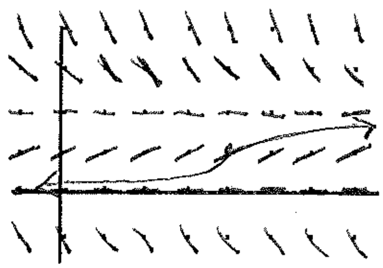
3 (c) Find $\frac{d^2y}{dx^2}$. For what values of x is the graph of the solution $y = f(x)$ concave up? Concave down? Explain.

$\frac{d^2y}{dx^2} = \frac{2(x^2+1)-(2x)(2x)}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2} = \frac{-2x^2+2}{(x^2+1)^2}$

---

20. Consider the logistic differential equation $\frac{dy}{dt} = \frac{1}{2}y(2-y)$;

1 (a) On the axes provided, sketch a slope field for the given differential equation.



$dy = y - \frac{1}{2}y^2 \, dt$

$\frac{1}{y-\frac{1}{2}y^2}\,dy = dt$

1 (b) Sketch a solution curve that passes through the point (4, 1) on your slope field.

skip (c) Show that $y = \frac{2}{1+2e^{-t}}$ satisfies the given differential equation.

1 (d) Find $\lim_{t\to\infty} y$ by using the solution curve given in part (c). $\lim_{t\to\infty} y = 2$

2 (e) Find $\frac{d^2y}{dt^2}$. For what values of y, $0<y<2$, does the graph of $y = f(t)$ have an inflection point? $\frac{d^2y}{dt^2} = 1\frac{dy}{dt} - y\frac{dy}{dt}$ $0=1-y$
$y=1$ is an inflection point

---

21. (a) On the slope field for $\frac{dP}{dt} = 3P - 3P^2$, sketch three solution curves showing different types of behavior for the population P.

SKIP?

(b) Is there a stable value of the population? If so, what is it?
(c) Describe the meaning of the shape of the solution curves for the population: Where is P increasing? Decreasing? What happens in the long run? Are there any inflection points? Where? What do they mean for the population?
(d) Sketch a graph of $\frac{dP}{dt}$ against P. Where is $\frac{dP}{dt}$ positive? Negative? Zero? Maximum? How do your observations about $\frac{dP}{dt}$ explain the shapes of your solution curves?

(Problem 21 is from Calculus (Third Edition) by Hughes-Hallett, Gleason, et al)

10/10

SLOPE FIELD CARD MATCH SOLUTIONS

Nancy Stephenson
Clements High School
Fort Bend ISD

Slope Fields	Differential Equations	Conclusions
SF 1	5	10
SF 2	9	8
SF 3	1	2
SF 4	7	6
SF 5	4	1
SF 6	2	3
SF 7	6	9
SF 8	3	5
SF 9	10	4
SF 10	8	7