## Section 5.8 Inverse Trig Derivatives — Day 1

**Opener:**
State the derivative rule for power rule, product rule, quotient rule, chain rule.

see Sec 5.4 day 3

| Function Name | f(x) | f'(x) |
|---|---|---|
| Arcsine | f(x) = arcsin (g(x)) | $f'(x) = \frac{f'(x)}{\sqrt{1-f(x)^2}}$ |
| Arccosine | f(x) = arccos (g(x)) | $f'(x) = \frac{-f'(x)}{\sqrt{1-f(x)^2}}$ |
| Arctangent | f(x) = arctan (g(x)) | $f'(x) = \frac{f'(x)}{1+f(x)^2}$ |

**Examples from 5-8A:** 41, 43

**Your Turn from 5-8A:** 45, 47

**Assignment:** Finish 5-6B

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## Differentiation - Natural Logs and Exponentials

Date________________ Period____

**Differentiate each function with respect to $x$.**

1) $y = \ln x^3$

2) $y = e^{2x^3}$

3) $y = \ln \ln 2x^4$

4) $y = \ln \ln 3x^3$

5) $y = \cos \ln 4x^3$

6) $y = e^{e^{3x^2}}$

7) $y = e^{(4x^3 + 5)^2}$

8) $y = \ln 4x^2 \cdot (-x^3 - 4)$

9) $y = \ln \left(-\frac{4x^4}{x^3 - 3}\right)^5$

10) $y = \frac{e^{5x^4}}{e^{4x^2 + 3}}$

In Exercises 39–58, find the derivative of the function.

39. $f(x) = e^{2x}$

41. $y = e^{-2x+x^2}$

43. $y = e^{\sqrt{x}}$

45. $g(t) = (e^{-t} + e^{t})^3$

47. $y = \ln(e^{x^2})$

49. $y = \ln(1 + e^{2x})$

55. $f(x) = e^{-x} \ln x$

57. $y = e^x(\sin x + \cos x)$

In Exercises 59 and 60, use implicit differentiation to find $dy/dx$.

59. $xe^y - 10x + 3y = 0$

60. $e^{xy} + x^2 - y^2 = 10$

In Exercises 61 and 62, find the second derivative of the function.

61. $f(x) = (3 + 2x)e^{-3x}$

In Exercises 87–108, find or evaluate the integral.

87. $\int e^{5x}(5)\, dx$

89. $\int_0^1 e^{-2x}\, dx$

91. $\int xe^{-x^2}\, dx$

93. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$

95. $\int \frac{e^{-x}}{1 + e^{-x}}\, dx$

97. $\int_1^3 \frac{e^{3/x}}{x^2}\, dx$

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# Differentiation - Inverse Trigonometric Functions

Date________________ Period____

**Differentiate each function with respect to *x*.**

1) $y = \cos^{-1} -5x^3$

2) $y = \sin^{-1} -2x^2$

3) $y = \tan^{-1} 2x^4$

4) $y = \csc^{-1} 4x^2$

5) $y = \left(\sin^{-1} 5x^2\right)^3$

6) $y = \sin^{-1} \left(3x^5 + 1\right)^3$

7) $y = \left(\cos^{-1} 4x^2\right)^2$

8) $y = \cos^{-1} \left(-2x^3 - 3\right)^3$

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# Differentiation - Inverse Trigonometric Functions

Date________________ Period____

**Differentiate each function with respect to *x*.**

1) $y = \cos^{-1} -5x^3$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1 - (-5x^3)^2}} \cdot -15x^2$$
$$= \frac{15x^2}{\sqrt{1 - 25x^6}}$$

2) $y = \sin^{-1} -2x^2$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - (-2x^2)^2}} \cdot -4x$$
$$= -\frac{4x}{\sqrt{1 - 4x^4}}$$

3) $y = \tan^{-1} 2x^4$

$$\frac{dy}{dx} = \frac{1}{(2x^4)^2 + 1} \cdot 8x^3$$
$$= \frac{8x^3}{4x^8 + 1}$$

4) $y = \csc^{-1} 4x^2$

$$\frac{dy}{dx} = -\frac{1}{|4x^2|\sqrt{(4x^2)^2 - 1}} \cdot 8x$$
$$= -\frac{2}{x\sqrt{16x^4 - 1}}$$

5) $y = (\sin^{-1} 5x^2)^3$

$$\frac{dy}{dx} = 3 \cdot (\sin^{-1} 5x^2)^2 \cdot \frac{1}{\sqrt{1 - (5x^2)^2}} \cdot 10x$$
$$= \frac{30x \cdot (\sin^{-1} 5x^2)^2}{\sqrt{1 - 25x^4}}$$

6) $y = \sin^{-1} (3x^5 + 1)^3$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - ((3x^5 + 1)^3)^2}} \cdot 3(3x^5 + 1)^2 \cdot 15x^4$$
$$= \frac{45x^4(3x^5 + 1)^2}{\sqrt{1 - (3x^5 + 1)^6}}$$

7) $y = (\cos^{-1} 4x^2)^2$

$$\frac{dy}{dx} = 2\cos^{-1} 4x^2 \cdot -\frac{1}{\sqrt{1 - (4x^2)^2}} \cdot 8x$$
$$= -\frac{16x\cos^{-1} 4x^2}{\sqrt{1 - 16x^4}}$$

8) $y = \cos^{-1} (-2x^3 - 3)^3$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1 - ((-2x^3 - 3)^3)^2}} \cdot 3(-2x^3 - 3)^2 \cdot -6x^2$$
$$= \frac{18x^2(-2x^3 - 3)^2}{\sqrt{1 - (-2x^3 - 3)^6}}$$

Calculus – Hyperbolic Functions

Name______________________

**30–45** Find the derivative. Simplify where possible.

**30.** $f(x) = e^x \cosh x$

**31.** $f(x) = \tanh \sqrt{x}$

**33.** $h(x) = \sinh(x^2)$

**35.** $G(t) = \sinh(\ln t)$

**36.** $y = \operatorname{sech} x\,(1 + \ln \operatorname{sech} x)$

**37.** $y = e^{\cosh 3x}$

**39.** $g(t) = t \coth \sqrt{t^2 + 1}$

**41.** $y = \cosh^{-1}\sqrt{x}$

**42.** $y = x \tanh^{-1}x + \ln \sqrt{1 - x^2}$

**43.** $y = x \sinh^{-1}(x/3) - \sqrt{9 + x^2}$

**44.** $y = \operatorname{sech}^{-1}(e^{-x})$

**45.** $y = \coth^{-1}(\sec x)$

**32.** $g(x) = \sinh^2 x$

**34.** $F(t) = \ln(\sinh t)$

**38.** $f(t) = \dfrac{1 + \sinh t}{1 - \sinh t}$

**40.** $y = \sinh^{-1}(\tan x)$

Integration and Inverse Trig.

(1) $\int \frac{1}{\sqrt{1-x^2}}\,dx$

(2) $\int \frac{1}{1+x^2}\,dx$

(3) $\int \frac{-1}{\sqrt{1-x^2}}\,dx$

(4) $\int \frac{2x}{\sqrt{1-x^4}}\,dx$

(5) $\int \frac{-x^2}{\sqrt{1-x^6}}\,dx$

(6) $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

(7) $\int \frac{-\csc^2 x}{1+\cot^2 x}$

(8) $\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{x\sqrt{1-\ln^2 x}}\,dx$

(9) $\int_{\frac{1}{2}}^{1} \frac{1}{2\sqrt{x}\,(1+x)}\,dx$

(10) $\int_{0}^{1} \frac{-8x^7}{\sqrt{1-x^{16}}}\,dx$

Solutions

$$\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\sin^{-1}(x^2) = \frac{2x}{\sqrt{1-x^4}}$$

$$\cos^{-1}(x^3) = \frac{-3x^2}{\sqrt{1-x^6}}$$

$$\tan^{-1}(e^x) = \frac{e^x}{1+e^{2x}}$$

$$\tan^{-1}(\cot x) \quad \frac{-\csc^2 x}{1+\cot^2 x}$$

$$\sin^{-1}(\ln x) \quad \frac{1}{x\sqrt{1-\ln^2 x}}dx$$

$$\cos^{-1}(x^8) \quad \frac{-8x^7}{\sqrt{1-x^{16}}}$$

$$\tan^{-1}(\sqrt{x}) = \frac{1}{2\sqrt{x}(1+x)}dx$$