Section 5.6 Differential Equations

State the general form of the differential equations.

$f(x + \Delta x) =$

Types of Variation

k is called the **Constant of Variation**

y varies **directly** as x means $y = k \cdot x$

y varies **indirectly** as x means $y = k / x$

z varies **jointly** as x and y means $z = k \cdot x \cdot y$

Proportional Rates of Change

In a situation where the rate of change of one variable is ***proportional*** to the rate of change of another:

$$y' = ky$$

Then $y = Ce^{kt}$ where C is the initial value and k is the proportionality constant.

Note: if k is positive, exponential growth will occur
if k is negative then exponential decay will occur.

Exponential Growth and Decay

Compounding n times per year, where P = principal, R = rate, T = years.

Future Value: $FV = P(1 + r/n)^{nt}$

Compounding continuously

Future Value: $FV = P \cdot e^{rt}$

Assignment: Finish 5-6A & B

Calculus Worksheet 5-6A Name______________________________ Pd______

In Exercises 1–10, solve the differential equation.

1. $\dfrac{dy}{dx} = x + 2$

3. $\dfrac{dy}{dx} = y + 2$

5. $y' = \dfrac{5x}{y}$

7. $y' = \sqrt{x}\,y$

9. $(1 + x^2)y' - 2xy = 0$

13. The rate of change of N with respect to s is proportional to $250 - s$.

14. The rate of change of y with respect to x varies jointly as x and $L - y$.

In Exercises 17–20, find the function $y = f(t)$ passing through the point $(0, 10)$ with the given first derivative. Use a graphing utility to graph the solution.

17. $\dfrac{dy}{dt} = \dfrac{1}{2}t$

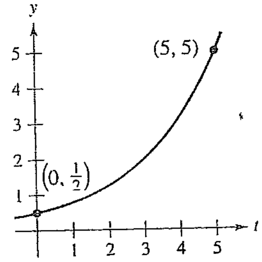
19. $\dfrac{dy}{dt} = -\dfrac{1}{2}y$

18. $\dfrac{dy}{dt} = -\dfrac{3}{4}\sqrt{t}$

20. $\dfrac{dy}{dt} = \dfrac{3}{4}y$

In Exercises 25–28, find the exponential function $y = Ce^{kt}$ that passes through the two given points.

25.



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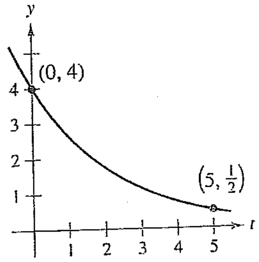
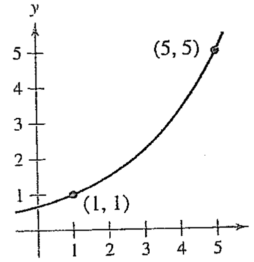
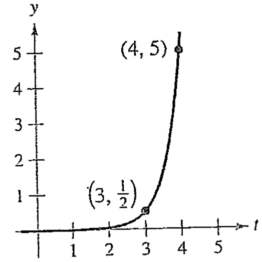


C 27.



C 28.



C

31. $\int_0^1 \tan x \, dx =$ 4.4

(A) 0

(B) $\frac{\tan^2 1}{2}$

(C) $\ln(\cos(1))$

(D) $\ln(\sec(1))$

(E) $\ln(\sec(1)) - 1$

6. An object moves with velocity $v(t) = t^2 - 8t + 7$.

   (a) Write a polynomial expression for the position of the particle at any time $t \geq 0$. 4.1 4.4
   (b) At what time(s) is the particle changing direction? 3.3
   (c) Find the total distance traveled by the particle from time $t = 0$ to $t = 4$. 4.4

C

44. A radioactive isotope, y, decays according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in seconds. If the half-life of y is 1 minute, then the value of k is 5.6

(A) −41.589 (B) −0.012 (C) 0.027 (D) 0.693 (E) 98.923

C

92. A population of bacteria given by $y(t)$ grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in minutes. If $y(10) = 10$ and $y(30) = 25$, what is the value of k? 5.6

(A) -2.079

(B) 0.046

(C) 0.107

(D) 0.125

(E) 0.230

~~AP~~ Calculus Worksheet 5-6A Name Key Pd______

In Exercises 1–10, solve the differential equation.

1. $\frac{dy}{dx} = x + 2$

$dy = (x+2)dx$

$y = \frac{1}{2}x^2 + 2x + C$

3. $\frac{dy}{dx} = y + 2$

$dy = (y+2)dx$

$\frac{1}{y+2}dy = dx$

$\ln|y+2| = x$

$y+2 = Ce^x$

$y = Ce^x - 2$

5. $y' = \frac{5x}{y}$ $\quad y' \to \frac{dy}{dx}$

$y\,dy = 5x\,dx$

$\frac{1}{2}y^2 = \frac{5}{2}x^2 + C$

$y^2 = 5x^2 + C$

$y = \pm\sqrt{5x^2 + C}$

7. $y' = \sqrt{x}\,y$

9. $(1 + x^2)y' - 2xy = 0$

(7) $\frac{dy}{dx} = \sqrt{x}y$

$\frac{1}{y}dy = \sqrt{x}dx$

$\ln|y| = \frac{2}{3}x^{3/2} + C$

$y = e^{2/3x^{3/2}+C} = Ce^{\frac{2}{3}x^{3/2}}$

(9) $(1+x^2)\frac{dy}{dx} = 2xy$

$\frac{dy}{dx} = \frac{2xy}{1+x^2}$

$dy = \frac{2xy}{1+x^2}dx$

$\frac{1}{y}dy = \frac{2x}{1+x^2}dx$

$\ln|y| = \ln|1+x^2| + C$

$y = e^{\ln|1+x^2| + C}$

$y = (1+x^2)e^C$

13. The rate of change of N with respect to s is proportional to $250 - s$.

$\frac{dN}{ds} = K(250 - s)$

$dN = K(250-s)ds$

$dN = (250K - sK)ds$

$N = 250K - \frac{K}{2}s^2 + C$

$N = \frac{-K}{2}(250-s)^2 + C$

14. The rate of change of y with respect to x varies jointly as x and $L - y$.

$\frac{dy}{dx} = K \cdot x \cdot (L-y)$

$\frac{1}{L-y}dy = Kx\,dx$

$\ln(L-y) = \frac{K}{2}x^2$

$\ln(L-y) = -\frac{K}{2}x^2$

$L - y = Ce^{-K/2 x^2}$

$-y = Ce^{-K/2 x^2} - L$

$y = Ce^{-K/2 x^2} + L$

In Exercises 17–20, find the function $y = f(t)$ passing through the point $(0, 10)$ with the given first derivative. Use a graphing utility to graph the solution.

17. $\frac{dy}{dt} = \frac{1}{2}t$

19. $\frac{dy}{dt} = -\frac{1}{2}y$

18. $\frac{dy}{dt} = -\frac{3}{4}\sqrt{t}$

20. $\frac{dy}{dt} = \frac{3}{4}y$

(17) $dy = \frac{1}{2}t\,dt$

$y = \frac{1}{4}t^2 + C$

$10 = \frac{1}{4}0^2 + C$

$y = \frac{1}{4}t^2 + 10$

(19) $dy = -\frac{1}{2}y\,dt$

$\frac{1}{y}dy = -\frac{1}{2}dt$

$\ln|y| = -\frac{1}{2}t + C$

$y = e^{-1/2 t + C}$

$10 = e^{-1/2 \cdot 0 + C}$

$10 = e^C$

$\ln 10 = C$

$y = e^{-1/2 t + \ln 10}$

or $y = 10e^{-1/2 t}$

(18) $dy = -\frac{3}{4}t^{1/2}dt$

$y = -\frac{3}{4}\cdot\frac{2}{3}t^{3/2} + C$

$y = -\frac{1}{2}t^{3/2} + C$

$10 = -\frac{1}{2}0^{3/2} + C$

$y = -\frac{1}{2}t^{3/2} + 10$

(20) $dy = \frac{3}{4}y\,dt$

$\frac{1}{y}dy = \frac{3}{4}dt$

$\ln|y| = \frac{3}{4}t + C$

$y = e^{\frac{3}{4}t + C}$

$10 = e^{\frac{3}{4}0 + C}$

$10 = e^C$

$\ln 10 = C$

$y = e^{\frac{3}{4}t + \ln 10}$

or $y = 10e^{\frac{3}{4}t}$

Compound Interest In Exercises 49–52, find the principal P that must be invested at rate r, compounded monthly, so that \$500,000 will be available for retirement in t years.

49. $r = 7\frac{1}{2}\%$, $t = 20$

51. $r = 8\%$, $t = 35$

50. $r = 6\%$, $t = 40$

52. $r = 9\%$, $t = 25$

$FV = P\left(1 + \frac{r}{n}\right)^{tn}$

(49) $500000 = P\left(1 + \frac{.075}{12}\right)^{12\cdot 20}$

$500000 = 4.460P$

$P = \$112,087$

(51) $500000 = P\left(1 + \frac{.08}{12}\right)^{12\cdot 35}$

$500000 = 16.292P$

$P = \$30,688.87$

(50) $500000 = P\left(1 + \frac{.06}{12}\right)^{12\cdot 40}$

$500000 = 10.957P$

$P = \$45631$

(52) $500000 = P\left(1 + \frac{.09}{12}\right)^{12\cdot 25}$

$500000 = 9.408P$

$P = \$53,143.92$

31. $\int_0^1 \tan x\, dx =$ 4.4

$\int_0^1 \frac{\sin x}{\cos x}\,dx$ $u = \cos x$, $du = -\sin x\,dx$

(A) 0 $\int_0^1 \frac{1}{u}(-du)$

(B) $\frac{\tan^2 1}{2}$ $[\ln|u|]_0^1$

(C) $\ln(\cos(1))$ — needs (—) $[\ln|\cos x|]_0^1$

(D) $\ln(\sec(1))$ - dec. match.

(E) $\ln(\sec(1)) - 1$

$\ln[\cos 1] - \ln[\cos 0]$

$\ln[\cos 1] - \ln[1]$

$\ln[\cos 1] - 0$

6. An object moves with velocity $v(t) = t^2 - 8t + 7$.

(a) Write a polynomial expression for the position of the particle at any time t ≥ 0. 4.1 4.4 $s(t) = \frac{1}{3}t^3 - 4t^2 + 7t + C$

(b) At what time(s) is the particle changing direction? $t = 1$ & 7 sec. 3.3

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 4$.

inc dec inc

$\int_0^4 (t^2 - 8t + 7) = \int_0^1 v(t)\,dt + \int_1^4 |v(t)|\,dt$ 4.4

21.3

$0 = t^2 - 8t + 7$

$0 = (t-7)(t-1)$

$t = 7$ $t = 1$

~~$\left[\frac{1}{3}(4)^3 - 4(4)^2 + 7(4)\right] - [0^2 - 0 + 0]$~~

~~$\frac{64}{3} - 64 + 28 - 0$~~

~~$21.3 - 36 = -14.6$ or $\frac{44}{3}$~~ ~~14.6 units in the neg. direction~~

C 5.6

44. A radioactive isotope, y, decays according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in seconds. If the half-life of y is 1 minute, then the value of k is

60 sec.

(A) −41.589 (B) −0.012 (C) 0.027 (D) 0.693 (E) 98.923

$dy = ky\,dt$

$\frac{1}{y}dy = k\,dt$

$\ln y = kt$

$y = e^{kt} = Ce^{kt}$

$y = Ce^{kt}$

$1 = 2e^{k \cdot 60}$

$\frac{1}{2} = e^{k \cdot 60}$

$\ln(\frac{1}{2}) = 60k$

$-.693 = 60k$

$-.0115 = k$

C 5.6

92. A population of bacteria given by $y(t)$ grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in minutes. If $y(10) = 10$ and $y(30) = 25$, what is the value of k?

(A) −2.079

(B) 0.046

(C) 0.107

(D) 0.125

(E) 0.230

$y = Ce^{kt}$

$10 = Ce^{k \cdot 10}$

$25 = Ce^{k \cdot 30}$

$\ln\frac{10}{C} = k \cdot 10$

$\ln\left(\frac{25}{C}\right) = k \cdot 30$

$\frac{1}{10}\ln(10/C) = K$

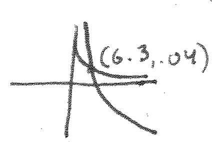
$\frac{1}{30}\ln(25/C) = K$

x Y x Y

Solve on Calc.

$0 < x < 50$

ZFIT



$Y = 6.324e^{.045t}$

~~Hon~~ Calculus Worksheet 5-6B Name______________________ Pd______

Solve the dif. eq.

2. $\dfrac{dy}{dx} = 4 - x$

4. $\dfrac{dy}{dx} = 4 - y$

6. $y' = \dfrac{\sqrt{x}}{3y}$

8. $y' = x(1 + y)$

10. $xy + y' = 100x$

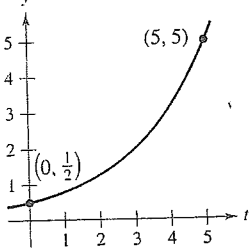
In Exercises 11–14, write and solve the differential equation that models the verbal statement.

11. The rate of change of Q with respect to t is inversely proportional to the square of t.

12. The rate of change of P with respect to t is proportional to $10 - t$.

In Exercises 25–28, find the exponential function $y = Ce^{kt}$ that passes through the two given points.

25.



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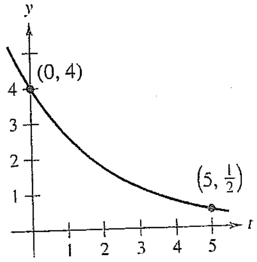
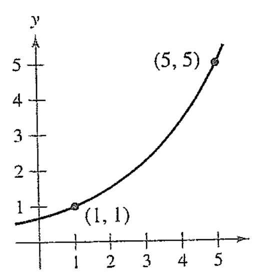
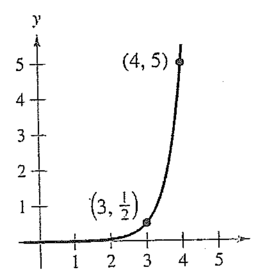


C **27.**



C **28.**



Compound Interest **In Exercises 53–56, find the time necessary for \$1000 to double if it is invested at a rate of r compounded (a) annually, (b) monthly, (c) daily, and (d) continuously.**

53. $r = 7\%$

55. $r = 8.5\%$

c

3. Sea grass grows on a lake. The rate of growth of the grass is $\frac{dG}{dt} = kG$, where k is a constant.

(a) Find an expression for G, the amount of grass in the lake (in tons), in terms of t, the number of years, if the amount of grass is 100 tons initially, and 120 tons after one year.

(b) In how many years will the amount of grass available be 300 tons?

(c) If fish are now introduced into the lake and consume a consistent 80 tons/year of sea grass, how long will it take for the lake to be completely free of sea grass?

now is at the end of year 1

c

2. At time $t = 0$ minutes, the temperature of a cup of coffee is 180 degrees Fahrenheit. Left in a room whose temperature is 70 degrees Fahrenheit, the coffee cools so that its temperature function $T(t)$, also measured in degrees Fahrenheit, satisfies the differential equation $\frac{dT}{dt} = -\frac{1}{2}T + 35$.

(a) Find an expression for $T(t)$ using the initial condition $T(0) = 180$.

(b) Find $\lim_{t\to\infty} T(t)$. Explain what this limit means in the context of the problem.

(c) At what time t is the temperature of the coffee decreasing at the rate of 15 degrees Fahrenheit per minute? How hot is the coffee at that point? Indicate units of measurement.

~~Pre~~-Calculus Worksheet 5-6B Name Key Pd______

Solve the dif. eq.

2. $\frac{dy}{dx} = 4 - x$

4. $\frac{dy}{dx} = 4 - y$

6. $y' = \frac{\sqrt{x}}{3y}$

8. $y' = x(1 + y)$

10. $xy + y' = 100x$

(2) $dy = (4-x)dx$

$y = 4x - \frac{1}{2}x^2 + C$

(4) $dy = (4-y)dx$

$\frac{1}{4-y}dy = dx$

$-\ln|4-y| = x + C$

$4 - y = e^{-x-C}$

$-y = e^{-x-C} - 4$

$y = 4 - e^{-x-C}$

$= 4 - Ce^{-x}$

(6) $3y\,dy = x^{1/2}dx$

$\left[\frac{3}{2}y^2 = \frac{2}{3}x^{3/2} + C\right]\frac{2}{3}$

$y^2 = \frac{4}{9}x^{3/2} + C$

$y = \pm\sqrt{\frac{4}{9}x^{3/2} + C}$

(8) $dy = (1+y)x\,dx$

$\frac{1}{1+y}dy = x\,dx$

$\ln|1+y| = \frac{1}{2}x^2 + C$

$1 + y = e^{\frac{1}{2}x^2 + C}$

$y = e^{\frac{1}{2}x^2 + C} - 1 = Ce^{\frac{1}{2}x^2} - 1$

(10) $\frac{dy}{dx} = 100x - xy$

$dy = x(100-y)dx$

$\frac{1}{100-y}dy = x\,dx$

$\ln|100-y| = \frac{1}{2}x^2 + C$

$100 - y = e^{\frac{1}{2}x^2 + C}$

$+y = -e^{\frac{1}{2}x^2 + C} + 100$

$= -Ce^{\frac{1}{2}x^2} + 100$

In Exercises 11–14, write and solve the differential equation that models the verbal statement.

11. The rate of change of Q with respect to t is inversely proportional to the square of t.

12. The rate of change of P with respect to t is proportional to $10 - t$.

(11) $\frac{dQ}{dt} = \frac{K}{t^2}$

$dQ = \frac{K}{t^2}dt$

$Q = \frac{-K}{t} + C$

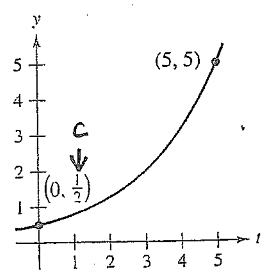
(12) $\frac{dP}{dt} = K(10-t)$

$dP = K(10-t)dt$

$P = \frac{-K}{2}(10-t)^2 + C$

In Exercises 25–28, find the exponential function $y = Ce^{kt}$ that passes through the two given points.

25.



$y = \frac{1}{2}e^{Kt}$

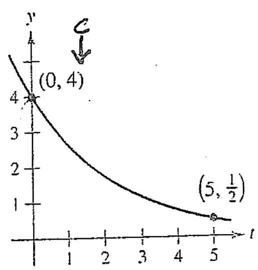
$5 = \frac{1}{2}e^{K\cdot 5}$

$10 = e^{K5}$

$\ln 10 = 5K$

$K = .460$

26.



$y = 4e^{Kt}$

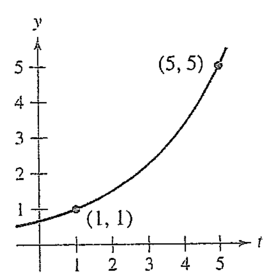
$\frac{1}{2} = 4e^{5K}$

$\frac{1}{8} = e^{5K}$

$\ln(1/8) = 5K$

$K = -.415$

C 27.



$1 = Ce^{K\cdot 1}$

$\frac{1}{e^K} = C$

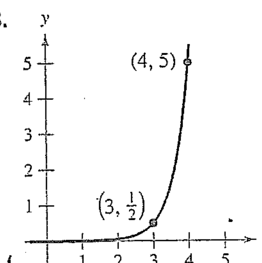
$5 = Ce^{K\cdot 5}$

$\frac{5}{e^{5K}} = C$

x y x y

$y = .668e^{.402t}$

C 28.



$\frac{1}{2} = Ce^{3K}$

$5 = Ce^{4K}$

$C = \frac{1/2}{e^{3K}}$

$C = \frac{5}{e^{4K}}$

$y = .0005e^{2.302t}$

Compound Interest **In Exercises 53–56, find the time necessary for \$1000 to double if it is invested at a rate of r compounded (a) annually, (b) monthly, ~~(c) daily,~~ and (d) continuously.**

53. $r = 7\%$

55. $r = 8.5\%$

54. ~~$r = 6\%$~~

56. ~~$r = 5.5\%$~~

(53) Annual

$2000 = 1000(1 + .07)^t$

$2 = 1.07^t$

$\ln 2 = t\ln 1.07$

$10.24\ \text{yrs} = t$

Month

$2000 = 1000\left(1 + \frac{.07}{12}\right)^{12t}$

$\ln 2 = 12t\ln(1.00583)$

$119.17 = 12t$

$9.930 = t$

Cont.

$FV = Pe^{Kt}$

$2000 = 1000e^{.07t}$

$\ln 2 = .07t$

$9.9 = t$

C

3. Sea grass grows on a lake. The rate of growth of the grass is $\frac{dG}{dt} = kG$, where k is a constant.

(a) Find an expression for G, the amount of grass in the lake (in tons), in terms of t, the number of years, if the amount of grass is 100 tons initially, and 120 tons after one year.

(b) In how many years will the amount of grass available be 300 tons?

(c) If fish are now introduced into the lake and consume a consistent 80 tons/year of sea grass, how long will it take for the lake to be completely free of sea grass?

now is at the end of year 1

a.
$G = Ce^{kt}$
$G = 100e^{kt}$
$120 = 100e^{k \cdot 1}$
$1.2 = e^{k}$
$\ln 1.2 = k$
$.182 = k$
$G = 100e^{.182t}$

b.
$300 = 100e^{.182t}$
$3 = e^{.182t}$
$\ln 3 = .182t$
$6.025 = t$
years

~~c. $G = 100e^{.182t} - 80t + 120$~~
~~$0 = 100e^{.182t} - 80t + 120$~~
~~$t = 2.265$ years~~

c. $120e^{.182t} - 80t = 0$
$t = 2.265$ years

X Double Checked

C

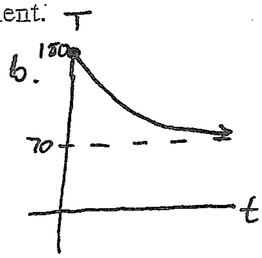
2. At time $t = 0$ minutes, the temperature of a cup of coffee is 180 degrees Fahrenheit. Left in a room whose temperature is 70 degrees Fahrenheit, the coffee cools so that its temperature function $T(t)$, also measured in degrees Fahrenheit, satisfies the differential equation $\frac{dT}{dt} = -\frac{1}{2}T + 35$.

(a) Find an expression for $T(t)$ using the initial condition $T(0) = 180$.

(b) Find $\lim_{t\to\infty} T(t)$. Explain what this limit means in the context of the problem.

(c) At what time t is the temperature of the coffee decreasing at the rate of 15 degrees Fahrenheit per minute? How hot is the coffee at that point? Indicate units of measurement.

a.
$dT = (-\frac{1}{2}T + 35)dt$
$\int \frac{1}{-\frac{1}{2}T + 35} dT = \int 1\, dt$
$-2\ln\left|-\frac{1}{2}T + 35\right| = t + C$
$\ln\left|-\frac{1}{2}T + 35\right| = -\frac{1}{2}t + C$
$-\frac{1}{2}T + 35 = e^{-\frac{1}{2}t + C}$
$-\frac{1}{2}T = Ce^{-\frac{1}{2}t} - 35$
$180 = Ce^{0} + 70$
$110 = C$
$T(t) = Ce^{-\frac{1}{2}t} + 70$
$T(t) = 110e^{-\frac{1}{2}t} + 70$

b.


decay model
$\lim_{t\to\infty} T(t) = 70$

c.
$\frac{dT}{dt}$
$-15 = -\frac{1}{2}T + 35$
$-50 = -\frac{1}{2}T$
$100 = T$ Temp $= 100°F$

$100 = 110e^{-\frac{1}{2}t} + 70$
$30 = 110e^{-\frac{1}{2}t}$
$\frac{3}{11} = e^{-\frac{1}{2}t}$
$\ln(3/11) = -\frac{1}{2}t$
$-2\ln(3/11) = t$
2.598 ~~3.1383~~ min $= t$ time.

Pre-Calculus Worksheet 5-7A Name________________________________ Pd______

In Exercises 1–6, verify the solution of the differential equation.

Solution	*Differential Equation*
1. $y = Ce^{4x}$	$y' = 4y$
2. $x^2 + y^2 = Cy$	$y' = 2xy/(x^2 - y^2)$
3. $y = C_1 \cos x + C_2 \sin x$	$y'' + y = 0$
4. $y = C_1 e^{-x} \cos x + C_2 e^{-x} \sin x$	$y'' + 2y' + 2y = 0$
5. $y = -\cos x \ln\lvert\sec x + \tan x\rvert$	$y'' + y = \tan x$
6. $y = \frac{2}{5}(e^{-2x} + e^x)$	$y'' + 2y' = 2e^x$

In Exercises 43–54, find the general solution of the differential equation.

43. $\dfrac{dy}{dx} = \dfrac{x}{y}$

45. $\dfrac{dr}{ds} = 0.05r$

47. $(2 + x)y' = 3y$

49. $yy' = \sin x$

51. $\sqrt{1 - 4x^2}\, y' = x$

53. $y \ln x - xy' = 0$

44. $\dfrac{dy}{dx} = \dfrac{x^2 + 2}{3y^2}$

46. $\dfrac{dr}{ds} = 0.05s$

48. $xy' = y$

50. $yy' = 6 \cos(\pi x)$

52. $\sqrt{x^2 - 9}\, y' = 5x$

54. $4yy' - 3e^x = 0$

6. If f is continuous for $a \le x \le b$, then at any point $x = c$, where $a < c < b$, which of the following must be true

(A) $f(c) = \dfrac{f(b) - f(a)}{b - a}$

(B) $f(a) = f(b)$

(C) $f(c) = 0$

(D) $\displaystyle\int_a^b f(x)\,dx = f(c)$

(E) $\displaystyle\lim_{x \to c} f(x) = f(c)$

4.4

8. Which of the following integrals correctly gives the area of the region consisting of all points above the x-axis and below the curve $y = 8 + 2x - x^2$?

4.4

(A) $\int_{-2}^{4} \left(x^2 - 2x - 8\right) dx$

(B) $\int_{-4}^{2} \left(8 + 2x - x^2\right) dx$

(C) $\int_{-2}^{4} \left(8 + 2x - x^2\right) dx$

(D) $\int_{-4}^{2} \left(x^2 - 2x - 8\right) dx$

(E) $\int_{2}^{4} \left(8 + 2x - x^2\right) dx$

4.4

C

32. If $\int_{30}^{100} f(x)dx = A$ and $\int_{50}^{100} f(x)dx = B$, then $\int_{30}^{50} f(x)dx =$

(A) $A + B$ (B) $A - B$ (C) 0 (D) $B - A$ (E) 20

C

40. Use differentials to approximate the change in the volume of a sphere when the radius is increased from 10 to 10.02 cm.

5.7

(A) 4213.973 (B) 1261.669 (C) 1256.637 (D) 25.233 (E) 25.133

Pre-Calculus Worksheet 5-7B Name________________________________ Pd_______

In Exercises 31–42, use integration to find a general solution of the differential equation.

31. $\dfrac{dy}{dx} = 3x^2$

33. $\dfrac{dy}{dx} = \dfrac{x}{1 + x^2}$

35. $\dfrac{dy}{dx} = \dfrac{x - 2}{x}$

37. $\dfrac{dy}{dx} = \sin 2x$

39. $\dfrac{dy}{dx} = x\sqrt{x - 3}$

41. $\dfrac{dy}{dx} = xe^{x^2}$

In Exercises 69–76, determine whether the function is homogeneous, and if it is, determine its degree.

69. $f(x, y) = x^3 - 4xy^2 + y^3$

71. $f(x, y) = \dfrac{x^2y^2}{\sqrt{x^2 + y^2}}$

73. $f(x, y) = 2 \ln xy$

75. $f(x, y) = 2 \ln \dfrac{x}{y}$

In Exercises 77–82, solve the homogeneous differential equation.

77. $y' = \dfrac{x + y}{2x}$

79. $y' = \dfrac{x - y}{x + y}$

81. $y' = \dfrac{xy}{x^2 - y^2}$

In Exercises 83–86, find the particular solution that satisfies the initial condition.

	Differential Equation	*Initial Condition*
83.	$x\,dy - (2xe^{-y/x} + y)\,dx = 0$	$y(1) = 0$
84.	$-y^2\,dx + x(x + y)\,dy = 0$	$y(1) = 1$
85.	$\left(x \sec \dfrac{y}{x} + y\right) dx - x\,dy = 0$	$y(1) = 0$
86.	$(2x^2 + y^2)\,dx + xy\,dy = 0$	$y(1) = 0$

c

2. A particle moves along the x-axis so that its acceleration at any time $t > 0$ is given by $a(t) = 12t - 18$. At time $t = 1$, the velocity of the particle is $v(1) = 0$ and the position is $x(1) = 9$.

   (a) Write an expression for the velocity of the particle $v(t)$.

   (b) At what values of t does the particle change direction? 4.4

   (c) Write an expression for the position $x(t)$ of the particle.

   (d) Find the total distance traveled by the particle from $t = \frac{3}{2}$ to $t = 6$.

4.4 or 5.2

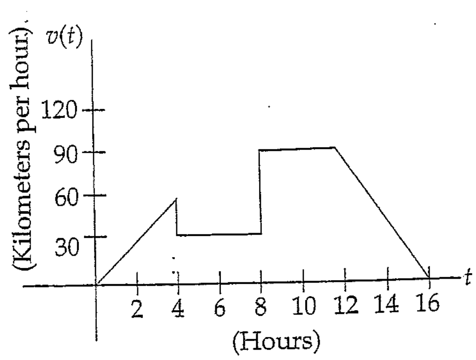
c 36. The average value of the function $f(x) = \ln^2 x$ on the interval $[2, 4]$ is

(A) -1.204 (B) 1.204 (C) 2.159 (D) 2.408 (E) 8.636

18. The average value of the function $f(x) = (x-1)^2$ on the interval from $x = 1$ to $x = 5$ is 4.4

(A) $-\frac{16}{3}$ (B) $\frac{16}{3}$ (C) $\frac{64}{3}$ (D) $\frac{66}{3}$ (E) $\frac{256}{3}$

10.



4.4

A car's velocity is shown on the graph above. Which of the following gives the total distance traveled from $t = 0$ to $t = 16$ (in kilometers)?

(A) 360 (B) 390 (C) 780 (D) 1000 (E) 1360