Section 5.4 Euler Number Derivatives and Integrals

Basics of exponentials:

- The domain of f(x) = $e^x$is (-∞, ∞), and the range is (0, ∞).
- The function f(x) = e^x is continuous, increasing, and one-to-one on its entire domain.
- The graph of f(x) = e^x is concave upward on its entire domain.
- $\lim_{x\to-\infty} e^x = 0 \text{ and } \lim_{x\to\infty} e^x = \infty$
- Graph it:

Useful Rules:

$$e^a e^b = e^{a+b}$$

$$\frac{e^a}{e^b} = e^{a-b}$$

Derivative Rule

Function Name	f(x)	f'(x)
e^x	$f(x)=e^{g(x)}$	$f(x)=g'(x)\bullet e^{g(x)}$

Assignment: Finish 5-4A, B, and C

Section 5.5 Common Log Derivatives

See the Basic rules of logs from section 5-1

Derivative Rule

Function Name	f(x)	f'(x)
Logarithm	$f(x)=\log_a g(x)$	$f'(x)=\dfrac{g'(x)}{g(x)\bullet\ln a}$
Exponential	$f(x)=a^{g(x)}$	$f'(x)=g'(x)\bullet a^{g(x)}\bullet\ln a$

Assignment: Finish 5-5A

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Name___________________________

Logarithmic Differentiation

Date________________ Period____

Use logarithmic differentiation to differentiate each function with respect to x.

1) $y = 2x^{2x}$

2) $y = 5x^{5x}$

3) $y = 3x^{3x}$

4) $y = 4x^{x^4}$

5) $y = (3x^4 + 4)^3\sqrt{5x^3 + 1}$

6) $y = (x^5 + 5)^2\sqrt{2x^2 + 3}$

7) $y = \dfrac{(3x^4 - 2)^5}{(3x^3 + 4)^2}$

8) $y = \sqrt{3x^2 + 1}\,(3x^4 + 1)^3$

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Name________________________________

Differentiation - Logs and Exponentials

Date________________ Period____

Differentiate each function with respect to x.

1) $y = 4^{4x^4}$

2) $y = 4^{-5x^3}$

3) $y = \log_3 3x^2$

4) $y = \log_2 4x^2$

5) $y = \log_3 (3x^5 + 5)^5$

6) $y = \log_5 (-5x^3 - 2)^3$

7) $y = (4^{x^3} + 2)^3$

8) $y = 3^{(x^4 + 1)^3}$

9) $y = 3^{\cos 3x^4}$

10) $y = \log_5 \tan 4x^4$

9) $y = \dfrac{\sqrt{2x^3 + 3}}{(x^4 - 3)^3}$

10) $y = (2x^2 - 5)^3 \sqrt{x^2 - 2}$

Use logarithmic differentiation to differentiate each function with respect to x. You do not need to simplify or substitute for y.

11) $y = \dfrac{(5x - 4)^4}{(3x^2 + 5)^5 \cdot (5x^4 - 3)^3}$

12) $y = (x + 2)^4 \cdot (2x - 5)^2 \cdot (5x + 1)^3$

13) $y = (5x^5 + 2)^2 \cdot (3x^3 - 1)^3 \cdot (3x - 1)^4$

14) $y = \dfrac{(x^2 + 3)^4}{(5x^5 - 2)^5 \cdot (3x^2 - 5)^2}$

15) $y = (3x^3 - 4)^5 \cdot (3x - 1)^3 \cdot (5x^3 - 2)^2 \cdot (x + 3)^4$

16) $y = \dfrac{(4x^2 - 5)^2}{(2x - 3)^4 \cdot (5x^4 - 2)^5 \cdot (3x^2 - 4)^3}$

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Name___________________

Logarithmic Differentiation

Date________________ Period____

Use logarithmic differentiation to differentiate each function with respect to x.

1) $y = 2x^{2x}$

$$\frac{dy}{dx} = y(2\ln x + 2)$$
$$= 4x^{2x}(\ln x + 1)$$

2) $y = 5x^{5x}$

$$\frac{dy}{dx} = y(5\ln x + 5)$$
$$= 25x^{5x}(\ln x + 1)$$

3) $y = 3x^{3x}$

$$\frac{dy}{dx} = y(3\ln x + 3)$$
$$= 9x^{3x}(\ln x + 1)$$

4) $y = 4x^{x^4}$

$$\frac{dy}{dx} = y(4x^3\ln x + x^3)$$
$$= 4x^{x^4+3}(4\ln x + 1)$$

5) $y = (3x^4 + 4)^3\sqrt{5x^3 + 1}$

$$\frac{dy}{dx} = y\left(\frac{36x^3}{3x^4 + 4} + \frac{15x^2}{10x^3 + 2}\right)$$
$$= \frac{3x^2(3x^4 + 4)^2(135x^4 + 24x + 20)}{2\sqrt{5x^3 + 1}}$$

6) $y = (x^5 + 5)^2\sqrt{2x^2 + 3}$

$$\frac{dy}{dx} = y\left(\frac{10x^4}{x^5 + 5} + \frac{2x}{2x^2 + 3}\right)$$
$$= \frac{2x(x^5 + 5)(11x^5 + 15x^3 + 5)}{\sqrt{2x^2 + 3}}$$

7) $y = \dfrac{(3x^4 - 2)^5}{(3x^3 + 4)^2}$

$$\frac{dy}{dx} = y\left(\frac{60x^3}{3x^4 - 2} - \frac{18x^2}{3x^3 + 4}\right)$$
$$= \frac{6x^2(3x^4 - 2)^4(21x^4 + 40x + 6)}{(3x^3 + 4)^3}$$

8) $y = \sqrt{3x^2 + 1}\,(3x^4 + 1)^3$

$$\frac{dy}{dx} = y\left(\frac{3x}{3x^2 + 1} + \frac{36x^3}{3x^4 + 1}\right)$$
$$= \frac{3x(3x^4 + 1)^2(39x^4 + 1 + 12x^2)}{\sqrt{3x^2 + 1}}$$

9) $y = \frac{\sqrt{2x^3 + 3}}{(x^4 - 3)^3}$

$$\frac{dy}{dx} = y\left(\frac{3x^2}{2x^3 + 3} - \frac{12x^3}{x^4 - 3}\right) = \frac{3x^2(-7x^4 - 3 - 12x)}{(x^4 - 3)^4\sqrt{2x^3 + 3}}$$

10) $y = (2x^2 - 5)^3\sqrt{x^2 - 2}$

$$\frac{dy}{dx} = y\left(\frac{12x}{2x^2 - 5} + \frac{x}{x^2 - 2}\right) = \frac{x(2x^2 - 5)^2(14x^2 - 29)}{\sqrt{x^2 - 2}}$$

Use logarithmic differentiation to differentiate each function with respect to *x*. You do not need to simplify or substitute for *y*.

11) $y = \frac{(5x - 4)^4}{(3x^2 + 5)^5 \cdot (5x^4 - 3)^3}$

$$\frac{dy}{dx} = y\left(\frac{20}{5x - 4} - \frac{30x}{3x^2 + 5} - \frac{60x^3}{5x^4 - 3}\right)$$

12) $y = (x + 2)^4 \cdot (2x - 5)^2 \cdot (5x + 1)^3$

$$\frac{dy}{dx} = y\left(\frac{4}{x + 2} + \frac{4}{2x - 5} + \frac{15}{5x + 1}\right)$$

13) $y = (5x^5 + 2)^2 \cdot (3x^3 - 1)^3 \cdot (3x - 1)^4$

$$\frac{dy}{dx} = y\left(\frac{50x^4}{5x^5 + 2} + \frac{27x^2}{3x^3 - 1} + \frac{12}{3x - 1}\right)$$

14) $y = \frac{(x^2 + 3)^4}{(5x^5 - 2)^5 \cdot (3x^2 - 5)^2}$

$$\frac{dy}{dx} = y\left(\frac{8x}{x^2 + 3} - \frac{125x^4}{5x^5 - 2} - \frac{12x}{3x^2 - 5}\right)$$

15) $y = (3x^3 - 4)^5 \cdot (3x - 1)^3 \cdot (5x^3 - 2)^2 \cdot (x + 3)^4$

$$\frac{dy}{dx} = y\left(\frac{45x^2}{3x^3 - 4} + \frac{9}{3x - 1} + \frac{30x^2}{5x^3 - 2} + \frac{4}{x + 3}\right)$$

16) $y = \frac{(4x^2 - 5)^2}{(2x - 3)^4 \cdot (5x^4 - 2)^5 \cdot (3x^2 - 4)^3}$

$$\frac{dy}{dx} = y\left(\frac{16x}{4x^2 - 5} - \frac{8}{2x - 3} - \frac{100x^3}{5x^4 - 2} - \frac{18x}{3x^2 - 4}\right)$$

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Name____________________

Differentiation - Logs and Exponentials

Date________________ Period____

Differentiate each function with respect to *x*.

1) $y = 4^{4x^4}$

$\frac{dy}{dx} = 4^{4x^4} \ln 4 \cdot 16x^3$

$= x^3 \cdot 4^{4x^4 + 2} \ln 4$

2) $y = 4^{-5x^3}$

$\frac{dy}{dx} = 4^{-5x^3} \ln 4 \cdot -15x^2$

$= -\frac{15x^2 \ln 4}{4^{5x^3}}$

3) $y = \log_3 3x^2$

$\frac{dy}{dx} = \frac{1}{3x^2 \ln 3} \cdot 6x$

$= \frac{2}{x \ln 3}$

4) $y = \log_2 4x^2$

$\frac{dy}{dx} = \frac{1}{4x^2 \ln 2} \cdot 8x$

$= \frac{2}{x \ln 2}$

5) $y = \log_3 (3x^5 + 5)^5$

$\frac{dy}{dx} = \frac{1}{(3x^5 + 5)^5 \ln 3} \cdot 5(3x^5 + 5)^4 \cdot 15x^4$

$= \frac{75x^4}{\ln 3 \cdot (3x^5 + 5)}$

6) $y = \log_5 (-5x^3 - 2)^3$

$\frac{dy}{dx} = \frac{1}{(-5x^3 - 2)^3 \ln 5} \cdot 3(-5x^3 - 2)^2 \cdot -15x^2$

$= -\frac{45x^2}{\ln 5 \cdot (-5x^3 - 2)}$

7) $y = (4^{x^3} + 2)^3$

$\frac{dy}{dx} = 3(4^{x^3} + 2)^2 \cdot 4^{x^3} \ln 4 \cdot 3x^2$

$= 9x^2(4^{x^3} + 2)^2 \cdot 4^{x^3} \ln 4$

8) $y = 3^{(x^4 + 1)^3}$

$\frac{dy}{dx} = 3^{(x^4 + 1)^3} \ln 3 \cdot 3(x^4 + 1)^2 \cdot 4x^3$

$= 4x^3 \cdot 3^{(x^4 + 1)^3 + 1} \cdot (x^4 + 1)^2 \ln 3$

9) $y = 3^{\cos 3x^4}$

$\frac{dy}{dx} = 3^{\cos 3x^4} \ln 3 \cdot -1 \sin 3x^4 \cdot 12x^3$

$= -4x^3 \cdot 3^{\cos 3x^4 + 1} \sin 3x^4 \cdot \ln 3$

10) $y = \log_5 \tan 4x^4$

$\frac{dy}{dx} = \frac{1}{\tan 4x^4 \cdot \ln 5} \cdot \sec^2 4x^4 \cdot 16x^3$

$= \frac{16x^3 \cdot \sec^2 4x^4}{\tan 4x^4 \cdot \ln 5}$

__Calculus Worksheet 5-5A Name______________________________ Pd______

In Exercises 19–24, solve for x or b.

19. (a) $\log_{10} 1000 = x$
(b) $\log_{10} 0.1 = x$

20. (a) $\log_3 \frac{1}{81} = x$
(b) $\log_6 36 = x$

In Exercises 41–56, find the derivative of the function.

41. $f(x) = 4^x$

43. $y = 5^{x-2}$

45. $g(t) = t^2 2^t$

47. $h(\theta) = 2^{-\theta} \cos \pi\theta$

49. $y = \log_3 x$

51. $f(x) = \log_2 \dfrac{x^2}{x-1}$

53. $y = \log_5 \sqrt{x^2 - 1}$

55. $g(t) = \dfrac{10 \log_4 t}{t}$

In Exercises 61–68, find or evaluate the integral.

61. $\int 3^x \, dx$

63. $\int_{-1}^{2} 2^x \, dx$

65. $\int x(5^{-x^2}) \, dx$

67. $\int \dfrac{3^{2x}}{1 + 3^{2x}} \, dx$

62. $\int 5^{-x} \, dx$

64. $\int_{-2}^{0} (3^3 - 5^2) \, dx$

66. $\int (3 - x)7^{(3-x)^2} \, dx$

68. $\int 2^{\sin x} \cos x \, dx$

5.5

19. If $f(x) = 3^{\pi x}$, then $f'(x) =$

(A) $\dfrac{3^{\pi x}}{\pi \ln 3}$

(B) $\dfrac{3^{\pi x}}{\ln 3}$

(C) $\dfrac{3^{\pi x}}{\pi}$

(D) $\pi\left(3^{\pi x - 1}\right)$

(E) $\pi \ln 3\left(3^{\pi x}\right)$

5.5

21. If $f(x) = 5^{3x}$ then $f'(x) =$

(A) $5^{3x}(\ln 125)$ (B) $\dfrac{5^{3x}}{3 \ln 5}$ (C) $3(5^{2x})$ (D) $3(5^{3x})$ (E) $3x(5^{3x-1})$

2.2 or 5.4

C 32. Let f be the function given by $f(x) = 3^x$. For what value of x is the slope of the line tangent to the curve at $(x, f(x))$ equal to 1?

(A) 1.099 (B) .086 (C) 0 (D) –.086 (E) –1.099

C 2. The temperature on New Year's Day in Hinterland was given by $T(H) = -A - B\cos\left(\frac{\pi H}{12}\right)$, where T is the temperature in degrees Fahrenheit and H is the number of hours from midnight ($0 \le T < 24$).

(a) The initial temperature at midnight was –15° F, and at noon of New Year's Day was 5° F. Find A and B.

(b) Find the average temperature for the first 10 hours.

(c) Use the Trapezoid Rule with 4 equal subdivisions to estimate $\int_6^8 T(H)\, dH$. 4.4

(d) Find an expression for the rate that the temperature is changing with respect to H.

~~Pre~~-Calculus Worksheet 5-5A Name Key Pd____

In Exercises 19–24, solve for x or b.

19. (a) $\log_{10} 1000 = x$
(b) $\log_{10} 0.1 = x$

$1000 = 10^x \quad x = 3$

$\frac{1}{10} = 10^x \quad x = -1$

20. (a) $\log_3 \frac{1}{81} = x$
(b) $\log_6 36 = x$

$\frac{1}{81} = 3^x \quad x = -4$

$36 = 6^x \quad x = 2$

In Exercises 41–56, find the derivative of the function.

41. $f(x) = 4^x$ $\quad f'(x) = 4^x \cdot 1 \cdot \ln 4$

43. $y = 5^{x-2}$ $\quad \longrightarrow y' = 5^{x-2}(1)\ln 5$

45. $g(t) = t^2 2^t$ $\quad g'(x) = [2t]2^t + t^2[2^t \cdot 1 \cdot \ln 2]$

47. $h(\theta) = 2^{-\theta}\cos \pi\theta$ $\quad \longrightarrow h'(\theta) = [2^{-\theta}(-1)\ln 2]\cos(\pi\theta) + 2^{-\theta}[-\sin(\pi\theta)\cdot\pi]$

49. $y = \log_3 x$ $\quad y' = \frac{1}{x\ln 3}$

51. $f(x) = \log_2 \frac{x^2}{x-1}$ $\quad \longrightarrow f'(x) = \frac{\frac{(x-1)2x - x^2(1)}{(x-1)^2}}{\frac{x^2}{x-1}\cdot\frac{\ln 2}{1}} = \frac{(1x^2-2x)}{(x-1)^2}\cdot\frac{(x-1)}{x^2\ln 2} = \frac{x(x-2)}{x(x-1)x\ln x} = \frac{x-2}{(x^2-x)\ln 2}$

53. $y = \log_5 \sqrt{x^2-1}$ $\quad y' = \frac{\frac{1}{2}(x^2-1)^{-1/2}(2x)}{\sqrt{x^2-1}\,\ln 5} = \frac{x}{(x^2-1)\ln 5}$

55. $g(t) = \frac{10\log_4 t}{t}$

$\longrightarrow g'(t) = \frac{t\left(10\frac{1}{t\ln 4}\right) - 10\log_4 t(1)}{t^2} = \frac{\frac{10t}{t\ln 4} - 10\log_4 t}{\frac{t^2}{1}} = \frac{10t^2}{\ln 4} - 10t^2\log_4 t$

In Exercises 61–68, find or evaluate the integral.

61. $\int 3^x\,dx$ $\quad = 3^x \cdot \frac{1}{\ln 3} + C$

63. $\int_{-1}^{2} 2^x\,dx$ $\quad = \left[2^x \cdot \frac{1}{\ln 2}\right]_{-1}^{2} = \frac{1}{\ln 2}\left[2^2 - 2^{-1}\right] = \frac{1}{\ln 2}\left[4 - \frac{1}{2}\right] = \frac{3.5}{\ln 2}$

65. $\int x(5^{-x^2})\,dx$

$u = -x^2$
$du = -2x\,dx$
$-\frac{1}{2}du = x\,dx$

$\int 5^u\left(-\frac{1}{2}du\right) = \frac{-1}{2\ln 5}\cdot 5^u = \frac{-1}{2\ln 5}\cdot 5^{-x^2} + C$

67. $\int \frac{3^{2x}}{1+3^{2x}}\,dx$

$u = 1 + 3^{2x}$
$du = 3^{2x}\cdot 2\ln 3\,dx$
$\frac{1}{2\ln 3}du = 3^{2x}dx$

$\int \frac{1}{u}\cdot\left(\frac{1}{2\ln 3}du\right) = \ln|u|\cdot\frac{1}{2\ln 3} + C = \ln|1+3^{2x}|\cdot\frac{1}{2\ln 3} + C$

62. $\int 5^{-x}\,dx$ $\quad = -5^{-x}\ln 5 + C$

64. $\int_{-2}^{0}(3^3 - 5^2)\,dx$ $\quad = \int_{-2}^{0}(27-25)dx = \int_{-2}^{0}(2)dx = [2x]_{-2}^{0} = 2(0) - 2(-2) = 4$

66. $\int (3-x)7^{(3-x)^2}\,dx$

$u = (3-x)^2$
$du = 2(3-x)(-1)dx$
$-\frac{1}{2}du = (3-x)dx$

$\int 7^u\left(-\frac{1}{2}du\right) = 7^u\cdot\left(-\frac{1}{2}\right)\left(\frac{1}{\ln 7}\right) + C = \frac{-1}{2\ln 7}\cdot 7^{(3-x)^2} + C$

68. $\int 2^{\sin x}\cos x\,dx$

$u = \sin x$
$du = \cos x\,dx$

$\int 2^u\,du = 2^u\frac{1}{\ln 2} + C = 2^{\sin x}\cdot\frac{1}{\ln 2} + C$

19. If $f(x) = 3^{\pi x}$, then $f'(x) =$ $3^{\pi x}(\pi)\ln 3$ 5.5

(A) $\dfrac{3^{\pi x}}{\pi \ln 3}$

(B) $\dfrac{3^{\pi x}}{\ln 3}$

(C) $\dfrac{3^{\pi x}}{\pi}$

(D) $\pi\left(3^{\pi x-1}\right)$

(E) $\pi \ln 3\left(3^{\pi x}\right)$

$y = 7^x$
$\ln y = \ln 7^x$
$\ln y = x \ln 7$

21. If $f(x) = 5^{3x}$ then $f'(x) =$ $5^{3x}(3)\ln 5 = 5^{3x}\ln 5^3 = 5^{3x}\ln 125$ 5.5

(A) $5^{3x}(\ln 125)$ (B) $\dfrac{5^{3x}}{3 \ln 5}$ (C) $3(5^{2x})$ (D) $3(5^{3x})$ (E) $3x(5^{3x-1})$

2.2 or 5.4

C

32. Let f be the function given by $f(x) = 3^x$. For what value of x is the slope of the line tangent to the curve at $(x, f(x))$ equal to 1?

$f'(x) = 3^x \ln 3$ $1 = 3^x \ln 3$ $x = .0856$

(A) 1.099 (B) .086 (C) 0 (D) −.086 (E) −1.099

C

2. The temperature on New Year's Day in Hinterland was given by $T(H) = -A - B\cos\left(\dfrac{\pi H}{12}\right)$, where T is the temperature in degrees Fahrenheit and H is the number of hours from midnight ($0 \le T < 24$).

(a) The initial temperature at midnight was −15° F, and at noon of New Year's Day was 5° F. Find A and B.

(b) Find the average temperature for the first 10 hours.

(c) Use the Trapezoid Rule with 4 equal subdivisions to estimate $\int_6^8 T(H)\,dH$. 4.4

(d) Find an expression for the rate that the temperature is changing with respect to H.

a) $T(0) = -A - B\cos(0)$
$-15 = -A - B$
$T(12) = -A - B\cos(\pi)$
$5 = -A + B$
$-15 = -A - B$
$-10 = -2A$
$5 = A$
$5 = -5 + B$
$10 = B$

b) $\frac{1}{10}\int_0^{10} T(H)\,dH$
$\frac{1}{10}\int_0^{10}\left(-5 - 10\cos\left(\frac{\pi H}{12}\right)\right)dH$
$\frac{1}{10}(-69.098) = -6.909°F$

c) -4.889

d) $T'(H) = +10\sin\left(\frac{\pi H}{12}\right)\cdot\frac{\pi}{12}$
$T'(H) = \frac{+10\pi}{12}\sin\left(\frac{\pi H}{12}\right)$
or
$+\frac{5\pi}{6}\sin\left(\frac{\pi H}{12}\right)$