Section 5.4 Euler Number Derivatives and Integrals

Basics of exponentials:

- The domain of f(x) = $e^x$is (-∞, ∞), and the range is (0, ∞).
- The function f(x) = e^x is continuous, increasing, and one-to-one on its entire domain.
- The graph of f(x) = e^x is concave upward on its entire domain.
- $\lim_{x\to-\infty} e^x = 0 \; and \; \lim_{x\to\infty} e^x = \infty$
- Graph it:

Useful Rules:

$$e^a e^b = e^{a+b}$$

$$\frac{e^a}{e^b} = e^{a-b}$$

Derivative Rule

Function Name	f(x)	f'(x)
e^x	$f(x)=e^{g(x)}$	$f(x)=g'(x)\bullet e^{g(x)}$

Assignment: Finish 5-4A, B, and C

Section 5.5 Common Log Derivatives

See the Basic rules of logs from section 5-1

Derivative Rule

Function Name	f(x)	f'(x)
Logarithm	$f(x) = \log_a g(x)$	$f'(x) = \dfrac{g'(x)}{g(x) \bullet \ln a}$
Exponential	$f(x)=a^{g(x)}$	$f'(x)=g'(x)\bullet a^{g(x)}\bullet \ln a$

Assignment: Finish 5-5A

Calculus Worksheet 5-4 **Name**______________________________ **Pd**______

Find the derivative

40. $f(x) = e^{1-x}$

42. $y = e^{-x^3}$

44. $y = x^2 e^{-x}$

46. $g(t) = e^{-3/t^2}$

48. $y = \ln\left(\frac{1 + e^x}{1 - e^x}\right)$

52. $y = \frac{e^x - e^{-x}}{2}$

54. $y = xe^x - e^x$

Integrate

88. $\int e^{-x^4}(-4x^3)\, dx$

90. $\int_3^4 e^{3-x}\, dx$

92. $\int x^2 e^{x^3/2}\, dx$

94. $\int \frac{e^{1/x^2}}{x^3}\, dx$

C

40. Find the distance traveled (to three decimal places) from $t = 1$ to $t = 5$ seconds, for a particle whose velocity is given by $v(t) = t + \ln t$.

4.4

(A) 6.000
(B) 1.609
(C) 16.047
(D) 0.800
(E) 148.413

Hint for #20: In trigonometry, the double angle theorem states that $\sin(2x) = 2\sin x \cos x$

5.5

20. $\int_0^{\frac{\pi}{2}} \sin(2x)e^{\sin^2 x}\,dx =$

(A) e (B) $e - 1$ (C) $1 - e$ (D) $e + 1$ (E) 1

23. $\int e^x(e^{3x})dx =$

5.4

(A) $\frac{1}{3}e^{3x} + C$
(B) $\frac{1}{4}e^{4x} + C$
(C) $\frac{1}{4}e^{5x} + C$
(D) $4e^{4x} + C$
(E) $4e^{5x} + C$

C

89. Which of the following is an equation of the tangent line to the graph of the function $f(x) = e^x + x^2$ at the point where $f'(x) = 2$?

5.4

(A) $y = 2x - 0.630$
(B) $y = 2x + 0.537$
(C) $y = 2x + 0.839$
(D) $y = 2x + 0.926$
(E) $y = 2x + 1.469$

~~Pre~~-Calculus Worksheet 5-4E Name Key Pd______

Find the derivative

40. $f(x) = e^{1-x}$ $\quad f'(x) = e^{1-x}(-1)$

42. $y = e^{-x^2}$ $\quad y' = e^{-x^2}(-2x)$

44. $y = x^2e^{-x}$ $\quad y' = 2xe^{-x} + x^2e^{-x}(-1)$

46. $g(t) = e^{-3/t^2}$ $\quad g'(t) = e^{-3/t^2}(6t^{-3})$

48. $y = \ln\left(\dfrac{1+e^x}{1-e^x}\right) = \dfrac{\dfrac{(1-e^x)(e^x)-(1+e^x)(-e^x)}{(1-e^x)^2}}{\left(\dfrac{1+e^x}{1-e^x}\right)}$

$\dfrac{2e^x}{1-e^{2x}}$

50. ~~$y = \ln\dfrac{e^x + e^{-x}}{2}$~~

52. $y = \dfrac{e^x - e^{-x}}{2} = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$ $\quad y' = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$

54. $y = xe^x - e^x$

$y' = 1e^x + xe^x - e^x = xe^x$

Integrate

88. $\displaystyle\int e^{-x^4}(-4x^3)\,dx$ $\quad u = -x^4$, $du = -4x^3dx$ $\quad \int e^u du = e^u = e^{-x^4} + C$

90. $\displaystyle\int_3^4 e^{3-x}\,dx = \left[-e^{3-x}\right]_3^4 = -e^{3-4} + e^0 = 1 - e^{-1}$

92. $\displaystyle\int x^2e^{x^3/2}\,dx$ $\quad u = \frac{x^3}{2}$, $du = \frac{3}{2}x^2dx$, $\frac{2}{3}du = x^2dx$ $\quad \int e^u\left(\frac{2}{3}du\right) = \frac{2}{3}e^u + C$, $\frac{2}{3}e^{x^3/2} + C$

94. $\displaystyle\int \frac{e^{1/x^2}}{x^3}\,dx$ $\quad u = x^{-2}$, $du = -2x^{-3}dx$, $-\frac{1}{2}du = \frac{1}{x^3}dx$ $\quad \int e^u\left(\frac{1}{2}du\right)$, $-\frac{1}{2}e^u + C$, $-\frac{1}{2}e^{x^{-2}} + C$

96. ~~$\displaystyle\int \frac{e^{2x}}{1 + e^{2x}}\,dx$~~

98. ~~$\displaystyle\int_0^{\sqrt{2}} xe^{-(x^2/2)}\,dx$~~

100. ~~$\displaystyle\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$~~

102. ~~$\displaystyle\int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2}\,dx$~~

~~**74.**~~ ***Area*** Perform the following steps to find the maximum area of the rectangle shown in the figure.

(a) Solve for c in the equation $f(c) = f(c + x)$.

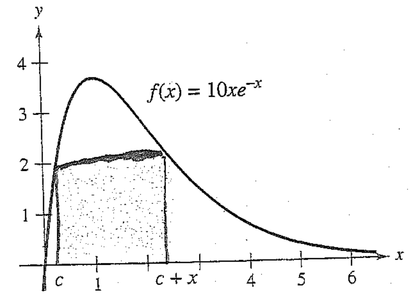
(b) Use the result in part (a) to write the area A as a function of x. [*Hint:* $A = xf(c)$]

(c) Use a graphing utility to graph the area function. Use the graph to approximate the dimensions of the rectangle of maximum area. Determine the maximum area.

(d) Use a graphing utility to graph the expression for c found in part (a). Use the graph to approximate

$$\lim_{x\to 0^+} c \quad \text{and} \quad \lim_{x\to\infty} c.$$

Use this result to describe the changes in dimensions and position of the rectangle for $0 < x < \infty$.



C

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(A) 6.000

(B) 1.609

(C) 16.047

(D) 0.800

(E) 148.413

$\int_1^5 (t + \ln t)\,dt$

16.047

4.4

$u = \sin^2 x = (\sin x)^2$

$du = 2\sin x \cdot \cos x\,dx$

$du = \sin(2x)\,dx$

$\int e^u\,du = e^u = e^{\sin^2 x}$

5.5

$\left[e^{\sin^2 x}\right]_0^{\pi/2} = e^{(\sin\frac{\pi}{2})^2} - e^{(\sin 0)^2}$

$e^1 - 1$

20. $\int_0^{\frac{\pi}{2}} \sin(2x)e^{\sin^2 x}\,dx =$ 1.718 $e^1 = 2.718$

(A) e (B) $e - 1$ (C) $1 - e$ (D) $e + 1$ (E) 1

23. $\int e^x(e^{3x})\,dx =$ $\int e^{x+3x}\,dx = \int e^{4x}\,dx = \frac{1}{4}e^{4x} + C$

5.4

(A) $\frac{1}{3}e^{3x} + C$

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$f'(x) = e^x + 2x$

$2 = e^x + 2x$

$x = .314$

$f(.314) = 1.469$

$y - 1.469 = 2(x - .314)$

$y = 2x - .628 + 1.469$

$y = 2x + .841$

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Integration by Substitution

Date________________ Period____

Evaluate each indefinite integral. Use the provided substitution.

1) $\int \frac{20x^4}{4x^5 + 3}\,dx;$

2) $\int 36x^2 e^{4x^3 + 3}\,dx;$

3) $\int 80x^3 \cdot 3^{5x^4 - 2}\,dx;$

4) $\int \frac{2}{x(-1 + \ln 4x)}\,dx;$

Evaluate each indefinite integral.

5) $\int \frac{12x^2}{x^3 + 2}\,dx$

6) $\int \frac{20e^{5x}}{e^{5x} + 3}\,dx$

7) $\int 10\sin -2x \cdot e^{\cos -2x}\,dx$

8) $\int \frac{5e^{-3 + \ln 3x}}{x}\,dx$