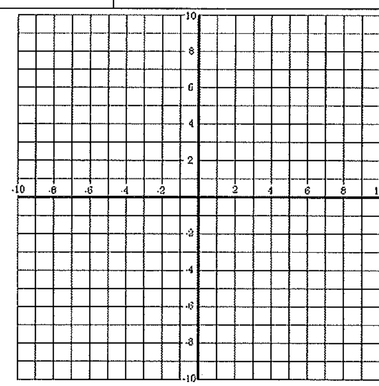
Section 5.3 Derivatives of Inverse Functions

Part 1

$f(x) = x^2 - 4$ find the inverse of f(x), which we will call g(x) g(x) = ____________________

Use your calculator to fill in the table.

x values	Ordered pair on f(x)	Slopes on f(x) at these points	Points that MUST exist on g(x)	Slopes at these points
1				
2				
3				

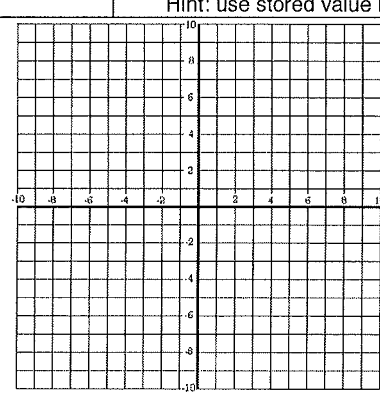
Sketch f(x), g(x), and y = x on the grid: 

Part 2

$f(x) = \ln(x + 3)$ find the inverse of f(x), which we will call g(x) g(x) = ____________________

Use your calculator to fill in the table.

x values	Ordered pair on f(x)	Slopes on f(x) at these points	Points that MUST exist on g(x)	Slopes at these points
-2				
-1	Hint: use store command Y→A		Hint: use stored value A	Hint: use stored value A
1	Hint: use store command Y→B		Hint: use stored value B	Hint: use stored value B

Sketch f(x), g(x), and y = x on the grid: 

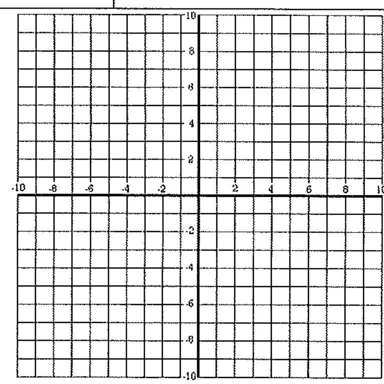
Part 3

$f(x) = \sqrt[3]{x+2} - 1$ find the inverse of f(x), which we will call g(x) g(x) = ______________

Use your calculator to fill in the table.

x values	Ordered pair on f(x)	Slopes on f(x) at these points	Points that MUST exist on g(x)	Slopes at these points
-2		Hint: TRUE answer is not 100		
-1				
6				

Sketch f(x), g(x), and y = x on the grid:



Summarize what is shown in the tables: ______________________________

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____

Check for Understanding

1. g(x) is the inverse of f(x). f(2) = 4 and f'(2) = -½ the ~~what~~ find g'(4)?

2. $f^{-1}(x)$ is the inverse of f(x). f(1) = 3 and f'(1) = 5. What tangent line must exist along $f^{-1}(x)$?

Book Definition for the Derivatives of Inverses

Function Name	f(x)	f'(x)
Inverses	f(x) and g(x) are inverse functions where $f^{-1}(x) = g(x)$ and $g^{-1}(x) = f(x)$	$g'(x) = \frac{1}{f'(g(x))}$ where the bottom is not zero

Extra Terms:

One-to-one: passes the vertical line test and horizontal line test. Required for inverse to exist.

Monotonic: always increase on the entire function or always decreasing on the entire function.

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Name___________________________________

Derivatives of Inverse Functions

Date________________ Period____

For each problem, find $(f^{-1})'(x)$ by direct computation.

1) $f(x) = -3x + 3$

2) $f(x) = -2x + 3$

For each problem, find $(f^{-1})'(x)$ by using the theorem $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$

3) $f(x) = -5x + 1$

4) $f(x) = -2x + 2$

5) $f(x) = \sqrt{-2x - 3}$

6) $f(x) = -4x^3 - 4$

For each problem, find $(f^{-1})'(x)$ by using the formula $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}}$, where $y = f^{-1}(x)$

7) $f(x) = x^7 + x - 3$

8) $f(x) = 3x^5 + 2x + 5$

Calculus Extra Practice on 5-3

Name ______________________

For each function, find the ordered pair and slope at $x=3$
Then find the inverse function and show the ordered pair & slope
of the matching point

Original	Inverse
c ① $f(x)=2x-4$	
c ② $f(x)=\sqrt{x}+3$	
c ③ $f(x)=(x-2)^3$	
c ④ $f(x)=\ln x$	

For This section, $f(x)$ and $g(x)$ are inverses

⑤ $f(x)=x^3-2$ find $g'(6)$

⑥ $f(x)=\sqrt{x^2-x+2}$ find $g'(2)$

⑦ $f(x)=\sin x$ on $[0,2\pi]$ find $g'(1)$

⑧ $f(x)=e^x$ find $g'(e)$

f & g are inverses
f is described in the table

x	$f(x)$	$f'(x)$
0	2	-4
1	0	$\frac{1}{2}$
2	1	5

find $g'(1) =$

$g'(2) =$

$g'(0) =$

Find the average value of $f(x) = -x^2 + 20$ on $[0, 2]$

Find the average value of $f(x) = \ln x$ on $[\frac{1}{2}, 2]$

Calculus Extra Practice on 5-3

Name Key

For each function, find the ordered pair and slope at $x=3$. Then find the inverse function and show the ordered pair & slope of the matching point

	Original	$x=3$	Inverse	$x=?$
c ①	$f(x)=2x-4$ $f'(x)=2$	$f(3)=6-4=2$ $f'(3)=2$	$x=2y-4$ $x+4=2y$ $y=\frac{1}{2}x+2$ $y'=\frac{1}{2}$	$x=2$ $g(2)=\frac{1}{2}(2)+2$ $=3$ $g'(2)=\frac{1}{2}$
c ②	$f(x)=\sqrt{x}+3$ $f'(x)=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$	$f(3)=\sqrt{3}+3\approx 4.732$ $f'(3)=\frac{1}{2\sqrt{3}}$	$x=\sqrt{y}+3$ $(x-3)^2=y$ $2(x-3)=y'$	$g(\sqrt{3}+3)=(\sqrt{3}+3-3)^2=3$ $g'(\sqrt{3}+3)=2(\sqrt{3}+3-3)$ $=2\sqrt{3}$
c ③	$f(x)=(x-2)^3$ $f'(x)=3(x-2)^2$	$f(3)=(3-2)^3=1$ $f'(3)=3(1)^2=3$	$x=(y-2)^3$ $\sqrt[3]{x}=y-2$ $2+\sqrt[3]{x}=y$ $\frac{1}{3}x^{-2/3}=y'$	$x=1$ $g(1)=2+\sqrt[3]{1}=3$ $g'(1)=\frac{1}{3}(1)^{-2/3}=\frac{1}{3}$
c ④	$f(x)=\ln x$ $f'(x)=\frac{1}{x}$	$f(3)=\ln(3)\approx 1.0986$ $f'(3)=\frac{1}{3}$	$x=\ln y$ $e^x=y$ $e^x=y'$	$x=\ln(3)$ $g(\ln 3)=e^{\ln(3)}=3$ $g'(\ln 3)=e^{\ln(3)}=3$

For this section, $f(x)$ and $g(x)$ are inverses

⑤ $f(x)=x^3-2$ find $g'(6)=\frac{1}{12}$

$(2,6)$

$m=12$

⑥ $f(x)=\sqrt{x^2-x+2}$ find $g'(2)=\pm\frac{4}{3}$

$(-1,2)$ $m=\frac{-3}{4}$

$(2,2)$ $m=\frac{3}{4}$

⑦ $f(x)=\sin x$ on $[0,2\pi]$ find $g'(1)=$ undef.

$(\frac{\pi}{2},1)$

$m=0$

⑧ $f(x)=e^x$ find $g'(e)=.066$

$(1,e)$

$m=15.154$

table

x	$f(x)$	$f'(x)$
0	2	-4
1	0	$\frac{1}{2}$
2	1	5

if f & g are inverses

find $g'(1) = \frac{1}{5}$

$g'(2) = -\frac{1}{4}$

$g'(0) = 2$

Find average value of $f(x) = -x^2 + 20$ on $[0, 2]$

$$\frac{1}{2}\int_0^2 f(x)\,dx = \frac{1}{2}(37.\overline{3}) = 18.\overline{6}$$

Find average value of $f(x) = \ln x$ on $[\frac{1}{2}, 2]$

$$\frac{1}{1.5}\int_{\frac{1}{2}}^{2} f(x)\,dx = \frac{1}{1.5}(.232) = .155$$