Calculus Chapter 5 Other Functions and Applications

Section 5.1 & 5.2 Natural Log Derivatives Day 1-3

Basic Natural Log Graphs?

- The domain is $(0, \infty)$.
- The graph passes through (1, 0)
- The graph has a vertical asymptote at x = 0.
- Graph it:

Useful rules:

The natural log is a log of base e (e = 2.718)

$$\ln(ab) = \ln(a) + \ln(b)$$

$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$

$$\ln(a^b) = b \bullet \ln(a)$$

$$e^{\ln x} = x$$

$$\ln(e^x) = x$$

Interesting Connection

The natural log can be defined:

$$\ln x = \int_1^x \frac{1}{t}dt \quad for\ x > 0$$

$$\ln e = \int_1^e \frac{1}{t}dt = 1$$

The derivative of the natural log:

Function Name	f(x)	f'(x)
Natural Log	f(x) = ln(g(x))	$f'(x) = \frac{g'(x)}{g(x)}$

Antiderivative using natural log:

Note the use of absolute value so the domain of 1/x and ln x are the same (all reals but zero).

$$\int \frac{1}{x}dx = \ln|x| + C$$

Assignment: Finish 5-1A, 5-2A, and 5-2B

Calculus Worksheet 5-1A Name________________________________ Pd________

In Exercises 7–10, match the function with its graph. [The graphs are labeled (a), (b), (c), and (d).]

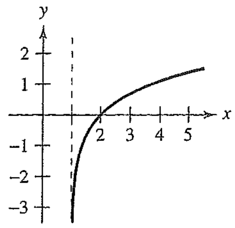
7. $f(x) = \ln x + 2$

8. $f(x) = -\ln x$

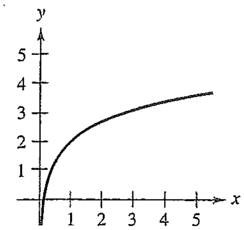
9. $f(x) = \ln(x - 1)$

10. $f(x) = -\ln(-x)$

(a)



(b)



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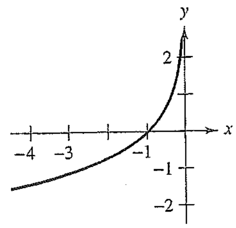
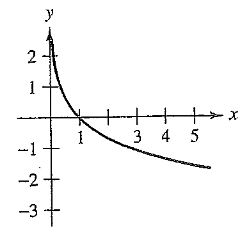
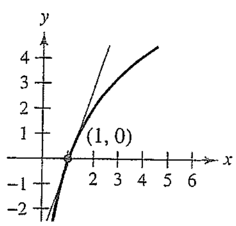


(d)

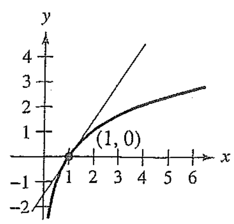


In Exercises 41–44, find the slope of the tangent line to the graph of the logarithmic function at the point (1, 0).

41. $y = \ln x^3$



42. $y = \ln x^{3/2}$



In Exercises 45–70, find the derivative of the function.

45. $g(x) = \ln x^2$

46. $h(x) = \ln(2x^2 + 1)$

47. $y = (\ln x)^4$

48. $y = x \ln x$

49. $y = \ln\left(x\sqrt{x^2 - 1}\right)$

50. $y = \ln\sqrt{x^2 - 4}$

51. $f(x) = \ln\left(\dfrac{x}{x^2 + 1}\right)$

54. $h(t) = \dfrac{\ln t}{t}$

53. $g(t) = \dfrac{\ln t}{t^2}$

56. $y = \ln(\ln x)$

55. $y = \ln(\ln x^2)$

57. $y = \ln\sqrt{\dfrac{x + 1}{x - 1}}$

In Exercises 77–82, locate any relative extrema and inflection points. Use a graphing utility to confirm your results.

77. $y = \dfrac{x^2}{2} - \ln x$

79. $y = x \ln x$

81. $y = \dfrac{x}{\ln x}$

4.4

4. A particle moves along the curve $y = \sqrt{x}$. As the particle passes the point $(9,\ 3)$, its x-coordinate is increasing at the rate of 2 units per minute.

(a) How fast is the y-coordinate of the particle changing at this instant? Indicate units of measurement.

(b) How fast is the distance from the particle to the origin changing at this instant? Indicate units of measurement.

(c) Let S be the region bounded by the graph of $y = \sqrt{x}$, the x-axis, and the vertical line drawn through the particle's position. How fast is the area of S changing as the particle moves through the point $(9,\ 3)$? Indicate units of measurement.

5.1

13. The slope of the line tangent to the graph of $3x^2 + 5 \ln y = 12$ at $(2, 1)$ is

(A) $-\frac{12}{5}$ (B) $\frac{12}{5}$ (C) $\frac{5}{12}$ (D) 12 (E) -7

5.1

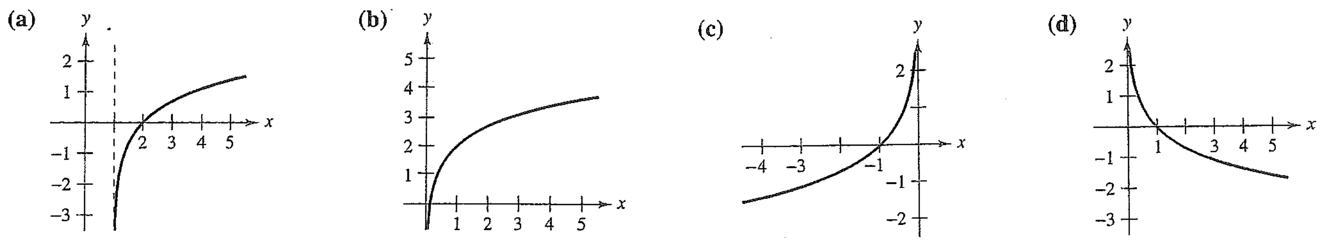
17. If $f(x) = \ln(\cos(3x))$, then $f'(x) =$

(A) $-3\csc(3x)$
(B) $3\sec(3x)$
(C) $3\tan(3x)$
(D) $-3\tan(3x)$
(E) $-3\cot(3x)$

Calculus Worksheet 5-1A Name Key Pd____

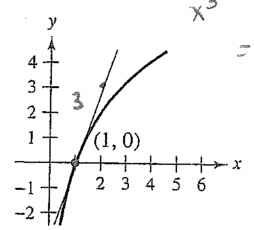
In Exercises 7–10, match the function with its graph. [The graphs are labeled (a), (b), (c), and (d).]

7. $f(x) = \ln x + 2$ b
8. $f(x) = -\ln x$ d
9. $f(x) = \ln(x - 1)$ a
10. $f(x) = -\ln(-x)$ c

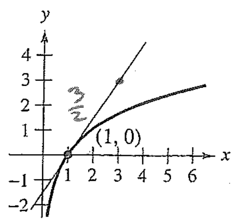


In Exercises 41–44, find the slope of the tangent line to the graph of the logarithmic function at the point (1, 0).

41. $y = \ln x^3$ $\quad \dfrac{3x^2}{x^3} = \dfrac{3}{x} = \dfrac{3}{1} = 3$



42. $y = \ln x^{3/2}$ $\quad \dfrac{\frac{3}{2}x^{1/2}}{x^{3/2}} = \dfrac{\frac{3}{2}\sqrt{x}}{x\sqrt{x}} = \dfrac{3}{2x}$



In Exercises 45–70, find the derivative of the function.

45. $g(x) = \ln x^2$ $\quad = \dfrac{2x}{x^2} = \dfrac{2}{x}$

46. $h(x) = \ln(2x^2 + 1)$ $\quad \dfrac{4x}{2x^2+1}$

47. $y = (\ln x)^4$ $\quad = 4(\ln x)^3 \cdot \dfrac{1}{x}$

48. $y = x \ln x$ $\quad = 1 \cdot \ln x + x\left(\dfrac{1}{x}\right)$

49. $y = \ln(x\sqrt{x^2 - 1})$ $\quad = \dfrac{1\sqrt{x^2-1} + x\frac{1}{2}(x^2-1)^{-1/2}(2x)}{x\sqrt{x^2-1}}$

50. $y = \ln\sqrt{x^2 - 4}$ $\quad \dfrac{\frac{1}{2}(x^2-4)^{-1/2}(2x)}{\sqrt{x^2-4}}$

51. $f(x) = \ln\left(\dfrac{x}{x^2 + 1}\right)$ $\quad = \dfrac{\left(\dfrac{1\cdot(x^2+1) - x(2x)}{(x^2+1)^2}\right)}{\left(\dfrac{x}{x^2+1}\right)}$

53. $g(t) = \dfrac{\ln t}{t^2}$ $\quad = \dfrac{\frac{1}{t}\cdot t^2 - \ln t \cdot 2t}{t^4}$

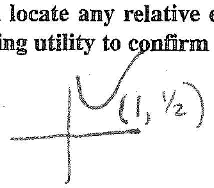
54. $h(t) = \dfrac{\ln t}{t}$ $\quad = \dfrac{\frac{1}{t}\cdot t - \ln t \cdot 1}{t^2}$

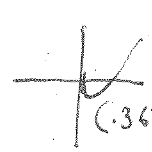
55. $y = \ln(\ln x^2)$ $\quad \dfrac{\frac{2x}{x^2}}{\ln x^2}$

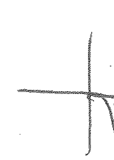
56. $y = \ln(\ln x)$ $\quad \dfrac{\frac{1}{x}}{\ln x} = \dfrac{1}{x \ln x}$

57. $y = \ln\sqrt{\dfrac{x + 1}{x - 1}}$ $\quad = \dfrac{\frac{1}{2}\left(\frac{x+1}{x-1}\right)^{-1/2}\left(\frac{1(x-1)-(x+1)\cdot 1}{(x-1)^2}\right)}{\sqrt{\frac{x+1}{x-1}}}$

In Exercises 77–82, locate any relative extrema and inflection points. Use a graphing utility to confirm your results.

77. $y = \dfrac{x^2}{2} - \ln x$  $(1, \frac{1}{2})$

79. $y = x \ln x$  $(.367, -.367)$

81. $y = \dfrac{x}{\ln x}$  $(2.71, 2.71)$

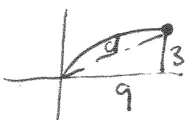
4. A particle moves along the curve $y = \sqrt{x}$. As the particle passes the point (9, 3), its x-coordinate is increasing at the rate of 2 units per minute.

$\frac{dy}{dt} = \frac{1}{2} x^{-1/2} \frac{dx}{dt} = \frac{1}{2\sqrt{x}} \cdot 2 = \frac{1}{\sqrt{9}} = \frac{1}{3}$

(a) How fast is the y-coordinate of the particle changing at this instant? Indicate units of measurement.

(b) How fast is the distance from the particle to the origin changing at this instant? Indicate units of measurement.

(c) Let S be the region bounded by the graph of $y = \sqrt{x}$, the x-axis, and the vertical line drawn through the particle's position. How fast is the area of S changing as the particle moves through the point (9, 3)? Indicate units of measurement.



$d^2 = x^2 + y^2$

$d^2 = x^2 + (\sqrt{x})^2$

$2d \frac{dd}{dt} = 2x \frac{dx}{dt} + \frac{dx}{dt}$

$d = \sqrt{90}$
$= 9.486$

$2(9.486) \frac{dd}{dt} = 2(9)(2) + (2) = 38$

$dd/dt = 38/2\sqrt{90} = 19/\sqrt{90}$

$A = \int_0^9 \sqrt{x}\,dx$

$\frac{dA}{dt} = \frac{d}{dt} \int_0^9 \sqrt{x}\,dx = \sqrt{9} = 3$

5.1

13. The slope of the line tangent to the graph of $3x^2 + 5 \ln y = 12$ at (2, 1) is

(A) $-\frac{12}{5}$ (B) $\frac{12}{5}$ (C) $\frac{5}{12}$ (D) 12 (E) −7

$6x + 5\left(\frac{1}{y}\right)dy = 0$

$6(2) + 5\left(\frac{1}{1}\right)dy = 0$

$dy = \frac{-12}{5}$

5.1

17. If $f(x) = \ln(\cos(3x))$, then $f'(x) =$

(A) $-3\csc(3x)$
(B) $3\sec(3x)$
(C) $3\tan(3x)$
(D) $-3\tan(3x)$
(E) $-3\cot(3x)$

$\frac{-\sin(3x) \cdot 3}{\cos(3x)} = -3\tan(3x)$

Bump - Dif Eq.

5.1

2. At time $t = 0$ minutes, the temperature of a cup of coffee is 180 degrees Fahrenheit. Left in a room whose temperature is 70 degrees Fahrenheit, the coffee cools so that its temperature function $T(t)$, also measured in degrees Fahrenheit, satisfies the differential equation $\frac{dT}{dt} = -\frac{1}{2}T + 35$.

$-2\frac{dT}{dt} = T - 70$

$-2\,dT = (T - 70)dt$

$\frac{-2}{T-70}dT = dt$

$\int \frac{1}{T-70}dT = -2t + C$

$\ln|T - 70| = -2t + C$

$T - 70 = e^{-2t+C}$

$T = e^{-2t+C} + 70$

(a) Find an expression for $T(t)$ using the initial condition $T(0) = 180$. $T(t) = e^{-2t + \ln 110} + 70$

(b) Find $\lim_{t\to\infty} T(t)$. Explain what this limit means in the context of the problem. the water reaching room temp.

(c) At what time t is the temperature of the coffee decreasing at the rate of 15 degrees Fahrenheit per minute? How hot is the coffee at that point? Indicate units of measurement.

$180 = e^{-2 \cdot 0 + C} + 70$

$110 = e^{C}$

$\ln 110 = C$

$100 = e^{-2t + \ln 110} + 70$

$30 = e^{-2t + \ln 110}$

$\ln 30 = -2t + \ln 110$

$\ln 30 - \ln 110 = -2t$

$\ln\left(\frac{30}{110}\right) = -2t$

$\ln(30/110)/-2 = t = .649$

$-15 = -\frac{1}{2}T + 35$

$-50 = -\frac{1}{2}T$

$100 = T$

at time $t = .649$ min, the rate of change is −15 deg/min and the temp. of the water is 100° F

Calculus Worksheet 5-2A Name________________________ Pd______

In Exercises 1–24, find the indefinite integral.

1. $\int \frac{5}{x}\,dx$

3. $\int \frac{1}{x+1}\,dx$

4. $\int \frac{1}{x-5}\,dx$

6. $\int \frac{1}{3x+2}\,dx$

7. $\int \frac{x}{x^2+1}\,dx$

8. $\int \frac{x^2}{3-x^3}\,dx$

9. $\int \frac{x^2-4}{x}\,dx$

10. $\int \frac{x}{\sqrt{9-x^2}}\,dx$

C

In Exercises 43–50, evaluate the definite integral. Use a graphing utility to verify your result.

43. $\int_0^4 \frac{5}{3x+1}\,dx$

45. $\int_1^e \frac{(1+\ln x)^2}{x}\,dx$

47. $\int_0^2 \frac{x^2-2}{x+1}\,dx$

49. $\int_1^2 \frac{1-\cos\theta}{\theta-\sin\theta}\,d\theta$

In Exercises 61–64, find $F'(x)$.

61. $F(x) = \int_1^x \frac{1}{t}\,dt$

63. $F(x) = \int_x^{3x} \frac{1}{t}\,dt$

22. $\lim_{x\to+\infty} \dfrac{2x^2+3x-5}{5x-3x^2-2}$ is

@2.5 a

(A) $-\dfrac{2}{3}$

(B) 0

(C) $\dfrac{2}{5}$

(D) 1

(E) nonexistent

6. An object moves along the x-axis starting off from the initial position $x(0) = 3$. The velocity of the object at time t is given by $v(t) = 4 - t^2$.

(a) Find the acceleration of the object at time $t = 3$. Is the object's velocity increasing or decreasing at $t = 3$? Is its speed increasing or decreasing at that point? Give a justification for your answers. 2.3

(b) At what times t, $0 \le t \le 3$, does the object change its direction? 2.3

(c) What is the total distance traveled by the object over the time interval $0 \le t \le 3$? 4.4

(d) What is the position of the object at time $t = 3$? 2.3

9. A particle moves along the x-axis so that its position at time $t > 0$ is given by $x(t) = 3\ln t - 6t + 7$. At what time $t > 0$ is the velocity of the particle equal to zero?

5.1

(A) $\ln \dfrac{1}{2}$

(B) $\dfrac{1}{2}$

(C) $\ln 2$

(D) 2

(E) 3

C

38. $\displaystyle\int \frac{\ln x}{3x}\, dx =$

5.2

(A) $6\ln^2|x| + C$

(B) $\dfrac{1}{6}\ln(\ln|x|) + C$

(C) $\dfrac{1}{3}\ln^2|x| + C$

(D) $\dfrac{1}{6}\ln^2|x| + C$

(E) $\dfrac{1}{3}\ln|x| + C$

~~Ww~~ Calculus Worksheet 5-2A Name Key Pd

In Exercises 1–24, find the indefinite integral.

1. $\int \frac{5}{x}\,dx$ $= 5\int \frac{1}{x} = 5\ln|x| + c$

3. $\int \frac{1}{x+1}\,dx$ $= \ln|x+1|$

4. $\int \frac{1}{x-5}\,dx$ $= \ln|x-5| + C$

6. $\int \frac{1}{3x+2}\,dx$ $= \ln|3x+2| \cdot \frac{1}{3} + c$

7. $\int \frac{x}{x^2+1}\,dx$ $= \frac{1}{2}\ln|x^2+1| + C$

8. $\int \frac{x^2}{3-x^3}\,dx$

$-3x^2$

$u = 3-x^3$

$du = -3x^2dx$

$-\frac{1}{3}du = x^2dx$

$\int \frac{1}{u}\left(-\frac{1}{3}du\right)$

$-\frac{1}{3}\ln|u| + C$

$\ln|3-x^3| \cdot \left(\frac{-1}{3}\right) + c$

9. $\int \frac{x^2-4}{x}\,dx$

$\int \left(x - \frac{4}{x}\right)dx = \frac{1}{2}x^2 - 4\ln|x| + c$

10. $\int \frac{x}{\sqrt{9-x^2}}\,dx$

$\int \frac{1}{u}u\,du$

$\int 1\,du$

$u + C$

$\sqrt{9-x^2} + C$

$u = \sqrt{9-x^2}$

$u^2 = 9-x^2$

$u^2+9 = -x^2$

$2u\,du = -2x\,dx$

$-u\,du = x\,dx$

$x = \sqrt{u^2+9}$

$\int \frac{1}{u}(-u\,du)$

$\int -1\,du$

$-u + C$

$-\sqrt{9-x^2} + C$

C

In Exercises 43–50, evaluate the definite integral. Use a graphing utility to verify your result.

43. $\int_0^4 \frac{5}{3x+1}\,dx$ $= 4.274$

45. $\int_1^e \frac{(1+\ln x)^2}{x}\,dx$ $= 2.\overline{3}$ or $\frac{7}{3}$

47. $\int_0^2 \frac{x^2-2}{x+1}\,dx$ $= -1.098$

49. $\int_1^2 \frac{1-\cos\theta}{\theta-\sin\theta}\,d\theta$ $= 1.928$

In Exercises 61–64, find $F'(x)$.

61. $F(x) = \int_1^x \frac{1}{t}\,dt$ $\frac{d}{dx}\int_1^x \frac{1}{t}dt$ $F'(x) = \frac{1}{x}$

63. $F(x) = \int_x^{3x} \frac{1}{t}\,dt$ $F(x) = \left[\ln|t|\right]_x^{3x} = \ln|3x| - \ln|x|$

$F'(x) = \frac{3}{3x} - \frac{1}{x} = \frac{1}{x} - \frac{1}{x} = 0$

22. $\lim_{x \to +\infty} \dfrac{2x^2+3x-5}{5x-3x^2-2}$ is $\frac{2}{-3}$

@2.5

(A) $-\dfrac{2}{3}$

(B) 0

(C) $\dfrac{2}{5}$

(D) 1

(E) nonexistent

6. An object moves along the x-axis starting off from the initial position $x(0) = 3$. The velocity of the object at time t is given by $v(t) = 4 - t^2$.

$a(t) = -2t$ $a(3) = -6$ vel. dec. Speed inc.

(a) Find the acceleration of the object at time $t = 3$. Is the object's velocity increasing or decreasing at $t = 3$? Is its speed increasing or decreasing at that point? Give a justification for your answers.

dec inc dec 2.3 2.3 4.4 2.3

(b) At what times t, $0 \le t \le 3$, does the object change its direction? $0 = 4 - t^2$ $t = \pm 2$

(c) What is the total distance traveled by the object over the time interval $0 \le t \le 3$?

(d) What is the position of the object at time $t = 3$?

total displacement: $\int_0^3 (4-t^2)\,dt = \left[4t - \frac{1}{3}t^3\right]_0^3$

$4(3) - \frac{1}{3}(3)^3 = 12 - 9 = 3$ Not total Distance

$s(t) = 4t - \frac{1}{3}t^3 + C$

$3 = 0 - 0 + C$

$\boxed{s(t) = 4t - \frac{1}{3}t^3 + 3}$

$s(3) = 4(3) - \frac{1}{3}(3)^3 + 3$

$12 - 9 + 3 = 6$

5.1

$\left|\int_0^2 (4-t^2)\,dt\right| + \left|\int_2^3 (4-t^2)\,dt\right|$

$\left|4t - \frac{1}{3}t^3\Big|_0^2\right| + \left|4t - \frac{1}{3}t^3\Big|_2^3\right|$

$\left|4(2) - \frac{1}{3}(2)^3 - 0\right| + \left|4 \cdot 3 - \frac{1}{3} \cdot 3^3 - (4(2) - \frac{1}{3}2^3)\right|$

$\left|8 - \frac{8}{3}\right| + \left|12 - 9 - (8 - \frac{8}{3})\right|$

$2.\bar{6}$

$5.\bar{3} + |3 - 5.\bar{3}|$ 5.2

$5.\bar{3} + 2.\bar{3} = 7.\bar{6}$ Total Distance

Dist. in Pos. Dir. Dist in Neg. Dir.

9. A particle moves along the x-axis so that its position at time $t > 0$ is given by $x(t) = 3\ln t - 6t + 7$. At what time $t > 0$ is the velocity of the particle equal to zero?

(A) $\ln \dfrac{1}{2}$

(B) $\dfrac{1}{2}$

(C) $\ln 2$

(D) 2

(E) 3

$x'(t) = 3\frac{1}{t} - 6$

$0 = \frac{3}{t} - 6$

$6 = \frac{3}{t}$

$6t = 3$

$t = \frac{1}{2}$

C

38. $\int \dfrac{\ln x}{3x}\,dx =$

(A) $6\ln^2|x| + C$ $6 \cdot 2\ln x \cdot \frac{1}{x}$ NO

(B) $\dfrac{1}{6}\ln(\ln|x|) + C$ $\frac{1}{6} \cdot \frac{\frac{1}{x}}{\ln|x|}$ NO

(C) $\dfrac{1}{3}\ln^2|x| + C$ $\frac{1}{3} \cdot 2\ln|x| \cdot \frac{1}{x}$ NO

(D) $\dfrac{1}{6}\ln^2|x| + C$ $\frac{1}{6} \cdot 2\ln|x| \cdot \frac{1}{x} = \frac{\ln x}{3x}$

(E) $\dfrac{1}{3}\ln|x| + C$ $\frac{1}{3}\frac{1}{x}$ $\frac{1}{3x}$

~~Hon~~ Calculus Worksheet 5-2B Name________________________ Pd______

Integrate

2. $\int \frac{10}{x}\,dx$

5. $\int \frac{1}{3-2x}\,dx$

19. $\int \frac{(\ln x)^2}{x}\,dx$

21. $\int \frac{1}{\sqrt{x+1}}\,dx$

23. $\int \frac{2x}{(x-1)^2}\,dx$

In Exercises 25–28, find the indefinite integral by *u*-substitution. (*Hint:* Let *u* be the denominator of the integrand.)

25. $\int \frac{1}{1+\sqrt{2x}}\,dx$

27. $\int \frac{\sqrt{x}}{\sqrt{x}-3}\,dx$

Integrate on Calculator.

C 44. $\int_{-1}^{1} \frac{1}{x+2}\,dx$

C 46. $\int_{e}^{e^2} \frac{1}{x\ln x}\,dx$

C 48. $\int_{0}^{1} \frac{x-1}{x+1}\,dx$

C 50. $\int_{0.1}^{0.2} (\csc 2\theta - \cot 2\theta)^2\,d\theta$

In Exercises 37–40, solve the differential equation. Use a graphing utility to graph three solutions, one of which passes through the indicated point.

37. $\frac{dy}{dx} = \frac{3}{2-x}, \quad (1, 0)$

39. $\frac{ds}{d\theta} = \tan 2\theta, \quad (0, 2)$

22. $\int_1^8 \frac{dx}{\sqrt[3]{x}} =$ 4.4

(A) $-\frac{63}{128}$

(B) $\frac{63}{128}$

(C) 1

(D) 3

(E) $\frac{9}{2}$

5.2

28. If $f(x) = \ln(\ln(1-x))$, then $f'(x) =$

(A) $-\frac{1}{\ln(1-x)}$

(B) $\frac{1}{(1-x)\ln(1-x)}$

(C) $\frac{1}{(1-x)^2}$

(D) $-\frac{1}{(1-x)\ln(1-x)}$

(E) $-\frac{1}{\ln(1-x)^2}$

C 5.2

78. If $g'(x) = x(\ln x)^2$ and $g(2) = 3$, what is $g(3)$?

(A) 2.163

(B) 2.660

(C) 2.780

(D) 5.163

(E) 5.660

C

41. $\int \ln 2x \ dx =$ 5.2

(A) $\frac{\ln 2x}{x} + C$

(B) $\frac{\ln 2x}{2x} + C$

(C) $x \ln x - x + C$

(D) $x \ln 2x - x + C$

(E) $2x \ln 2x - 2x + C$

Pre-Calculus Worksheet 5-2B Name Key Pd

Integrate

2. $\int \frac{10}{x}\,dx$ $\quad 10\ln|x|+C$

5. $\int \frac{1}{3-2x}\,dx$

$u=3-2x$

$du=-2dx$

$-\frac{1}{2}du=dx$

$\int \frac{1}{u}\left(-\frac{1}{2}du\right)$

$-\frac{1}{2}\ln|u|+C$

$-\frac{1}{2}\ln|3-2x|+C$

~~17.~~ $\cancel{\int \frac{x^4+x-4}{x^2+2}\,dx}$

19. $\int \frac{(\ln x)^2}{x}\,dx$

$u=\ln x$

$du=\frac{1}{x}dx$

$\int u^2\,du$

$\frac{1}{3}u^3+C$

$\frac{1}{3}\ln^3 x+C$

21. $\int \frac{1}{\sqrt{x+1}}\,dx$ $=$ $\cancel{\ln|\sqrt{x+1}|+C}$

$u=\sqrt{x+1}$

$\int \frac{1}{u}(2u\,du)$

$2\sqrt{x+1}+C$

23. $\int \frac{2x}{(x-1)^2}\,dx$

$u=x-1$

$u+1=x$

$du=dx$

$\int \frac{2(u+1)}{u^2}\,du$

$\int\left(\frac{2u}{u^2}+\frac{2}{u^2}\right)du$

$\int(2u^{-1}+2u^{-2})\,du$

$2\ln|u|+-2u^{-1}+C$

$2\ln|u|-\frac{2}{u}+C$

In Exercises 25–28, find the indefinite integral by u-substitution. (*Hint:* Let u be the denominator of the integrand.)

25. $\int \frac{1}{1+\sqrt{2x}}\,dx$

$u=1+\sqrt{2x}$

$(u-1)^2=2x$

$2(u-1)du=2dx$

$(u-1)du=dx$

$\int \frac{1}{u}(u-1)\,du$

$\int\left(1-\frac{1}{u}\right)du$

$u-\ln|u|+C$

$1+\sqrt{2x}-\ln|1+\sqrt{2x}|+C$

~~26.~~ $\cancel{\int \frac{1}{1+\sqrt{3x}}\,dx}$

27. $\int \frac{\sqrt{x}}{\sqrt{x}-3}\,dx$

$u=\sqrt{x}-3$

$u+3=\sqrt{x}$

$(u+3)^2=x$

$2(u+3)du=dx$

$\int \frac{u+3}{u}(2(u+3)du)$

$\int\left(\left(1+\frac{3}{u}\right)(2u+6)\right)du$

$\int\left(2u+6+6+\frac{18}{u}\right)du$

$u^2+12u+18\ln|u|+C$

$(\sqrt{x}-3)^2+12(\sqrt{x}-3)+18\ln|\sqrt{x}-3|+C$

~~28.~~ $\cancel{\int \frac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\,dx}$

Integrate on Calculator.

C 44. $\int_{-1}^{1} \frac{1}{x+2}\,dx$ $\quad 1.098$

C 46. $\int_{e}^{e^2} \frac{1}{x\ln x}\,dx$ $\quad .693$

C 48. $\int_{0}^{1} \frac{x-1}{x+1}\,dx$ $\quad -.386$

C 50. $\int_{0.1}^{0.2} (\csc 2\theta - \cot 2\theta)^2\,d\theta$ $\quad .002$

In Exercises 37–40, solve the differential equation. Use a graphing utility to graph three solutions, one of which passes through the indicated point.

37. $\frac{dy}{dx} = \frac{3}{2-x}$, $(1, 0)$ $\quad x\ y$

39. $\frac{ds}{d\theta} = \tan 2\theta$, $(0, 2)$ $\quad x\ y$

37

$u=2-x$

$du=-1dx$

$-3du=3dx$

$dy(2-x)=3dx$

$dy=\frac{3}{2-x}dx$

$\int dy=\int \frac{3}{2-x}dx$

$\int dy=\int \frac{1}{u}(-3du)=-3\ln|2-x|+C$

$y=-3\ln|2-x|+C$

$0=-3\ln|2-1|+C$

$0=0+C$

$y=-3\ln|2-x|+0$

39

$ds=\tan 2\theta\,d\theta$

$u=\cos 2\theta$

$du=-\sin 2\theta\cdot 2d\theta$

$-\frac{1}{2}du=\sin 2\theta\,d\theta$

$\int ds=\int \frac{\sin 2\theta}{\cos 2\theta}d\theta$

$s=\int \frac{1}{u}\cdot\left(-\frac{1}{2}du\right)$

$s=-\frac{1}{2}\ln|\cos 2\theta|+C$

$2=-\frac{1}{2}\ln|\cos 2\cdot 0|+C$

$2=-\frac{1}{2}\ln|1|+C$

$2=0+C$

$s=\frac{1}{2}\ln|\cos 2\theta|+$

22. $\int_1^8 \frac{dx}{\sqrt[3]{x}} =$

$\int_1^8 x^{-1/3}dx = \left[\frac{3}{2}x^{2/3}\right]_1^8 = \frac{3}{2}\cdot 8^{2/3} - \frac{3}{2}\cdot 1^{2/3}$ 4.4

$= \frac{3}{2}\cdot 4 - \frac{3}{2}\cdot 1$

$6 - 1.5 = 4.5$

(A) $-\frac{63}{128}$

(B) $\frac{63}{128}$

(C) 1

(D) 3

(E) $\frac{9}{2}$

work outside in

5.2

28. If $f(x) = \ln(\underbrace{\ln(1-x)})$, then $f'(x) =$

$f'(x) = \frac{\frac{-1}{1-x}}{\ln(1-x)} = \frac{\frac{-1}{1-x}}{\frac{\ln(1-x)}{1}} = \frac{-1}{(1-x)}\cdot\frac{1}{\ln(1-x)} = \frac{-1}{(1-x)\ln(1-x)}$

(A) $-\frac{1}{\ln(1-x)}$

(B) $\frac{1}{(1-x)\ln(1-x)}$

(C) $\frac{1}{(1-x)^2}$

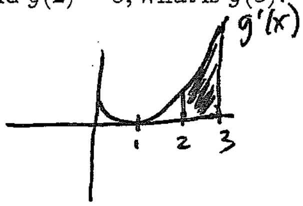
(D) $-\frac{1}{(1-x)\ln(1-x)}$

(E) $-\frac{1}{\ln(1-x)^2}$

C

5.2

78. If $g'(x) = x(\ln x)^2$ and $g(2) = 3$, what is $g(3)$?



$\underbrace{\int_2^3 g'(x)\,dx}_{\text{calculator}} = g(3) - g(2)$

$2.162 = g(3) - 3$

$5.162 = g(3)$

(A) 2.163

(B) 2.660

(C) 2.780

(D) 5.163

(E) 5.660

C

41. $\int \ln 2x \; dx =$

5.2

~~(A)~~ $\frac{\ln 2x}{x} + C$ $\quad \frac{x\left(\frac{2}{2x}\right) - 1\cdot\ln(2x)}{x^2} = \frac{1-\ln(2x)}{x^2}$ No

~~(B)~~ $\frac{\ln 2x}{2x} + C$ $\quad \frac{(2x)\left(\frac{2}{2x}\right) - 2\ln(2x)}{(2x)^2} = \frac{2-2\ln(2x)}{4x^2}$ No

~~(C)~~ $x\ln x - x + C$ $\quad 1\cdot\ln x + x\left(\frac{1}{x}\right) - 1 = \ln x$ No

(D) $x\ln 2x - x + C$ $\quad 1\cdot\ln(2x) + x\left(\frac{2}{2x}\right) - 1 = \ln(2x) + 1 - 1 = \ln(2x)$ Yes

~~(E)~~ $2x\ln 2x - 2x + C$ $\quad 2\cdot\ln(2x) + 2x\left(\frac{2}{2x}\right) - 2 = 2\ln(2x) + 2 - 2 = 2\ln(2x)$ No

4.4

5. $\int_0^1 x(\sqrt{x}+1)\,dx =$

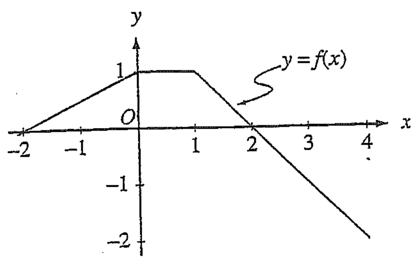
(A) 0

(B) $\frac{5}{6}$

(C) $\frac{9}{10}$

(D) 1

(E) 2



4.4

25. The graph of the piecewise linear function f on the closed interval $[-2, 4]$ is shown above. What is $\int_{-1}^{2} f(2x)\,dx$?

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) 1

(D) $\frac{9}{4}$

(E) $\frac{9}{2}$

C

36. The average value of the function $f(x) = \ln^2 x$ on the interval $[2, 4]$ is

(A) −1.204 (B) 1.204 (C) 2.159 (D) 2.408 (E) 8.636

4.4

5.2

4

5.2