Calculus Chapter 3 Applications of Derivatives

Section 3.1 Extrema and Critical Numbers

Types of Extrema?

- Global Min (Absolute min): the lowest point on the domain.
- Local Min (Relative min): the lowest point in a region, but not for the overall domain.
- Global Max (Absolute max): the highest point on the domain.
- Local Max (Relative max): the highest point in a region, but not for the overall domain.

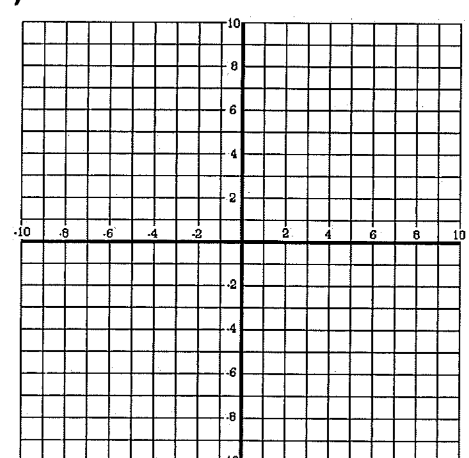
Note: The x value must be defined for the min or max to exist.

Picture:

Investigation 1 (Calculator):

Original function: f(x) = (1/4)x^4 + (2/3)x^3 – (5/2)x^2 – 6x – 1

Original graph:



State all extremes:

Global min(s):

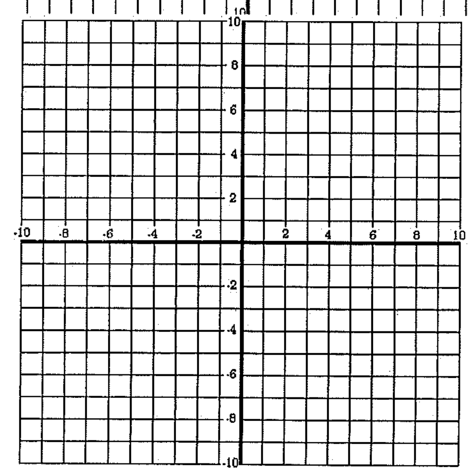
Global max(s):

Local min(s):

Local Max(s):

Derivative Function:

Derivative Graph:



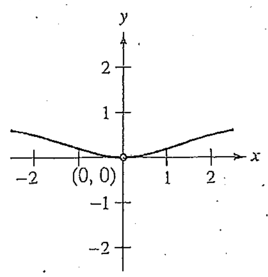
Roots:

Undefined Values:

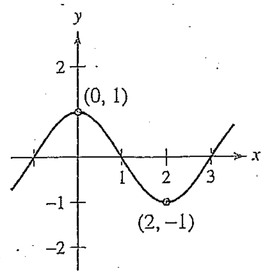
~~Ath~~-Calculus Worksheet 3-1A Name________________ Pd______

In Exercises 1–6, find the value of the derivative (if it exists) at each indicated extremum.

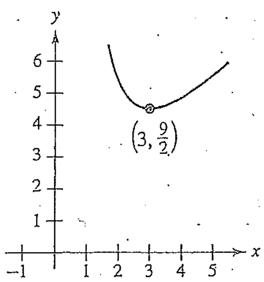
1. $f(x) = \dfrac{x^2}{x^2 + 4}$



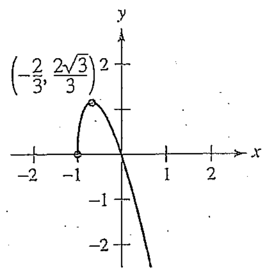
2. $f(x) = \cos \dfrac{\pi x}{2}$



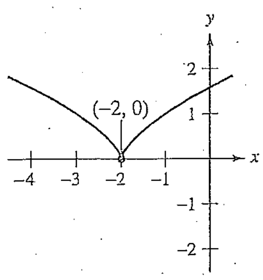
3. $f(x) = x + \dfrac{27}{2x^2}$



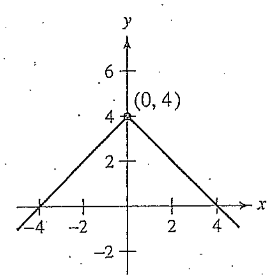
4. $f(x) = -3x\sqrt{x + 1}$



5. $f(x) = (x + 2)^{2/3}$

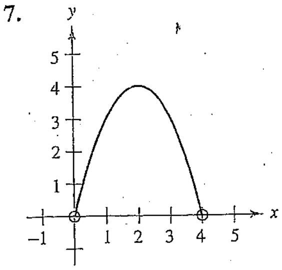


6. $f(x) = 4 - |x|$



In Exercises 7–10, approximate the critical numbers of the function shown in the graph. Determine whether the function has a relative maximum, relative minimum, absolute maximum, absolute minimum, or none of these at each critical number on the interval shown.

7.



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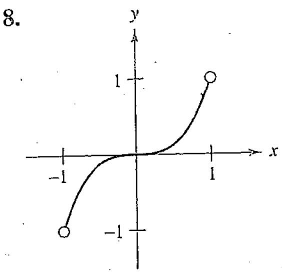

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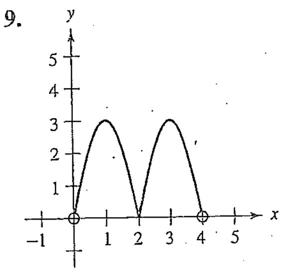

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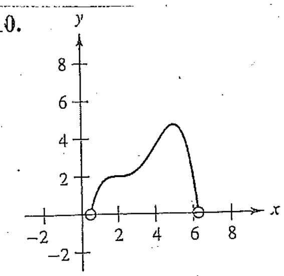


In Exercises 11–16, find any critical numbers of the function.

11. $f(x) = x^2(x - 3)$

12. $g(x) = x^2(x^2 - 4)$

C13. $g(t) = t\sqrt{4 - t},\ t < 3$

C14. $f(x) = \dfrac{4x}{x^2 + 1}$

2.4

13. If $g(x) = f(x^2)$, what is $g''(x)$?

(A) $2xf''(x^2)$

(B) $4x^2 f''(x^2)$

(C) $(4x^2 + 2x)f''(x^2)$

(D) $4x^2 f''(x^2) + 2f'(x^2)$

(E) $4x^2 f''(x^2) + 2xf'(x^2)$

2.3

18. A particle's position is given by $s = t^3 - 6t^2 + 9t$. What is its acceleration at time $t = 4$?

(A) 0
(B) 9
(C) -9
(D) -12
(E) 12

C ← requires weird trig identity $\cos n \cos m - \sin n \sin m = \cos(n+m)$

2.3 OR

$2 \sin t \cos t = \sin(2t)$

21. If $f(x) = \sin^2 x$, find $f'''(x)$.

(A) $-\sin^2 x$
(B) $2\cos 2x$
(C) $\cos 2x$
(D) $-4\sin 2x$
(E) $-\sin 2x$

C

43. If $f(x) = \left(1 + \frac{x}{20}\right)^5$, find $f''(40)$

(A) 0.068
(B) 1.350
(C) 5.400
(D) 6.750
(E) 540.000

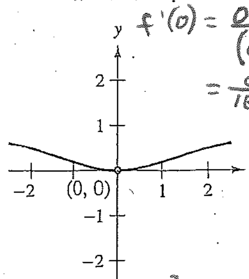
Calculus Worksheet 3-1A Name Key Pd____

In Exercises 1–6, find the value of the derivative (if it exists) at each indicated extremum.

1. $f(x) = \dfrac{x^2}{x^2+4}$

$f'(x) = \dfrac{2x(x^2+4) - x^2(2x)}{(x^2+4)^2}$

$f'(0) = \dfrac{0-0}{(0^2+4)^2} = \dfrac{0}{16} = 0$



2. $f(x) = \cos\dfrac{\pi x}{2}$

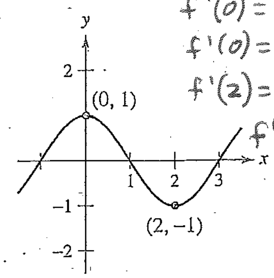
$f'(x) = -\sin\left(\frac{\pi}{2}x\right)\frac{\pi}{2}$

$f'(0) = -\frac{\pi}{2}\sin 0$

$f'(0) = 0$

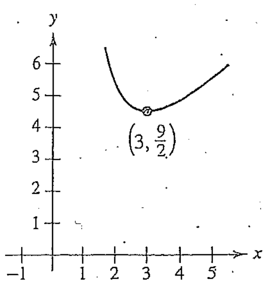
$f'(2) = -\frac{\pi}{2}\sin\pi$

$f'(2) = -\frac{\pi}{2}(0) = 0$

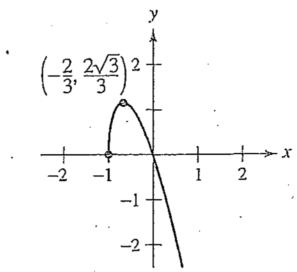


3. $f(x) = x + \dfrac{27}{2x^2}$

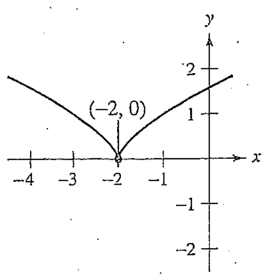
$x + \frac{27}{2}x^{-2}$



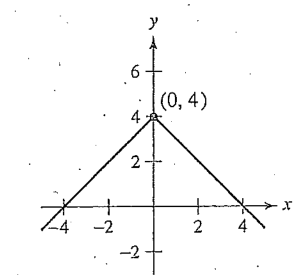
4. $f(x) = -3x\sqrt{x+1}$



5. $f(x) = (x+2)^{2/3}$



6. $f(x) = 4 - |x|$



In Exercises 7–10, approximate the critical numbers of the function shown in the graph. Determine whether the function has a relative maximum, relative minimum, absolute maximum, absolute minimum, or none of these at each critical number on the interval shown.

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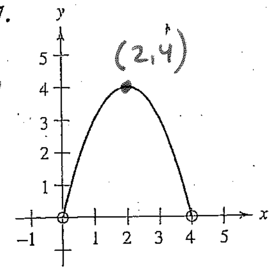

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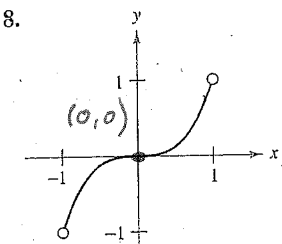

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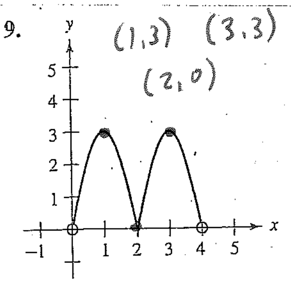

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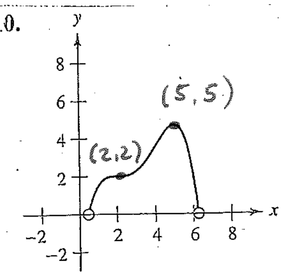


③ $f'(x) = 1 + -27x^{-3} = 1 - \dfrac{27}{x^3}$

$f'(3) = 1 - \dfrac{27}{3^3} = 1 - 1 = 0$

④ $f'(x) = -3\sqrt{x+1} + -3x\left(\tfrac{1}{2}\right)(x+1)^{-1/2}(1)$

$f'(-2/3) = -3\sqrt{-\tfrac{2}{3}+1} - 3\left(-\tfrac{2}{3}\right)\left(\tfrac{1}{2}\right)\left(-\tfrac{2}{3}+1\right)^{-1/2}$

$= \dfrac{-3\sqrt{1}}{\sqrt{3}} + \dfrac{1}{\frac{\sqrt{1}}{\sqrt{3}}} = \dfrac{-3\sqrt{3}}{3} + \dfrac{\sqrt{3}}{1}$

$= -\sqrt{3} + \sqrt{3} = 0$

⑤ $f'(x) = \frac{2}{3}(x+2)^{-1/3}(1)$

$f'(-2) = \frac{2}{3}(-2+2)^{-1/3}(1) = \dfrac{2}{3(0)^{1/3}} = \text{undef.}$

⑥ undef. corner at (0,4)

In Exercises 11–16, find any critical numbers of the function.

11. $f(x) = x^2(x-3)$

12. $g(x) = x^2(x^2-4)$

13. $g(t) = t\sqrt{4-t},\ t < 3$

14. $f(x) = \dfrac{4x}{x^2+1}$

⑪ $f'(x) = 2x(x-3) + x^2(1)$

$f'(x) = 2x^2 - 6x + x^2$

$f'(x) = 3x^2 - 6x$

$0 = 3x(x-2)$

$x = 0$ $x = 2$

⑫ $g'(x) = 2x(x^2-4) + x^2(2x)$

$g'(x) = 2x^3 - 8x + 2x^3$

$g'(x) = 4x^3 - 8x$

$0 = 4x(x^2-2)$

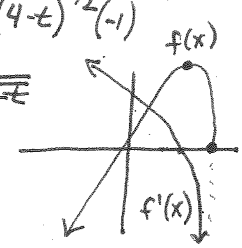
$x = 0$ $x = \pm\sqrt{2}$

⑬ $g'(t) = \sqrt{4-t} + t\left(\tfrac{1}{2}\right)(4-t)^{-1/2}(-1)$

$g'(t) = \sqrt{4-t} + \dfrac{-t}{2\sqrt{4-t}}$

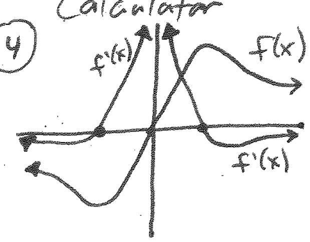
calculator:

$x = 2.\bar{6}$ or $\frac{8}{3}$



Calculator

⑭



$x = -1, 0, 1$

Chain Rule 2.4

13. If $g(x) = f(x^2)$, what is $g''(x)$?

(A) $2xf''(x^2)$

(B) $4x^2 f''(x^2)$

(C) $(4x^2 + 2x)f''(x^2)$

(D) $4x^2 f''(x^2) + 2f'(x^2)$

(E) $4x^2 f''(x^2) + 2xf'(x^2)$

$g'(x) = f'(x^2) \cdot 2x$

$g''(x) = [f''(x^2)(2x)](2x) + f'(x^2) \cdot 2$

$= 4x^2 f''(x^2) + 2f'(x^2)$

2.3

18. A particle's position is given by $s = t^3 - 6t^2 + 9t$. What is its acceleration at time $t = 4$?

(A) 0
(B) 9
(C) -9
(D) -12
(E) 12

$v = 3t^2 - 12t + 9$

$a = 6t - 12$

$a = 6(4) - 12$

$24 - 12$

12

C $[\sin x]^2$

21. If $f(x) = \sin^2 x$, find $f'''(x)$.

(A) $-\sin^2 x$
(B) $2\cos 2x$
(C) $\cos 2x$
(D) $-4\sin 2x$
(E) $-\sin 2x$

2.3

$f'(x) = 2\sin x \cdot \cos x$

$f''(x) = 2\cos x \cdot \cos x + 2\sin x(-\sin x)$

$= 2\cos^2 x - 2\sin^2 x \rightarrow 2(\cos^2 x - \sin^2 x)$, $2(\cos(x+x))$

$f'''(x) = 4\cos x(-\sin x) - 4\sin x(\cos x)$

$= -4\sin x\cos x - 4\sin x\cos x$

$= -8\sin x\cos x = -4(2\sin x\cos x) = -4\sin 2x$

$f''(x) = 2\cos(2x)$

$f'''(x) = 2(-\sin(2x))(2)$

$= -4\sin(2x)$

C

43. If $f(x) = \left(1 + \frac{x}{20}\right)^5$, find $f''(40)$.

(A) 0.068
(B) 1.350
(C) 5.400
(D) 6.750
(E) 540.000

2.4

$f'(x) = 5\left(1 + \frac{x}{20}\right)^4\left(\frac{1}{20}\right) = \frac{1}{4}\left(1 + \frac{x}{20}\right)^4$

$f''(x) = \frac{1}{4}(4)\left(1 + \frac{x}{20}\right)^3\left(\frac{1}{20}\right)$

$f''(40) = \frac{1}{20}\left(1 + \frac{40}{20}\right)^3 = \frac{1}{20}(3)^3 = \frac{27}{20} = 1.35$

~~Pre~~Calculus Worksheet 3-13 Name________________ Pd______

In Exercises 17–32, locate the absolute extrema of the function on the closed interval.

17. $f(x) = 2(3 - x),\ [-1, 2]$

19. $f(x) = -x^2 + 3x,\ [0, 3]$

21. $f(x) = x^3 - \frac{3}{2}x^2,\ [-1, 2]$

C 18. $f(x) = \frac{2x + 5}{3},\ [0, 5]$

C 20. $f(x) = x^2 + 2x - 4,\ [-1, 1]$

C 22. $f(x) = x^3 - 12x,\ [0, 4]$

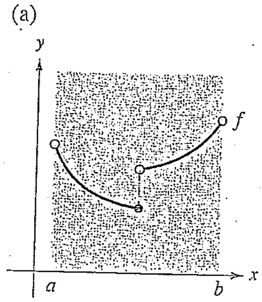
In Exercises 49 and 50, graph a function on the interval $[-2, 5]$ having the given characteristics.

49. Absolute maximum at $x = -2$
    Absolute minimum at $x = 1$
    Relative maximum at $x = 3$

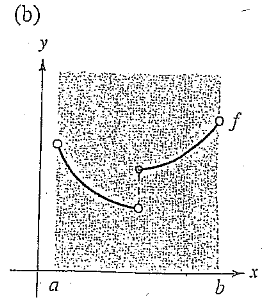
50. Relative minimum at $x = -1$
    Critical number at $x = 0$, but no extrema
    Absolute maximum at $x = 2$
    Absolute minimum at $x = 5$

In Exercises 51–54, determine from the graph whether f has a minimum in the open interval (a, b).

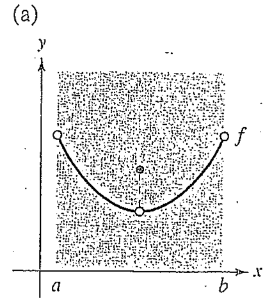
51. (a)



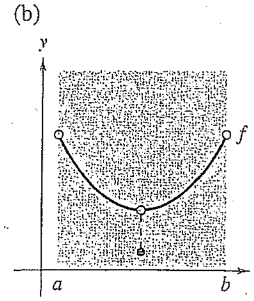
(b)



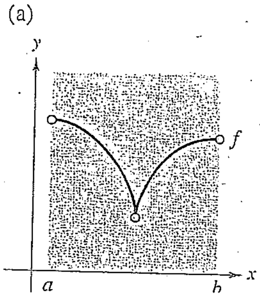
53. (a)



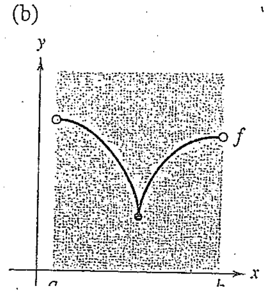
(b)



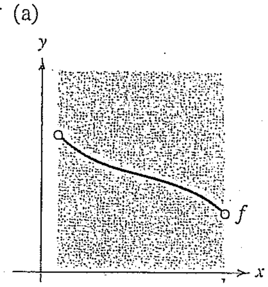
52. (a)



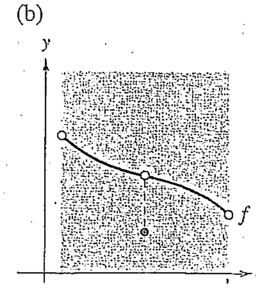
(b)



54. (a)



(b)



t (hours)	0	2	5	7	8	10
$v(t)$ (miles per hour)	50	55	60	70	65	75

C

3. The table above gives the velocity $v(t)$ at selected times t of a car traveling along a straight road. 2.2

(a) Use the values of the table to approximate the acceleration of the car at time $t = 6$. Show the work that leads to your answer and indicate units of measure.

6. An object moves along the x-axis starting off from the initial position $x(0) = 3$. The velocity of the object at time t is given by $v(t) = 4 - t^2$.

(a) Find the acceleration of the object at time $t = 3$. Is the object's velocity increasing or decreasing at $t = 3$? Is its speed increasing or decreasing at that point? Give a justification for your answers. 2.3

(b) At what times t, $0 \le t \le 3$, does the object change its direction? 2.3

~~(c)~~ What is the total distance traveled by the object over the time interval $0 \le t \le 3$? 4.4

~~(d)~~ What is the position of the object at time $t = 3$? 2.3

2.2 or 5.8

10. If $f(x) = \cos^2 x$, then $f''(\pi) =$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 2π

~~Adv~~ Calculus Worksheet 3-1B Name Key Pd

In Exercises 17–32, locate the absolute extrema of the function on the closed interval.

17. $f(x) = 2(3 - x)$, $[-1, 2]$ $\quad 6-2x$ $\quad f'(x) = -2$ no critical points $\quad f(-1) = 8$ abs max $\quad f(2) = 2$ abs min

19. $f(x) = -x^2 + 3x$, $[0, 3]$ $\quad f'(x) = -2x+3$, $0 = -2x+3$ $\quad$ crit. pt $x = 1.5$ $\quad f(0) = 0$ min $\quad f(1.5) = -2.25+4.5 = 2.25$ abs max $\quad f(3) = -9+9 = 0$ min

21. $f(x) = x^3 - \frac{3}{2}x^2$, $[-1, 2]$ $\quad f'(x) = 3x^2 - 3x$, $0 = 3x(x-1)$ $\quad$ crit. pt. $x = 0 \,\&\, 1$ $\quad f(-1) = -1 - 1.5 = -2.5$ abs min

$f(0) = 0$

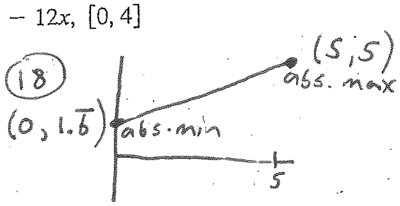
$f(1) = 1 - 1.5 = -.5$

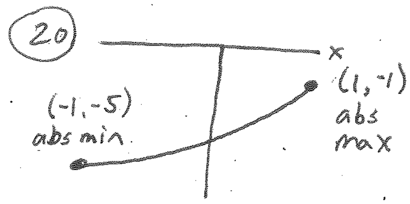
$f(2) = 8 - 6 = 2$ abs max

C 18. $f(x) = \dfrac{2x+5}{3}$, $[0, 5]$

C 20. $f(x) = x^2 + 2x - 4$, $[-1, 1]$

C 22. $f(x) = x^3 - 12x$, $[0, 4]$





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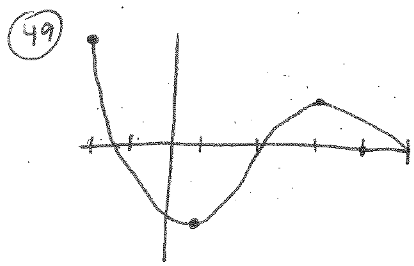

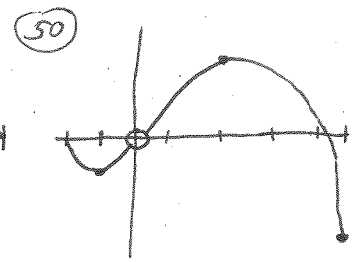

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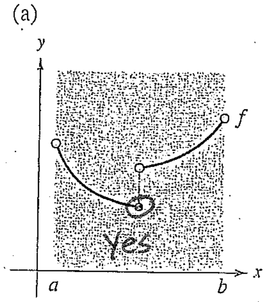
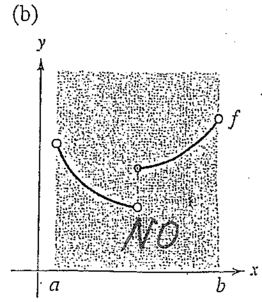
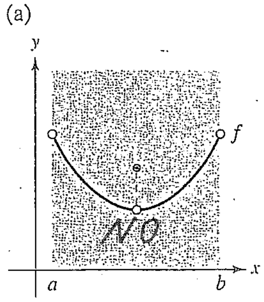
51. (a)



(b)



53. (a)



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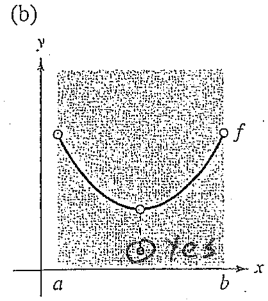
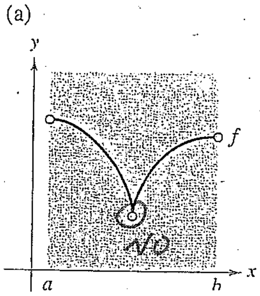
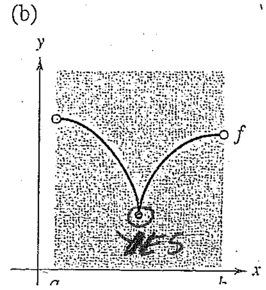


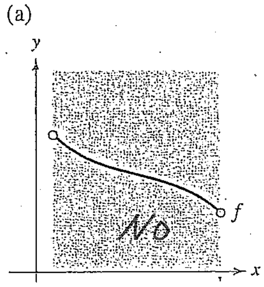
52. (a)



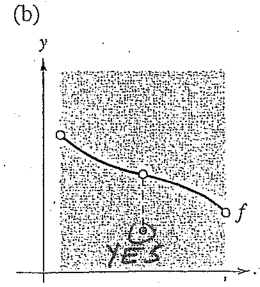
(b)



54. (a)



(b)



Find Slope

x	t (hours)	0	2	5	7	8	10
y	$v(t)$ (miles per hour)	50	55	60	70	65	75

C

3. The table above gives the velocity $v(t)$ at selected times t of a car traveling along a straight road. 2.2

(a) Use the values of the table to approximate the acceleration of the car at time $t = 6$. Show the work that leads to your answer and indicate units of measure.

$\frac{70-60}{7-5} = \frac{10}{2} = 5$ mph/h

accel. is the rate of change (slope) of velocity. Finding the slope of velocity at (5,60) and (7,70) will approx. accel. at $t=6$. The accel is 5 mph/h

6. An object moves along the x-axis starting off from the initial position $x(0) = 3$. The velocity of the object at time t is given by $v(t) = 4 - t^2$.

(a) Find the acceleration of the object at time $t = 3$. Is the object's velocity increasing or decreasing at $t = 3$? Is its speed increasing or decreasing at that point? Give a justification for your answers.

(b) At what times t, $0 \leq t \leq 3$, does the object change its direction? 2.3 2.3

a) $a(t) = -2t$ $a(3) = -6$

$0 = -2t$

$0 = t$

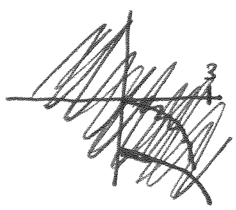
velocity is decreasing because accel is neg.
Speed is increasing because velocity is getting increasingly negative and speed = |velocity|

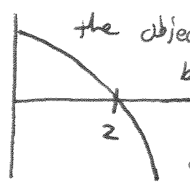
b) $v(t)$ graph:

$0 = 4 - t^2$

$4 = t^2$

$\pm 2 = t$





the object changes direction at $t=2$ because it is moving in the pos. direction $0<t<2$ and in the neg direction $2<t<3$ based on the velocity graph.

$[\cos x]^2$

10. If $f(x) = \cos^2 x$, then $f''(\pi) =$ 2.2 or 5.8

(A) -2 (B) 0 (C) 1 (D) 2 (E) 2π

$f'(x) = 2\cos x \cdot (-\sin x)$

$f''(x) = 2(-\sin x)(-\sin x) + 2\cos x(-\cos x)$

$2\sin^2 x - 2\cos^2 x$

$2(\sin\pi)^2 - 2(\cos\pi)^2$

$2(0)^2 - 2(-1)^2 = 0 - 2 = -2$