AP Calculus **Section 2-5 A** Name________________________ Pd____

Find d/dx for the following

1. $y^2 = 3x^4$

2. $y^5 = \sin x$

3. $\cos y = \sqrt{x}$

4. $\sqrt{y} = 3x^5$

5. $xy = \tan x + x$

6. $\frac{1}{y} = x(x-1)^3$

7. $x^2y = 7\sqrt{x}$

8. $\sin(x)\cos(y) = \frac{1}{x^4}$

Extra Practice – CALCULATOR SECTION – State the Equation of the tangent line

1. $h(x) = 3[\cos(x)]^2$ at $x = 2\pi$

2. $j(x) = \frac{1}{x+1} + \frac{x}{\sqrt{x}}$ at $x = 1$

3. $k(x) = \frac{2x}{x^2-2} + \sqrt{x}$ at $x = 1$

AP Calculus **Section 2-5 A** Name Key Pd____

Find d/dx for the following

1. $y^2 = 3x^4$

$$2y\frac{dy}{dx} = 12x^3$$

$$\frac{dy}{dx} = \frac{6x^3}{y}$$

2. $y^5 = \sin x$

3. $\cos y = \sqrt{x}$

$$-\sin y \cdot \frac{dy}{dx} = \frac{1}{2}x^{-1/2}$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{x}\,\sin y}$$

4. $\sqrt{y} = 3x^5$

5. $xy = \tan x + x$

$$1y + x\frac{dy}{dx} = \sec^2 x + 1$$

$$\frac{dy}{dx} = \frac{\sec^2 x + 1 - y}{x}$$

6. $\frac{1}{y} = x(x-1)^3$

7. $x^2y = 7\sqrt{x}$

$$2xy + x^2\frac{dy}{dx} = 7\left(\tfrac{1}{2}\right)x^{-1/2}$$

$$x^2\frac{dy}{dx} = \frac{7}{2\sqrt{x}} - 2xy$$

$$\frac{dy}{dx} = \frac{\frac{7}{2\sqrt{x}} - 2xy}{x^2}$$

8. $\sin(x)\cos(y) = \frac{1}{x^4}$

Extra Practice – CALCULATOR SECTION – State the Equation of the tangent line

1. $h(x) = 3[\cos(x)]^2$ at $x = 2\pi$

$y - 3 = 0(x - 2\pi)$

2. $j(x) = \frac{1}{x+1} + \frac{x}{\sqrt{x}}$ at $x = 1$

$y - 1.5 = \frac{1}{4}(x - 1)$

3. $k(x) = \frac{2x}{x^2-2} + \sqrt{x}$ at $x = 1$

$y + 1 = \frac{-11}{2}(x - 1)$

AP Calculus **Section 2-5 B** Name______________________ Pd____

Find dy/dx by implicit differentiation:

1. $x^2 + y^2 = 36$

2. $x^2 - y^2 = 16$

3. $x^{1/2} + y^{1/2} = 9$

4. $x^3 + y^3 = 8$

5. $x^3 - xy + y^2 = 4$

6. $x^2y + y^2x = -2$

7. $x^3y^3 - y = x$

8. $\sqrt{xy} = x - 2y$

9. $x^3 - 3x^2y + 2xy^2 = 12$

10. $2 \sin x \cos y = 1$

Find dy/dx by implicit differentiation and evaluate the derivative at the indicated point.

	Equation	*Point*
21.	$xy = 4$	$(-4, -1)$
22.	$x^2 - y^3 = 0$	$(1, 1)$
23.	$y^2 = \frac{x^2 - 4}{x^2 + 4}$	$(2, 0)$

6. What is the slope of the tangent line to the graph of $\frac{xy+1}{y+2} = 1$ at the point (2, 1)?

(A) -1

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 1

(E) nonexistent

16. If $y^2 + xy = 6$, what is $\frac{dy}{dx}$ at the point $(-1, 3)$?

(A) $-\frac{3}{5}$

(B) $-\frac{3}{7}$

(C) $\frac{3}{7}$

(D) $\frac{3}{5}$

(E) $\frac{6}{5}$

20. If $f(x) = \sqrt{(x^3 + 5x + 121)}\,(x^2 + x + 11)$ then $f'(0) =$

(A) $\frac{5}{2}$ (B) $\frac{27}{2}$ (C) 22 (D) $22 + \frac{2}{\sqrt{5}}$ (E) $\frac{247}{2}$

22. Find the slope of the normal line to $y = x + \cos xy$ at (0, 1).

(A) 1

(B) -1

(C) 0

(D) 2

(E) Undefined

AP Calculus **Section 2-5 B** Name________________________ Pd____

Find dy/dx by implicit differentiation:

1. $x^2 + y^2 = 36$

$2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{x}{y}$

2. $x^2 - y^2 = 16$

3. $x^{1/2} + y^{1/2} = 9$

$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}\frac{dy}{dx} = 0$

$\frac{1}{2\sqrt{y}}\frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$

$dy/dx = -\frac{\sqrt{y}}{\sqrt{x}}$

4. $x^3 + y^3 = 8$

5. $x^3 - xy + y^2 = 4$

$3x^2 - y - x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2y - x) = y - 3x^2$

$= \frac{y - 3x^2}{2y - x}$

6. $x^2y + y^2x = -2$

7. $x^3y^3 - y = x$

$3x^2y^3 + x^3 3y^2\frac{dy}{dx} - \frac{dy}{dx} = 1$

$\frac{dy}{dx}(3y^2 - 1) = 1 - 3x^2y^3$

$= \frac{1 - 3x^2y^3}{3y^2 - 1}$

8. $\sqrt{xy} = x - 2y$

9. $x^3 - 3x^2y + 2xy^2 = 12$

$3x^2 - 6xy - 3x^2\frac{dy}{dx} + 2y^2 + 2x\,2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(4xy - 3x^2) = 6xy - 3x^2 - 2y^2$

$\frac{dy}{dx} = \frac{6xy - 3x^2 - 2y^2}{4xy - 3x^2}$

10. $2 \sin x \cos y = 1$

$2\cos x \cos y + -2\sin x \sin y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{\cos x \cos y}{\sin x \sin y} = \cot x \cot y$

Find dy/dx by implicit differentiation and evaluate the derivative at the indicated point.

	Equation	*Point*
21.	$xy = 4$	$(\overset{x}{-4}, \overset{y}{-1})$
22.	$x^2 - y^3 = 0$	$(1, 1)$
23.	$y^2 = \frac{x^2 - 4}{x^2 + 4}$	$\overset{x\ y}{(2, 0)}$

$1y + xy' = 0$

$y' = \frac{-y}{x}$ $y' = -\frac{-1}{-4} = -\frac{1}{4}$

$x^2 - y^3 = 0$

$2x - 3y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{2x}{3y^2} = \frac{2(1)}{3(1)^2} = \frac{2}{3}$

$y' = \frac{(x^2+4)(2x) - (x^2-4)(2x)}{(x^2+4)^2\, 2y}$

$y' =$ undef.

6. What is the slope of the tangent line to the graph of $\frac{xy+1}{y+2} = 1$ at the point (2, 1)?

[✓ (A)] -1

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 1

(E) nonexistent

$$xy+1 = y+2$$
$$1y+xy'+0 = y'+0$$
$$xy'-y' = -y$$
$$y'(x-1) = -y$$
$$y' = \frac{-y}{x-1}$$
$$= \frac{-1}{2-1} = \frac{-1}{1} = -1$$

16. If $y^2 + xy = 6$, what is $\frac{dy}{dx}$ at the point $(-1, 3)$?

[✓ (A)] $-\frac{3}{5}$

(B) $-\frac{3}{7}$

(C) $\frac{3}{7}$

(D) $\frac{3}{5}$

(E) $\frac{6}{5}$

$$2yy'+y+xy'=0$$
$$y'(2y+x) = -y$$
$$y' = \frac{-y}{2y+x}$$
$$= \frac{-3}{2(3)+-1} = \frac{-3}{5}$$

20. If $f(x) = \sqrt{(x^3+5x+121)}\,(x^2+x+11)$ then $f'(0) =$

(A) $\frac{5}{2}$ [✓ (B)] $\frac{27}{2}$ (C) 22 (D) $22 + \frac{2}{\sqrt{5}}$ (E) $\frac{247}{2}$

$$f(x) = (x^3+5x+121)^{\frac{1}{2}}(x^2+x+11)$$
$$f'(x) = \tfrac{1}{2}(x^3+5x+121)^{-1/2}\cdot(3x^2+5)\cdot(x^2+x+11) + (x^3+5x+121)^{1/2}(2x+1)$$

$x = 0$

$$\tfrac{1}{2}(121)^{-1/2}(5)(11)+(121)^{\frac{1}{2}}(1)$$
$$\tfrac{1}{2}\cdot\tfrac{1}{11}\cdot 55 + 11$$
$$\frac{55}{22}+11$$
$$\frac{5}{2}+11 = 13.5$$

2.5

or

$\frac{27}{2}$

22. Find the slope of the normal line to $y = x + \cos xy$ at (0, 1).

(A) 1

[✓ (B)] -1

(C) 0

(D) 2

(E) Undefined

$$y' = 1 + -\sin(xy)\cdot(1y+xy')$$
$$y' = 1 - \cdot\sin(0)\cdot(1\cdot1+0y')$$
$$y' = 1$$

Normal = perp.

so Normal is -1

AP Calculus **Section 2-5 C** Name________________________ Pd____

Find dy/dx by implicit differentiation.

11. $\sin x + 2\cos 2y = 1$

12. $(\sin \pi x + \cos \pi y)^2 = 2$

13. $\sin x = x(1 + \tan y)$

14. $\cot y = x - y$

15. $y = \sin(xy)$

16. $x = \sec \frac{1}{y}$

17. $x^2 + y^2 = 16$

18. $x^2 + y^2 - 4x + 6y + 9 = 0$

19. $9x^2 + 16y^2 = 144$

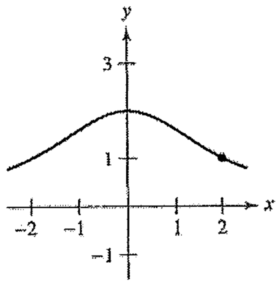
20. $9y^2 - x^2 = 9$

In Exercises 29–32, find the slope of the tangent line to the graph at the indicated point.

29. Witch of Agnesi:

$(x^2 + 4)y = 8$

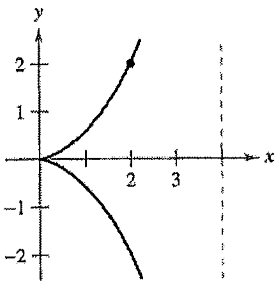
Point: (2, 1)



30. Cissoid:

$(4 - x)y^2 = x^3$

Point: (2, 2)



41. $\sqrt{x} + \sqrt{y} = 4, \quad (9, 1)$ with calculator

3. If $3x^2 - 2xy + 3y = 1$, then when $x = 2$, $\frac{dy}{dx} =$

(A) -12 (B) -10 (C) $-\frac{10}{7}$ (D) 12 (E) 32

5. If $x^2 - 2xy + 3y^2 = 8$, then $\frac{dy}{dx} =$

(A) $\frac{8+2y-2x}{6y-2x}$

(B) $\frac{3y-x}{y-x}$

(C) $\frac{2x-2y}{6y-2x}$

(D) $\frac{1}{3}$

(E) $\frac{y-x}{3y-x}$

14. Find $\frac{dy}{dx}$ if $x^3y + xy^3 = -10$.

(A) $(3x^2 + 3xy^2)$

(B) $-(3x^2 + 3xy^2)$

(C) $\frac{(3x^2y + y^3)}{(3xy^2 + x^3)}$

(D) $-\frac{(3x^2y + y^3)}{(3xy^2 + x^3)}$

(E) $-\frac{(x^2y + y^3)}{(xy^2 + x^3)}$

17. Find the equation of the tangent line to $9x^2 + 16y^2 = 52$ through $(2, -1)$.

(A) $-9x + 8y - 26 = 0$

(B) $9x - 8y - 26 = 0$

(C) $9x - 8y - 106 = 0$

(D) $8x + 9y - 17 = 0$

(E) $9x + 16y - 2 = 0$

C

33. Find the value(s) of $\frac{dy}{dx}$ of $x^2y + y^2 = 5$ at $y = 1$.

(A) $-\frac{3}{2}$ only (B) $-\frac{2}{3}$ only (C) $\frac{2}{3}$ only (D) $\pm\frac{2}{3}$ (E) $\pm\frac{3}{2}$

AP Calculus **Section 2-5 C** Name Key Pd____

Find dy/dx by implicit differentiation.

11. $\sin x + 2\cos 2y = 1$

$\cos x + -2\sin(2y)\cdot 2y' = 0$

$y' = \frac{+\cos x}{4\sin(2y)}$

12. $(\sin \pi x + \cos \pi y)^2 = 2$

13. $\sin x = x(1 + \tan y)$

$\cos x = (1 + \tan y) + x(0 + \sec^2 y\, y')$

$\cos x - 1 - \tan y = x\sec^2 y\, y'$

$\frac{\cos x - 1 - \tan y}{x\sec^2 y} = y'$

14. $\cot y = x - y$

15. $y = \sin(xy)$

$\frac{dy}{dx} = \cos(xy)\left(y + x\frac{dy}{dx}\right)$

$\frac{dy}{dx}(1 - \cos(xy)\cdot x) = \frac{y\cos(xy)}{1 - \cos(xy)\cdot x}$

16. $x = \sec \frac{1}{y}$

17. $x^2 + y^2 = 16$

$2x + 2yy' = 0$

$y' = -\frac{x}{y}$

18. $x^2 + y^2 - 4x + 6y + 9 = 0$

19. $9x^2 + 16y^2 = 144$

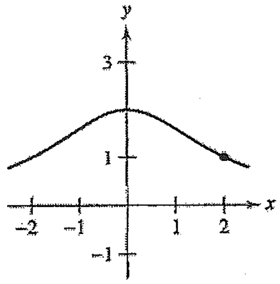
$18x + 32yy' = 0$

$y' = \frac{-18x}{32y} = \frac{-9x}{16y}$

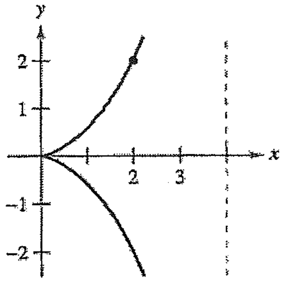
20. $9y^2 - x^2 = 9$

In Exercises 29–32, find the slope of the tangent line to the graph at the indicated point.

29. Witch of Agnesi:
$(x^2 + 4)y = 8$
Point: (2, 1)



30. Cissoid:
$(4 - x)y^2 = x^3$
Point: (2, 2)



(29) $y = \frac{8}{x^2+4}$

$y' = \frac{(x^2+4)\cdot 0 - 8(2x)}{(x^2+4)^2}$

at $x = 2$

$y' = \frac{-16(2)}{(2^2+4)^2} = \frac{-32}{64} = -\frac{1}{2}$

tan: $y - 1 = \frac{-1}{2}(x - 2)$

41. $\sqrt{x} + \sqrt{y} = 4, \quad (9, 1)$ with calculator

$3(4)-4y+3y=1$
$-y=-11$
$y=11$

3. If $3x^2-2xy+3y=1$, then when $x=2$, $\frac{dy}{dx}=$

(A) -12 [✓ (B) -10] (C) $-\frac{10}{7}$ (D) 12 (E) 32

$6x-2y-2xy'+3y'=0$
$6(2)-2(11)-2(2)y'+3y'=0$
$-y'=10$
$y'=-10$

5. If $x^2-2xy+3y^2=8$, then $\frac{dy}{dx}=$

(A) $\frac{8+2y-2x}{6y-2x}$ (D) $\frac{1}{3}$

(B) $\frac{3y-x}{y-x}$ (E) $\frac{y-x}{3y-x}$

[✓ (C) $\frac{2x-2y}{6y-2x}$]

$2x-2y-2xy'+6yy'=0$
$y'(6y-2x)=2x-2y$
$y'=\frac{2x-2y}{6y-2x}=\frac{x-y}{3y-x}\left(\frac{-1}{-1}\right)$
$\frac{y-x}{x-3y}$

14. Find $\frac{dy}{dx}$ if $x^3y+xy^3=-10$.

(A) $(3x^2+3xy^2)$

(B) $-(3x^2+3xy^2)$

(C) $\frac{(3x^2y+y^3)}{(3xy^2+x^3)}$

[✓ (D) $-\frac{(3x^2y+y^3)}{(3xy^2+x^3)}$]

(E) $-\frac{(x^2y+y^3)}{(xy^2+x^3)}$

$3x^2y+x^3y'+1y^3+x3y^2y'=0$
$y'(x^3+3xy^2)=-y^3-3x^2y$
$y'=\frac{-y^3-3x^2y}{x^3+3xy^2}=-\frac{3x^2y+y^3}{x^3+3xy^2}$

17. Find the equation of the tangent line to $9x^2+16y^2=52$ through $(2,-1)$.

(A) $-9x+8y-26=0$

[✓ (B) $9x-8y-26=0$]

(C) $9x-8y-106=0$

(D) $8x+9y-17=0$

(E) $9x+16y-2=0$

$18x+32yy'=0$
$18(2)+32(-1)y'=0$
$-32y'=-36$
$y'=\frac{36}{32}=\frac{9}{8}$

$y+1=\frac{9}{8}(x-2)$ $(\times 8)$
$8y+8=9(x-2)$
$8y+8=9x-18$
$-9x+8y+26=0$
$9x-8y-26=0$

C

33. Find the value(s) of $\frac{dy}{dx}$ of $x^2y+y^2=5$ at $y=1$.

(A) $-\frac{3}{2}$ only (B) $-\frac{2}{3}$ only (C) $\frac{2}{3}$ only [✓ (D) $\pm\frac{2}{3}$] (E) $\pm\frac{3}{2}$

$x^2+1=5$
$x^2=4$
$x=\pm 2$

$2xy+x^2y'+2yy'=0$
$y'(x^2+2y)=-2xy$
$y'=\frac{-2xy}{x^2+2y}$

$\frac{-2(2)(1)}{2^2+2(1)}=\frac{-4}{6}$ $\frac{-2(-2)(1)}{(-2)^2+2(1)}=\frac{4}{6}$

AP Calculus **Section 2-5 D** Name________________________ Pd____

1. Find $\frac{dy}{dx}$ for $2\sin x \bullet \cos y = 1$

2. Find $\frac{dy}{dx}$ for $9y^2 = x^2 + 9$

3. Find the equation of the tangent line for $(4 - x)y^2 = x^3$ at (2, 2)

4. Find the equation of the tangent line for $x \bullet \cos y = 1$ at $(2, \pi/3)$

5. Find the equation of the tangent line for $x^3 + y^3 - 6xy = 0$ at (4/3, 8/3)

Find $\frac{d^2y}{dx^2}$ for $3x^2 - 2xy + 3y = 1$ at (2, 11)

Find $\frac{d^2y}{dx^2}$ for $x^2y + y^2 = 1$ at (2, 1)

Extra Practice – CALCULATOR SECTION – State the Equation of the tangent line

1. $h(x) = e^{\sqrt{x+2}} - x$ at x = 4.2

2. $j(x) = \ln(x^3 - 2) + \sec(x)$ at x = 3.1

3. $k(x) = \sqrt{\ln(x+2)} + \left(\frac{1}{x+1}\right)^2$ at x = 4.9

AP Calculus **Section 2-5 D** Name Key Pd____

1. Find $\frac{dy}{dx}$ for $2\sin x \bullet \cos y = 1$

$$2\cos x \cdot \cos y + 2\sin x(-\sin y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = + \frac{2\cos x \cos y}{2\sin x \sin y} = \cot x \cot y$$

2. Find $\frac{dy}{dx}$ for $9y^2 = x^2 + 9$

$$18y\ y' = 2x$$

$$y' = \frac{2x}{18y} = \frac{x}{9y}$$

3. Find the equation of the tangent line for $(4 - x)y^2 = x^3$ at (2, 2)

$$y - 2 = 2(x - 2)$$

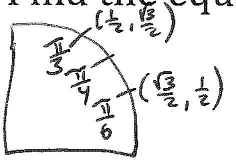
$$-1y^2 + (4-x)2y\frac{dy}{dx} = 3x^2$$

$$-1(2)^2 + (4-2)2(2)\frac{dy}{dy} = 3(2)^2$$

$$-4 + 8\frac{dy}{dx} = 12$$

$$\frac{dy}{dx} = 2$$

4. Find the equation of the tangent line for $x \bullet \cos y = 1$ at $(2, \pi/3)$



$$y - \frac{\pi}{3} = \frac{1}{2\sqrt{3}}(x - 2)$$

$$1\cos y + x(-\sin y)\frac{dy}{dx} = 0$$

$$\cos(\tfrac{\pi}{3}) - 2(\sin(\tfrac{\pi}{3}))\frac{dy}{dx} = 0$$

$$\frac{1}{2} - 2(\tfrac{\sqrt{3}}{2})\frac{dy}{dx} = 0$$

$$-\sqrt{3}\frac{dy}{dx} = -\frac{1}{2} \qquad \frac{dy}{dx} = \frac{1}{2\sqrt{3}}$$

5. Find the equation of the tangent line for $x^3 + y^3 - 6xy = 0$ at (4/3, 8/3)

$$y - \frac{8}{3} = \frac{4}{5}(x - \tfrac{4}{3})$$

$$3x^2 + 3y^2y' - 6y - 6xy' = 0$$

$$3(\tfrac{4}{3})^2 + 3(\tfrac{8}{3})^2y' - 6(\tfrac{8}{3}) - 6(\tfrac{4}{3})y' = 0$$

$$\frac{16}{3} + \frac{64}{3}y' - 16 - 8y' = 0$$

$$\frac{40}{3}y' = \frac{32}{3}$$

$$y' = \frac{32}{3} \cdot \frac{3}{40} = \frac{4}{5}$$

Find $\frac{d^2y}{dx^2}$ for $3x^2 - 2xy + 3y = 1$ at (2, 11)

Find $\frac{d^2y}{dx^2}$ for $x^2y + y^2 = 1$ at (2, 1)

Extra Practice – CALCULATOR SECTION – State the Equation of the tangent line

1. $h(x) = e^{\sqrt{x+2}} - x$ at $x = 4.2$

2. $j(x) = \ln(x^3 - 2) + \sec(x)$ at $x = 3.1$

3. $k(x) = \sqrt{\ln(x+2)} + \left(\frac{1}{x+1}\right)^2$ at $x = 4.9$