Calculus **Section 2-4A** Name________________________ Pd____

Original Functions	*Inside*	*Outside*	*Derivative*
1. $y = (6x - 5)^4$			
2. $y = \dfrac{1}{\sqrt{x+1}}$			
3. $y = \sqrt{x^2 - 1}$			
4. $y = 3\tan(\pi x^2)$			
5. $y = \csc^3 x$			
6. $y = \cos\dfrac{3x}{2}$			

Find the derivative:

7. $y = (2x - 7)^3$

8. $y = (2x^3 + 1)^2$

9. $g(x) = 3(4 - 9x)^4$

10. $y = 3(4 - x^2)^5$

11. $f(x) = (9 - x^2)^{2/3}$

12. $f(t) = (9t + 2)^{2/3}$

13. $f(t) = \sqrt{1 - t}$

14. $g(x) = \sqrt{5 - 3x}$

15. $y = \sqrt[3]{9x^2 + 4}$

16. $g(x) = \sqrt{x^2 - 2x + 1}$

17. $y = 2\sqrt[4]{4 - x^2}$

18. $f(x) = -3\sqrt[4]{2 - 9x}$

19. $y = \dfrac{1}{x - 2}$

20. $s(t) = \dfrac{1}{t^2 + 3t - 1}$

1. If $f(x) = \frac{x+3}{x^2+1}$, then $f'(-2) =$

(A) $-\frac{9}{25}$

(B) $-\frac{1}{4}$

(C) $\frac{1}{25}$

(D) $\frac{1}{4}$

(E) $\frac{9}{25}$

7. If $f(x) = x^2\sqrt{3x+1}$, then $f'(x) =$

(A) $\frac{-3x^2-2x}{\sqrt{3x+1}}$

(B) $\frac{9x^2+2x}{\sqrt{3x+1}}$

(C) $\frac{-9x^2+4x}{2\sqrt{3x+1}}$

(D) $\frac{15x^2+4x}{2\sqrt{3x+1}}$

(E) $\frac{-9x^2-4x}{2\sqrt{3x+1}}$

8. An equation of the line normal to the graph of $y = \sqrt{(3x^2+2x)}$ at (2, 4) is

(A) $-4x + y = 20$ (B) $4x + 7y = 20$ (C) $-7x + 4y = 2$ (D) $7x + 4y = 30$ (E) $4x + 7y = 36$

26. If $y = \left(\frac{x^3-2}{2x^5-1}\right)^4$, find $\frac{dy}{dx}$ at $x = 1$.

(A) -52 (B) -28 (C) -13 (D) 13 (E) 52

Calculus **Section 2-4A** Name Key Pd____

Original Functions	*Inside*	*Outside*	*Derivative*
1. $y = (6x - 5)^4$			
2. $y = \dfrac{1}{\sqrt{x+1}}$			
3. $y = \sqrt{x^2 - 1}$			
4. $y = 3\tan(\pi x^2)$			
5. $y = \csc^3 x$			
6. $y = \cos \dfrac{3x}{2}$			

Find the derivative:

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8. $y = (2x^3 + 1)^2$

9. $g(x) = 3(4 - 9x)^4$

10. $y = 3(4 - x^2)^5$

11. $f(x) = (9 - x^2)^{2/3}$

12. $f(t) = (9t + 2)^{2/3}$

13. $f(t) = \sqrt{1 - t}$

14. $g(x) = \sqrt{5 - 3x}$

15. $y = \sqrt[3]{9x^2 + 4}$

16. $g(x) = \sqrt{x^2 - 2x + 1}$

17. $y = 2\sqrt[4]{4 - x^2}$

18. $f(x) = -3\sqrt[4]{2 - 9x}$

19. $y = \dfrac{1}{x - 2}$

20. $s(t) = \dfrac{1}{t^2 + 3t - 1}$

1. If $f(x) = \dfrac{x+3}{x^2+1}$, then $f'(-2) =$

(A) $-\dfrac{9}{25}$

(B) $-\dfrac{1}{4}$

(C) $\dfrac{1}{25}$

(D) $\dfrac{1}{4}$

(E) $\dfrac{9}{25}$

$$\frac{(x^2+1)(1)-(x+3)(2x)}{(x^2+1)^2}$$

at $x=-2$

$$\frac{(4+1)-(-2+3)(2(-2))}{(4+1)^2}$$

$$\frac{5+4}{5^2} = \frac{9}{25}$$

7. If $f(x) = x^2\sqrt{3x+1}$, then $f'(x) =$

$(3x+1)^{1/2}$

(A) $\dfrac{-3x^2-2x}{\sqrt{3x+1}}$

(B) $\dfrac{9x^2+2x}{\sqrt{3x+1}}$

(C) $\dfrac{-9x^2+4x}{2\sqrt{3x+1}}$

(D) $\dfrac{15x^2+4x}{2\sqrt{3x+1}}$

(E) $\dfrac{-9x^2-4x}{2\sqrt{3x+1}}$

See the product rule

$$2x \cdot \sqrt{3x+1} + x^2 \cdot \tfrac{1}{2}(3x+1)^{-1/2}(3)$$

$$2x\sqrt{3x+1} + \frac{3}{2}\frac{x^2}{\sqrt{3x+1}}$$

$$\frac{2x\sqrt{3x+1}}{1} \cdot \frac{2\sqrt{3x+1}}{2\sqrt{3x+1}}$$

$$\frac{4x(3x+1)+3x^2}{2\sqrt{3x+1}}$$

$$12x^2+4x+3x^2$$

$$\frac{15x^2+4x}{2\sqrt{3x+1}}$$

$\perp$ to tangent

8. An equation of the line normal to the graph of $y = \sqrt{(3x^2+2x)}$ at (2, 4) is

(A) $-4x+y=20$ (B) $4x+7y=20$ (C) $-7x+4y=2$ (D) $7x+4y=30$ (E) $4x+7y=36$

Find tangent slope

$$y = (3x^2+2x)^{1/2}$$

$$y' = \tfrac{1}{2}(3x^2+2x)^{-1/2}(6x+2)$$

at $x=2$

$$y' = \tfrac{1}{2}(3\cdot4+2\cdot2)^{-1/2}(6\cdot2+2)$$

$$= \frac{14}{2\sqrt{16}} = \frac{14}{8} = \frac{7}{4}$$

$\perp$ to $\frac{7}{4}$ is $-\frac{4}{7}$

$$y-4 = -\tfrac{4}{7}(x-2)$$

$$7\left[y-4 = -\tfrac{4}{7}x+\tfrac{8}{7}\right]$$

$$7y-28 = -4x+8$$

$$7y+4x=36$$

26. If $y = \left(\dfrac{x^3-2}{2x^5-1}\right)^4$, find $\dfrac{dy}{dx}$ at $x=1$.

(A) -52 (B) -28 (C) -13 (D) 13 (E) 52

$$4\left(\frac{x^3-2}{2x^5-1}\right)^3 \frac{(2x^5-1)(3x^2)-(x^3-2)(10x^4)}{(2x^5-1)^2}$$

at $x=1$

$$4\left(\frac{-1}{1}\right)^3 \frac{(1)(3)-(-1)(10)}{(1)^2}$$

$$-4\cdot(3+10)$$

$$-4(13) = -52$$

Calculus **Section 2-4B** Name_______________ Pd_____

Find the derivative:

21. $f(t) = \left(\frac{1}{t-3}\right)^2$

22. $y = -\frac{5}{(t+3)^3}$

23. $y = \frac{1}{\sqrt{x+2}}$

24. $g(t) = \sqrt{\frac{1}{t^2-2}}$

25. $f(x) = x^2(x-2)^4$

26. $f(x) = x(3x-9)^3$

27. $y = x\sqrt{1-x^2}$

28. $y = \frac{1}{2}x^2\sqrt{16-x^2}$

29. $y = \frac{x}{\sqrt{x^2+1}}$

30. $y = \frac{x}{\sqrt{x^4+4}}$

31. $g(x) = \left(\frac{x+5}{x^2+2}\right)^2$

32. $h(t) = \left(\frac{t^2}{t^3+2}\right)^2$

33. $f(v) = \left(\frac{1-2v}{1+v}\right)^3$

34. $g(x) = \left(\frac{3x^2-2}{2x+3}\right)^3$

47. $y = \cos 3x$

49. $g(x) = 3 \tan 4x$

51. $y = \sin(\pi x)^2$

53. $h(x) = \sin 2x \cos 2x$

55. $f(x) = \dfrac{\cot x}{\sin x}$

57. $y = 4 \sec^2 x$

59. $f(\theta) = \frac{1}{4} \sin^2 2\theta$

61. $f(t) = 3 \sec^2(\pi t - 1)$

63. $y = \sqrt{x} + \frac{1}{4} \sin(2x)^2$

65. $y = \sin(\cos x)$

48. $y = \sin \pi x$

50. $h(x) = \sec x^2$

52. $y = \cos(1 - 2x)^2$

54. $g(\theta) = \sec(\frac{1}{2}\theta) \tan(\frac{1}{2}\theta)$

56. $g(v) = \dfrac{\cos v}{\csc v}$

58. $y = 2 \tan^3 x$

60. $g(t) = 5 \cos^2 \pi t$

62. $h(t) = 2 \cot^2(\pi t + 2)$

64. $y = 3x - 5 \cos(\pi x)^2$

66. $y = \sin \sqrt[3]{x} + \sqrt[3]{\sin x}$

Calculus Section 2-4B Name Key Pd____

Find the derivative:

21. $f(t) = \left(\dfrac{1}{t-3}\right)^2$

$2\left(\dfrac{1}{t-3}\right)\cdot\left(\dfrac{(t-3)\cdot 0 - 1(1)}{(t-3)^2}\right)$

OR $(t-3)^{-2}$ $-2(t-3)^{-3}\cdot 1$

22. $y = -\dfrac{5}{(t+3)^3}$

23. $y = \dfrac{1}{\sqrt{x+2}} = (x+2)^{-1/2}$

$-\frac{1}{2}(x+2)^{-3/2}(1)$

24. $g(t) = \sqrt{\dfrac{1}{t^2-2}}$

25. $f(x) = x^2(x-2)^4$

$2x(x-2)^4 + x^2(4)(x-2)^3$

26. $f(x) = x(3x-9)^3$

27. $y = x\sqrt{1-x^2} = x(1-x^2)^{1/2}$

$1(1-x^2)^{1/2} + x\left(\frac{1}{2}\right)(1-x^2)^{-1/2}(-2x)$

28. $y = \frac{1}{2}x^2\sqrt{16-x^2}$

29. $y = \dfrac{x}{\sqrt{x^2+1}}$

$\dfrac{\sqrt{x^2+1}\,(1) - x\left(\frac{1}{2}\right)(x^2+1)^{-1/2}(2x)}{(x^2+1)}$

30. $y = \dfrac{x}{\sqrt{x^4+4}}$

31. $g(x) = \left(\dfrac{x+5}{x^2+2}\right)^2$

$2\left(\dfrac{x+5}{x^2+2}\right)\left(\dfrac{(x^2+2)(1)-(x+5)(2x)}{(x^2+2)^2}\right)$

32. $h(t) = \left(\dfrac{t^2}{t^3+2}\right)^2$

33. $f(v) = \left(\dfrac{1-2v}{1+v}\right)^3$

$3\left(\dfrac{1-2v}{1+v}\right)^2\cdot\left(\dfrac{(1+v)(-2)-(1-2v)(1)}{(1+v)^2}\right)$

34. $g(x) = \left(\dfrac{3x^2-2}{2x+3}\right)^3$

47. $y = \cos 3x$ $\quad y' = -\sin(3x)(3)$

49. $g(x) = 3 \tan 4x$

51. $y = \sin(\pi x)^2$ $\quad y' = 2\sin(\pi x)\cos(\pi x)\pi$

53. $h(x) = \sin 2x \cos 2x$

55. $f(x) = \dfrac{\cot x}{\sin x}$ $\quad f'(x) = \dfrac{\sin x(-\csc^2 x) - \cot x \cos x}{\sin^2 x}$

57. $y = 4 \sec^2 x$

59. $f(\theta) = \frac{1}{4} \sin^2 2\theta$ $\quad f'(\theta) = \frac{1}{4}(2)\sin(2\theta)\cos(2\theta)(2)$

61. $f(t) = 3 \sec^2(\pi t - 1)$

63. $y = \sqrt{x} + \frac{1}{4} \sin(2x)^2$ $\quad y' = \frac{1}{2}x^{-1/2} + \frac{1}{4}(2)\sin(2x)\cos(2x)(2)$

65. $y = \sin(\cos x)$

48. $y = \sin \pi x$ $\quad y' = -\cos(\pi x)(\pi)$

50. $h(x) = \sec x^2$

52. $y = \cos(1 - 2x)^2$ $\quad y' = 2\cos(1-2x)(-\sin(1-2x))(-2)$

54. $g(\theta) = \sec\left(\frac{1}{2}\theta\right) \tan\left(\frac{1}{2}\theta\right)$

56. $g(v) = \dfrac{\cos v}{\csc v}$ $\quad g'(v) = \dfrac{\csc v(-\sin v) - \cos v(-\csc v \cot v)}{\csc^2 v}$

58. $y = 2 \tan^3 x$

60. $g(t) = 5 \cos^2 \pi t$ $\quad g'(t) = 10\cos(\pi t)(-\sin(\pi t))(\pi)$

62. $h(t) = 2 \cot^2(\pi t + 2)$

64. $y = 3x - 5 \cos(\pi x)^2$ $\quad y' = 3 - 10\cos(\pi x)(-\sin(\pi x))(\pi$

66. $y = \sin \sqrt[3]{x} + \sqrt[3]{\sin x}$

Calculus **Section 2-4C** Name______________________________ Pd_____

State the equation of the tangent line at the given point:

Function	*Point*
67. $s(t) = \sqrt{t^2 + 2t + 8}$	$(2, 4)$
68. $y = \sqrt[5]{3x^3 + 4x}$	$(2, 2)$
69. $f(x) = \dfrac{3}{x^3 - 4}$	$\left(-1, -\dfrac{3}{5}\right)$
70. $f(x) = \dfrac{1}{(x^2 - 3x)^2}$	$\left(4, \dfrac{1}{16}\right)$

Use your calculator to find the equation of the tangent line.

Function	x value	y-value	Slope	Equation of the tangent line
71. $f(t) = \dfrac{3t + 2}{t - 1}$	x = 0			
72. $f(x) = \dfrac{x + 1}{2x - 3}$	x = 2			
73. $y = 37 - \sec^3(2x)$	x = 0			
74. $y = \dfrac{1}{x} + \sqrt{\cos x}$	x = π/2			

9. If $f(x) = x^2 \cos 2x$, find $f'(x)$.

(A) $2x\sin 2x$

(B) $-2x\cos 2x + 2x^2 \sin 2x$

(C) $-4x\sin 2x$

(D) $2x\cos 2x - 2x^2 \sin 2x$

(E) $2x - 2\sin 2x$

15. If $f(x) = \sqrt{1+\sqrt{x}}$, find $f'(x)$.

(A) $\dfrac{-1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$

(B) $\dfrac{1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$

(C) $\dfrac{1}{4\sqrt{1+\sqrt{x}}}$

(D) $\dfrac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$

(E) $\dfrac{-1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$

24. $\lim_{x\to 0}\frac{\tan^3(2x)}{x^3}=$

(A) -8
(B) -2
(C) 2
(D) 8
(E) The limit does not exist.

26. If $f(x)=\cos^3(x+1)$ then $f'(\pi)=$

(A) $-3\cos^2(\pi+1)\sin(\pi+1)$
(B) $3\cos^2(\pi+1)$
(C) $3\cos^2(\pi+1)\sin(\pi+1)$
(D) $3\pi\cos^2(\pi+1)$
(E) 0

12. What is the equation of the line tangent to the graph of y = $\sin^2 x$ at $x=\frac{\pi}{4}$?

(A) $y-\frac{1}{2}=-\left(x-\frac{\pi}{4}\right)$

(B) $y-\frac{1}{2}=\left(x-\frac{\pi}{4}\right)$

(C) $y-\frac{1}{\sqrt{2}}=\left(x-\frac{\pi}{4}\right)$

(D) $y-\frac{1}{\sqrt{2}}=\frac{1}{2}\left(x-\frac{\pi}{4}\right)$

(E) $y-\frac{1}{2}=\frac{1}{2}\left(x-\frac{\pi}{4}\right)$

20. If $f(x)=\sqrt{(x^3+5x+121)}\,(x^2+x+11)$ then $f'(0)=$

(A) $\frac{5}{2}$ (B) $\frac{27}{2}$ (C) 22 (D) $22+\frac{2}{\sqrt{5}}$ (E) $\frac{247}{2}$

C
43. Two particles leave the origin at the same time and move along the y-axis with their respective positions determined by the functions $y_1=\cos 2t$ and $y_2=4\sin t$ for $0<t<6$. For how many values of t do the particles have the same acceleration? 2

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Calculus **Section 2-4C** Name Key Pd____

State the equation of the tangent line at the given point:

Function	*Point*
67. $s(t) = \sqrt{t^2 + 2t + 8}$	$(2, 4)$
68. $y = \sqrt[5]{3x^3 + 4x}$	$(2, 2)$
69. $f(x) = \dfrac{3}{x^3 - 4}$	$\left(-1, -\dfrac{3}{5}\right)$
70. $f(x) = \dfrac{1}{(x^2 - 3x)^2}$	$\left(4, \dfrac{1}{16}\right)$

Use your calculator to find the equation of the tangent line.

Function	x value	y-value	Slope	Equation of the tangent line
71. $f(t) = \dfrac{3t + 2}{t - 1}$	x = 0			
72. $f(x) = \dfrac{x + 1}{2x - 3}$	x = 2			
73. $y = 37 - \sec^3(2x)$	x = 0			
74. $y = \dfrac{1}{x} + \sqrt{\cos x}$	x = π/2			

9. If $f(x) = x^2 \cos 2x$, find $f'(x)$.

(A) $2x \sin 2x$

(B) $-2x \cos 2x + 2x^2 \sin 2x$

(C) $-4x \sin 2x$

(D) $2x \cos 2x - 2x^2 \sin 2x$

(E) $2x - 2\sin 2x$

$2x \cos 2x + x^2(-\sin 2x\,(2))$

$2x \cos 2x - 2x^2 \sin 2x$

15. If $f(x) = \sqrt{1 + \sqrt{x}}$, find $f'(x)$.

$(1 + x^{1/2})^{1/2}$

$\frac{1}{2}(1 + x^{1/2})^{-1/2}\left(\frac{1}{2}x^{-1/2}\right)$

$\dfrac{1}{2\sqrt{1+\sqrt{x}}} \cdot \dfrac{1}{2\sqrt{x}}$

$\dfrac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$

(A) $\dfrac{-1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$

(B) $\dfrac{1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$

(C) $\dfrac{1}{4\sqrt{1+\sqrt{x}}}$

(D) $\dfrac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$

(E) $\dfrac{-1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$

24. $\lim_{x\to 0}\frac{\tan^3(2x)}{x^3} =$ $\left(\frac{\tan 2x}{x}\right)^3 = \frac{2}{1}$

(A) -8
(B) -2
(C) 2
(D) 8
(E) The limit does not exist.

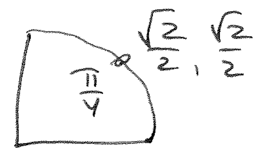
26. If $f(x)=\cos^3(x+1)$ then $f'(\pi)=$

(A) $-3\cos^2(\pi+1)\sin(\pi+1)$
(B) $3\cos^2(\pi+1)$
(C) $3\cos^2(\pi+1)\sin(\pi+1)$
(D) $3\pi\cos^2(\pi+1)$
(E) 0

$3(\cos(x+1))^2 \cdot (-\sin(x+1))\cdot 1$

$-3\cos^2(\pi+1)\sin(\pi+1)$

12. What is the equation of the line tangent to the graph of $y=\sin^2 x$ at $x=\frac{\pi}{4}$?



(A) $y-\frac{1}{2}=-\left(x-\frac{\pi}{4}\right)$
(B) $y-\frac{1}{2}=\left(x-\frac{\pi}{4}\right)$
(C) $y-\frac{1}{\sqrt2}=\left(x-\frac{\pi}{4}\right)$
(D) $y-\frac{1}{\sqrt2}=\frac{1}{2}\left(x-\frac{\pi}{4}\right)$
(E) $y-\frac{1}{2}=\frac{1}{2}\left(x-\frac{\pi}{4}\right)$

$2\sin x\cos x$

$\frac{2}{1}\cdot\left(\frac{\sqrt2}{2}\right)\left(\frac{\sqrt2}{2}\right)=1$

slope $=1$

$\left(\sin\frac{\pi}{4}\right)^2 = \left(\frac{\sqrt2}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$

y-value $=\frac{1}{2}$

$y-\frac{1}{2}=1\left(x-\frac{\pi}{4}\right)$

20. If $f(x)=\sqrt{(x^3+5x+121)}\,(x^2+x+11)$ then $f'(0)=$

(A) $\frac{5}{2}$ (B) $\frac{27}{2}$ (C) 22 (D) $22+\frac{2}{\sqrt5}$ (E) $\frac{247}{2}$

$\frac{1}{2}(x^3+5x+121)^{-1/2}(3x^2+5)(x^2+x+11)+\sqrt{x^3+5x+121}\,(2x+1)$

$x=0$ $\quad \frac{1\cdot5\cdot11}{2\sqrt{121}}+\sqrt{121}\cdot1 = \frac{55}{22}+11 = \frac{5}{2}+\frac{22}{2}=\frac{27}{2}$

C

43. Two particles leave the origin at the same time and move along the y-axis with their respective positions determined by the functions $y_1=\cos 2t$ and $y_2=4\sin t$ for $0<t<6$. For how many values of t do the particles have the same acceleration? 2

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

$y_1'=-\sin 2t\cdot 2$
$y_1''=-2\cos 2t\cdot 2$
$=-4\cos 2t$

$y_2'=4\cos t$
$y_2''=-4\sin t$