Calculus Test 3 Prep

Review for Quiz 9 (Sec 4-2 to 4-4)

Riemann Sums

9–12 Use the Midpoint Rule with the given value of n to approximate the integral. Round the answer to four decimal places.

9. $\int_0^8 \sin\sqrt{x}\, dx, \quad n = 4$

Note: be ready for Left, Right and Trap Rule as well.

Evaluate the integral by interpreting it in terms of areas.

37. $\int_{-3}^{0} \left(1 + \sqrt{9 - x^2}\right) dx$

Fundamental Theorem of Calculus

19–42 Evaluate the integral.

19. $\int_{-2}^{3} (x^2 - 3)\, dx$

21. $\int_{-2}^{0} \left(\frac{1}{2}t^4 + \frac{1}{4}t^3 - t\right) dt$

29. $\int_{1}^{4} \sqrt{\frac{5}{x}}\, dx$

34. $\int_{0}^{\pi/3} \frac{\sin\theta + \sin\theta \tan^2\theta}{\sec^2\theta}\, d\theta$

Fundamental Theorem of Calculus part 2

EXAMPLE 2 Find the derivative of the function $g(x) = \int_{0}^{x} \sqrt{1 + t^2}\, dt$.

11. $F(x) = \int_{x}^{0} \sqrt{1 + \sec t}\, dt$

find F'(x)

56. $g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2 + t^4}}\, dt$

Find g'(x)

Evaluate the following Integrals:

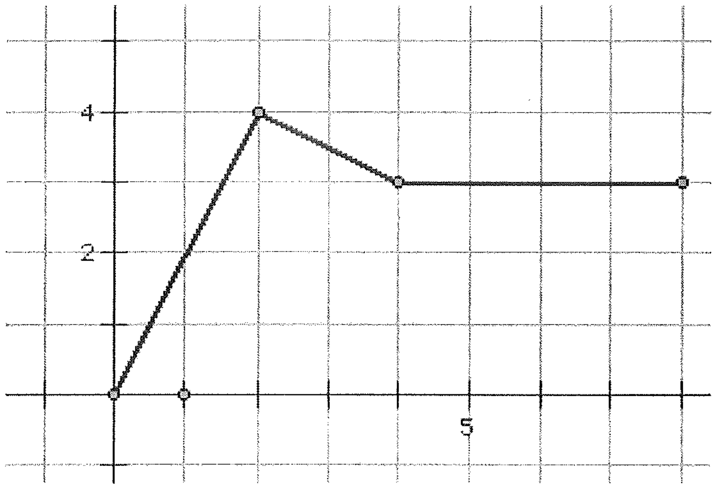
1. $\int(\cos x + x^3)dx$

2. $\int\left(x + \frac{3}{x^4} + 6\sqrt{x}\right)dx$

3. $\int 4\csc\theta\cot\theta\, d\theta$ *given that* $\left(\frac{\pi}{2}, -2\right)$ *is on the original*

4. $\int \frac{\sqrt{x}+8}{x^2}dx$ given that $(4, -7)$ is on the original

Car 1 has a velocity that is shown in the graph below
where t is in seconds and the rate is in feet per second.



Car 2 has a a velocity that is defined by v(t) = $2\sqrt{t}$

a. Find the velocity of car 1 and car 2 at t = 5 seconds. Include units.

b. Find the acceleration of car 1 and car 2 at t = 1 second. Include units.

c. Find the total distance travelled by car 1 and the total distance travelled by car 2 over the time interval $0 \leq t \leq 8$. Indicate units.

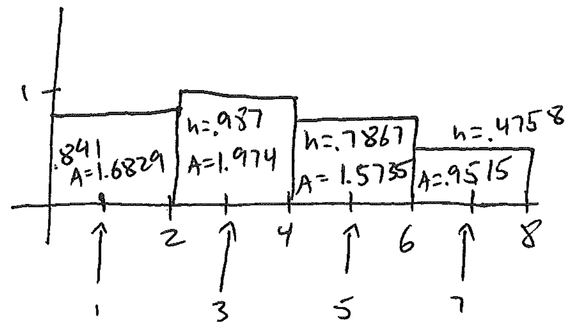
Calculus Test 3 Prep

Review for Quiz 9 (Sec 4-2 to 4-4)

Riemann Sums

9–12 Use the Midpoint Rule with the given value of n to approximate the integral. Round the answer to four decimal places.

9. $\int_0^8 \sin\sqrt{x}\,dx, \quad n = 4$



f(1) f(3) f(5) f(7)

.841 .987 .7867 .4758

total Area=6.1819

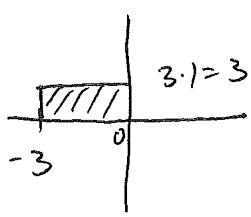
Note: be ready for Left, Right and Trap Rule as well.

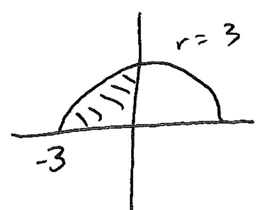
Evaluate the integral by interpreting it in terms of areas.

$x^2+y^2=r^2$

37. $\int_{-3}^{0} \left(1 + \sqrt{9 - x^2}\right) dx$

rect. circle





$\frac{1}{4}\pi 3^2 = \frac{9}{4}\pi$

$3 + \frac{9}{4}\pi = 10.068$

Fundamental Theorem of Calculus

19–42 Evaluate the integral.

19. $\int_{-2}^{3} (x^2 - 3)\, dx$ $\quad \frac{1}{3}x^3 - 3x \Big|_{-2}^{3} = \left[\frac{1}{3}(3)^3 - 3(3)\right] - \left[\frac{1}{3}(-2)^3 - 3(-2)\right]$

$-3\frac{1}{3}$

* **21.** $\int_{-2}^{0} \left(\frac{1}{2}t^4 + \frac{1}{4}t^3 - t\right) dt$

$\frac{1}{2}\cdot\frac{1}{5}t^5 + \frac{1}{4}\cdot\frac{1}{4}t^4 - \frac{1}{2}t^2 \Big|_{-2}^{0} = [0] - \left[\frac{1}{10}(-2)^5 + \frac{1}{16}(-2)^4 - \frac{1}{2}(-2)^2\right] = 4.2$

*

$\frac{\sqrt{5}}{\sqrt{x}} = \sqrt{5}\, x^{-1/2}$

29. $\int_{1}^{4} \sqrt{\frac{5}{x}}\, dx = \sqrt{5}\int_1^4 x^{-1/2} dx = \sqrt{5}\left[2x^{\frac{1}{2}}\right]\Big|_1^4 = \sqrt{5}\left[2\sqrt{4} - 2\sqrt{1}\right]$

$4 - 2$

$= 2\sqrt{5}$

4.472

$\frac{\sin^2 + \cos^2 = 1}{\cos^2}$

$\tan^2 + 1 = \sec^2$

$\frac{\sin\theta(1 + \tan^2\theta)}{\sec^2\theta}$

34. $\int_{0}^{\pi/3} \frac{\sin\theta + \sin\theta\, \tan^2\theta}{\sec^2\theta}\, d\theta$ $\quad \frac{\sin\theta(\sec^2\theta)}{\sec^2\theta} = \sin\theta$

$\int_0^{\pi/3} \sin\theta\, d\theta = -\cos\theta \Big|_0^{\frac{\pi}{3}} = -\cos\frac{\pi}{3} - -\cos 0 = .5$

*

Fundamental Theorem of Calculus part 2

EXAMPLE 2 Find the derivative of the function $g(x) = \int_0^x \underbrace{\sqrt{1 + t^2}}_{\text{in terms of } t}\, dt$. $= \sqrt{1+x^2}$

x

constant

11. $F(x) = \int_x^0 \sqrt{1 + \sec t}\, dt$ find F'(x) $\longrightarrow -\int_0^x \sqrt{1+\sec t}\, dt$

$-\sqrt{1+\sec x}$

56. $g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2 + t^4}}\, dt$ Find g'(x) $\longrightarrow$ $\text{anti}\Big|_{\tan x}^{x^2} = \text{anti}(x^2) - \text{anti}(\tan x)$

$g'(x) = \frac{1}{\sqrt{2 + (x^2)^4}}(2x) - \frac{1}{\sqrt{2 + (\tan x)^4}}\sec^2(x)$

$\frac{1}{\sqrt{2+x^8}}(2x) - \frac{1}{\sqrt{2+\tan^4 x}}\sec^2(x)$

Evaluate the following Integrals:

1. $\int(\cos x + x^3)dx$ $= \sin x + \frac{1}{4}x^4 + C$

2. $\int\left(x + \frac{3}{x^4} + 6\sqrt{x}\right)dx$

$\int\left(x + 3x^{-4} + 6x^{1/2}\right)dx = \frac{1}{2}x^2 + \frac{3}{-3}x^{-3} + \frac{6}{1}\cdot\frac{2}{3}x^{\frac{3}{2}} + C$

$\frac{1}{2}x^2 - \frac{1}{x^3} + 4x^{3/2} + C$

3. $\int 4\csc\theta\cot\theta\, d\theta$ *given that* $\left(\frac{\pi}{2}, -2\right)$ *is on the original*

$\theta \quad y$

*

* $\csc x$

$-\csc x \cot x$

$y = -4\csc\theta + C$

$-2 = -4\csc\left(\frac{\pi}{2}\right) + C$

$-2 = -4 + C$

$2 = C$

$y = -4\csc\theta + 2$

4. $\int\frac{\sqrt{x}+8}{x^2}dx$ given that (4, −7)is on the original

$x \quad y$

$\int\left(\frac{x^{\frac{1}{2}}}{x^2} + \frac{8}{x^2}\right)dx$

$\int\left(x^{-\frac{3}{2}} + 8x^{-2}\right)dx$

$y = -2x^{-\frac{1}{2}} - 8x^{-1} + C$

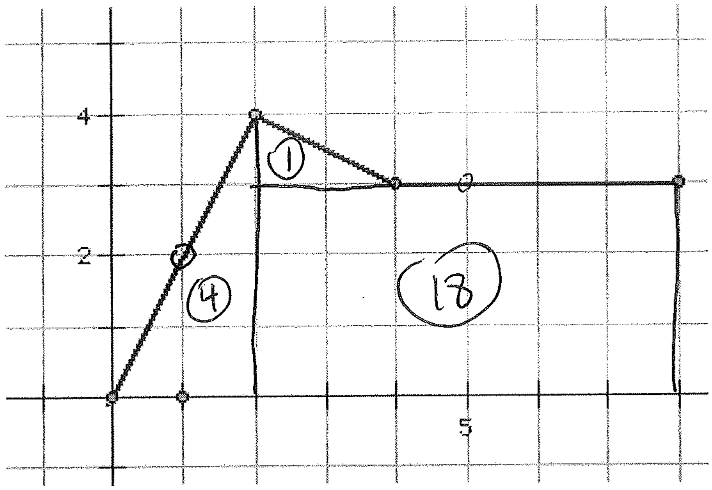
$-7 = -2(4)^{-1/2} - 8(4)^{-1} + C$

$-7 = -3 + C$

$-4 = C$

$y = \frac{-2}{\sqrt{x}} - \frac{8}{x} - 4$

Car 1 has a velocity that is shown in the graph below where t is in seconds and the rate is in <u>feet per second</u>.



Car 2 has a a velocity that is defined by $v(t) = 2\sqrt{t}$

a. Find the velocity of car 1 and car 2 at t = 5 seconds. Include units.

$\downarrow$

3 ft/sec

$v(5) = 2\sqrt{5} = 4.47$ ft/sec

b. Find the acceleration of car 1 and car 2 at t = 1 second. Include units.

$\downarrow$

2 ft/sec^2

$v(t) = 2t^{1/2}$

$a(t) = t^{-1/2}$

$a(1) = 1$^-.5 $= 1$ ft/sec^2

c. Find the total distance travelled by car 1 and the total distance travelled by car 2 over the time interval $0 \le t \le 8$. Indicate units.

$\int_0^8 v(t)\,dt = s(8) - s(0)$ = total displacement

Car 1

$\int_0^8 v(t)\,dt$ = area under the curve.

23 ft

Car 2

$\int_0^8 2t^{1/2}\,dt$

$\frac{2}{1} \cdot \frac{2}{3} t^{3/2} = \frac{4}{3} t^{3/2} \Big|_0^8$

$\frac{4}{3}(8)^{3/2} - \frac{4}{3}(0)^{3/2}$

30.169 ft