Calculus Test 3 Prep

Review for Quiz 8 (Sec 3-5 to 4-1)

Optimization:

1. If $y = x - 1$, then minimize the function $g(x) = xy + 5x$

2. A rectangle has a perimeter of 10. Maximize the area.

27. Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side L if one side of the rectangle lies on the base of the triangle.

48. A boat leaves a dock at 2:00 PM and travels due south at a speed of 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 3:00 PM. At what time were the two boats closest together?

<u>Differential Equations</u> (All will quiz on this topic, Only AP will test later on this topic)

1. Use differential equations to find the approximate value of f(3.01) given that f(3) = -1 and f′(3) = 2

2. Use the tangent line at x = -1 to approximate the value of g(-1.1) along g(x) = x^3 + x^2 – 2x

3. Use differential equations to find the approximate value of g(1.9) given that g(2) = 6 and g′(2) = -4

4. Use the tangent line at x = 2 to approximate the value x = 2.04 along the curve f(x) = $x^2 - 4x$

Calculus Test 3 Prep

Review for Quiz 8 (Sec 3-5 to 4-1)

Optimization:

1. If y = x – 1, then minimize the function g(x) = xy + 5x

$g(x) = x(x-1) + 5x = x^2 - x + 5x = x^2 + 4x$

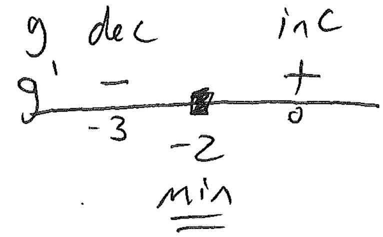
$y = -2 - 1$

$y = -3$

$g(x) = x^2 + 4x$

$g'(x) = 2x + 4$

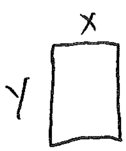
C.P. $x = -2$



$g(-2) = (-2)^2 + 4(-2) = -4$

$x = -2,\ y = -3$ min of $g(x)$ is -4

2. A rectangle has a perimeter of 10. Maximize the area.



$10 = 2x + 2y$

$5 = x + y$

$y = 5 - x$

Domain:

$0 \le x \le 5$

$A(x) = xy = x(5-x)$

$A(x) = 5x - x^2$

$A'(x) = 5 - 2x$

C.P. at $x = 2.5$

L $A(0) = 0$

CP $A(2.5) = 6.25$

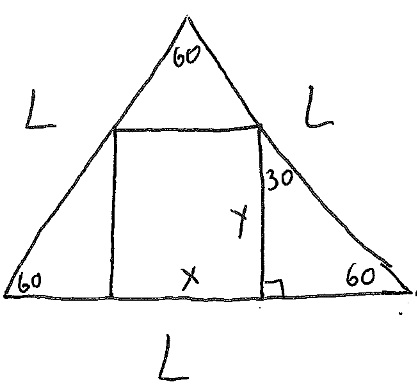
R $A(5) = 0$

ATQ. Max area is at

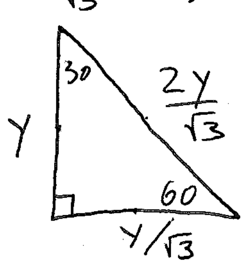
$x = 2.5 \quad y = 2.5$

Area is 6.25

* **27.** Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side L if one side of the rectangle lies on the base of the triangle.



$L = x + \frac{2y}{\sqrt{3}} \rightarrow x = L - \frac{2y}{\sqrt{3}}$



$A(y) = xy = \left(L - \frac{2y}{\sqrt{3}}\right)y$

$A(y) = Ly - \frac{2}{\sqrt{3}}y^2$

$A'(y) = L - \frac{4}{\sqrt{3}}y$

$0 = L - \frac{4}{\sqrt{3}}y$

$\left(\frac{-\sqrt{3}}{4}\right)(-L) = \frac{-4}{\sqrt{3}}y\left(\frac{-\sqrt{3}}{4}\right)$

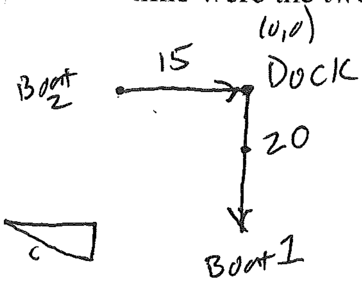
$\frac{\sqrt{3}}{4}L = y$ this is the C.P.

Frowning Parabola ∴ CP is at a Max

$x = L - \frac{2}{\sqrt{3}}\left(\frac{\sqrt{3}}{4}L\right) = L - \frac{1}{2}L = \frac{1}{2}L$

$A(y) = x \cdot y = \frac{1}{2}L\left(\frac{\sqrt{3}}{4}L\right) = \frac{\sqrt{3}}{8}L^2$

$\ast$ **48.** A boat leaves a dock at 2:00 PM and travels due south at a speed of 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 3:00 PM. At what time were the two boats closest together?



Boat 1: $d = 20t$, pair: $(0, -20t)$

Boat 2: $d = 15t$, pair: $(-15+15t, 0)$

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$d = \sqrt{(-15+15t-0)^2 + (0- -20t)^2}$

$d = \sqrt{225 - 450t + 225t^2 + 400t^2}$

$d = \sqrt{625t^2 - 450t + 225}$

$d'(t) = \frac{1}{2}(625t^2 - 450t + 225)^{-1/2}(1250t - 450)$

$d'(t) = \dfrac{1250t - 450}{2\sqrt{625t^2 - 450t + 225}}$

$1250t - 450 = 0$

$t = .36$

domain: $0 \le t \le 1$

L $d(0) = 15$

CP $d(.36) = 12$ min at time

R $d(1) = 20$

$\boxed{2:22}$

Differential Equations (All will quiz on this topic, Only AP will test later on this topic)

1. Use differential equations to find the approximate value of f(3.01) given that f(3) = -1 and f'(3) = 2

$\underset{y}{y - \underline{-1}} = \underset{m}{\underline{2}}(x - \underset{x}{\underline{3}})$

$x = 3.01 \quad y + 1 = 2(3.01 - 3)$

$y + 1 = .02$

$y = -.98$

2. Use the tangent line at x = -1 to approximate the value of g(-1.1) along g(x) = x^3 + x^2 – 2x

$y - \underline{2} = \underline{-1}(x - \underline{-1})$ $\quad g(-1) = 2$

$x = -1.1 \quad y - 2 = -1(-1.1 + 1)$ $\quad g'(x) = 3x^2 + 2x - 2$

$g'(-1) = -1$

$y - 2 = .1 \quad y = 1.9$

3. Use differential equations to find the approximate value of g(1.9) given that g(2) = 6 and g'(2) = -4

$y - \underline{6} = \underline{-4}(x - 2)$

$x = 1.9 \quad y - 6 = -4(1.9 - 2)$

$y - 6 = .4$

$y = 6.4$

4. Use the tangent line at x = 2 to approximate the value x = 2.04 along the curve f(x) = x² – 4x

$y - (-4) = 0(x - 2)$ $\quad f(2) = -4$

$y + 4 = 0$ $\quad f'(x) = 2x - 4$

$x = 2.04 \quad y = -4$ $\quad f'(2) = 0$