Calculus Test 2 Prep

** *Review for Quiz 7* **

Find the absolute maximum and minimum values of f on the given interval.

47. $f(x) = 2x^3 - 3x^2 - 12x + 1, \quad [-2, 3]$

53. $f(t) = t - \sqrt[3]{t}, \quad [-1, 4]$

55. $f(t) = 2\cos t + \sin 2t, \quad [0, \pi/2]$

11–14 Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

11. $f(x) = 2x^2 - 3x + 1$, $[0, 2]$

11. $f(x) = x^4 - 2x^2 + 3$

(a) Find the intervals on which f is increasing or decreasing.
(b) Find the local maximum and minimum values of f.
(c) Find the intervals of concavity and the inflection points.

45. Suppose the derivative of a function f is $f'(x) = (x + 1)^2(x - 3)^5(x - 6)^4$. On what interval is f increasing?

Use the function $f(x) = \frac{x^2-4}{x^2+6x+8}$ to answer the questions on this page.

1. State the intervals of increasing and decreasing.

2. State the Local Extremes (Min/Max) on the interval [-10, 10].

3. Find the absolute minimum on the interval [-10, 10].

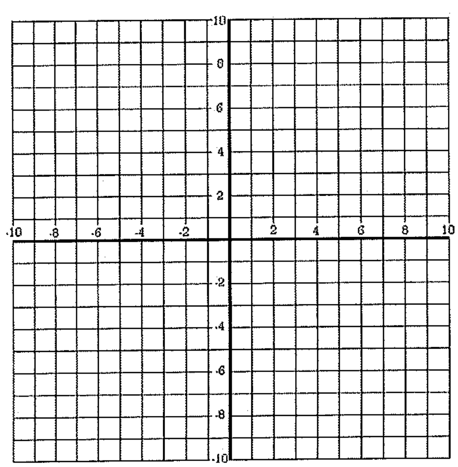
4. State the intervals of concavity on the interval [-10. 10].

5. State the location of any inflection point(s). Explain your answer.

6. Describe the end behavior of f(x) at ±infinity. Find any horizontal asymptotes.

7. Find the y-intercept, x-intercept(s), and any vertical asymptotes.

8. Sketch the curve using your solutions from 1-7.



Use the function $g(x) = (1/3)x^3 - (1/2)x^2 - 6x$ to answer the questions on this page.

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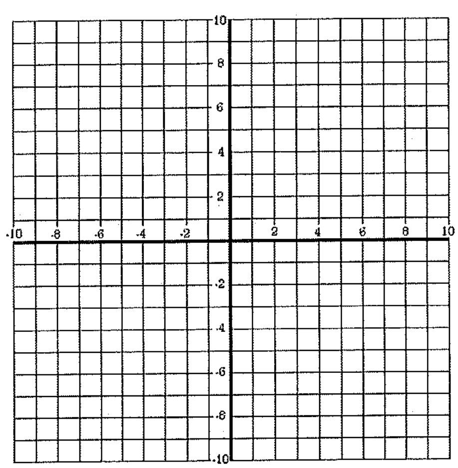

Use the function $h(x) = x \cdot \tan(x)$ on interval $[-\pi/2, \pi/2]$ to answer the questions on this page.

1. State the intervals of increasing and decreasing.

2. State the Local Extremes (Min/Max)

3. Find the absolute minimum and maximum

4. State the intervals of concavity

5. State the location of any inflection point(s). Explain your answer.

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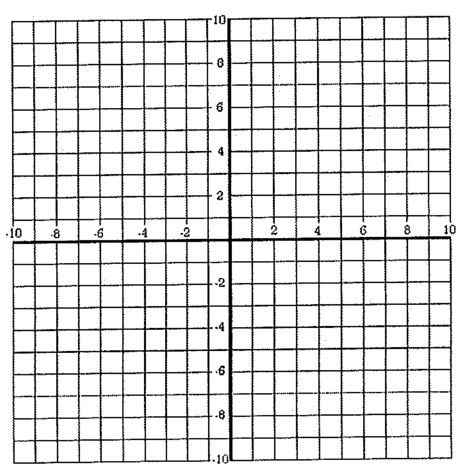


Calculus Test 2 Prep

*** Review for Quiz 7 ***

Find the absolute maximum and minimum values of f on the given interval.

47. $f(x) = 2x^3 - 3x^2 - 12x + 1, \quad [-2, 3]$

$f'(x) = 6x^2 - 6x - 12$

$0 = 6(x^2 - x - 2)$

$0 = (x-2)(x+1)$

critical pts

$x = 2 \,\&\, -1$

Test

L $f(-2) = -3$

cp $f(-1) = 8$ abs max

cp $f(2) = -19$ abs min

R $f(3) = -8$

---

53. $f(t) = t - \sqrt[3]{t}, \quad [-1, 4]$

$f'(t) = 1 - \frac{1}{3}t^{-2/3}$

$f'(t) = 1 - \frac{1}{3\sqrt[3]{t^2}}$

Critical Pts

$t = 0$

$t = \sqrt{1/27}$

$t = -\sqrt{1/27}$

$0 = 1 - \frac{1}{3}t^{-2/3}$

$t^{-2/3} = 3$

$t^{-2} = 27$

$\frac{1}{t^2} = \frac{27}{1}$

$27t^2 = 1 \quad t = \pm\sqrt{\frac{1}{27}}$

L $f(-1) = 0$

CP $f(-\sqrt{\frac{1}{27}}) = .385$

CP $f(0) = 0$

CP $f(\sqrt{\frac{1}{27}}) = -.385$ abs min

R $f(4) = 2.413$ abs Max

55. $f(t) = 2\cos t + \sin 2t, \quad [0, \pi/2]$

$f'(t) = -2\sin t + 2\cos(2t)$

$0 = -2\sin t + 2\cos(2t)$

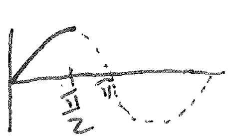
$2\sin t = 2\cos(2t)$

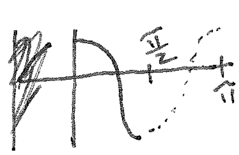
$\sin t = \cos(2t)$

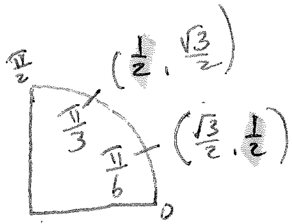
L: $f(0) = 2$

CP: $f(\frac{\pi}{6}) = 2.6$ max

R: $f(\frac{\pi}{2}) = 0$ min







$\sin t = \cos t$ at $t = \frac{\pi}{4}$

$t = .524$

$\sin\frac{\pi}{6} = \frac{1}{2}$

$\cos(2\cdot\frac{\pi}{6}) = \frac{1}{2}$

$\cos(\frac{\pi}{3}) = \frac{1}{2}$

Critical: $\frac{\pi}{6}$

(55)

Alternate Solution

Double Angle Theorem
$\sin(2x) = 2\sin x \cdot \cos x$

Pythagorean Identity

$\sin^2 x + \cos^2 x = 1$

so

$\cos^2 x = 1 - \sin^2 x$

$$f(t) = 2\cos t + \sin(2t)$$
$$f(t) = 2\cos t + 2\sin t\cos t$$
$$f'(t) = -2\sin t + 2\cos t\,\cancel{\cos}\,t - 2\sin t\sin t$$
$$f'(t) = -2\sin t + 2\cos^2 t - 2\sin^2 t$$
$$0 = -2\sin t + 2\cos^2 t - 2\sin^2 t$$

$\downarrow$

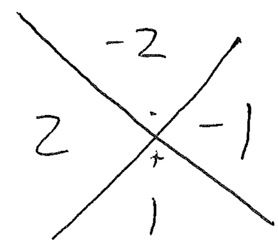
~~divide by~~

$$0 = -2\sin t + 2(1 - \sin^2 t) - 2\sin^2 t$$
$$0 = -2\sin t + 2 - 2\sin^2 t - 2\sin^2 t$$
$$0 = -4\sin^2 t - 2\sin t + 2$$

Divide by -2

$$0 = 2\sin^2 t + 1\sin t - 1$$

Factor Diamond



$$0 = 2\sin^2 t + 2\sin t - 1\sin t - 1$$
$$0 = 2\sin t(\sin t + 1) - 1(\sin t + 1)$$
$$0 = (2\sin t - 1)(\sin t + 1)$$

$2\sin t - 1 = 0$

$\sin t = \frac{1}{2}$

$t = \sin^{-1}\left(\frac{1}{2}\right)$

$t = \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin t + 1 = 0$

$\sin t = -1$

$t = \sin^{-1}(-1)$

$t = \frac{3\pi}{2}$

only $\frac{\pi}{6}$ is on the interval $[0, \frac{\pi}{2}]$

L $f(0) = 2$

CP $f\left(\frac{\pi}{6}\right) = 2.6 \leftarrow$ max

R $f\left(\frac{\pi}{2}\right) = 0 \leftarrow$ min

11–14 Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

11. $f(x) = 2x^2 - 3x + 1, \quad [0, 2]$

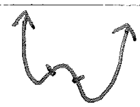
$\left.\begin{array}{l} f(0) = 1 \\ f(2) = 3 \end{array}\right\}$ secant slope $\frac{3-1}{2-0} = \frac{2}{2} = 1$

$f'(x) = 4x - 3$

$1 = 4x - 3$

$4 = 4x$

$1 = x$



11. $f(x) = x^4 - 2x^2 + 3$

(a) Find the intervals on which f is increasing or decreasing.

(b) Find the local maximum and minimum values of f.

(c) Find the intervals of concavity and the inflection points.

$f'(x) = 4x^3 - 4x$

$0 = 4x(x^2 - 1)$

$0 = 4x(x+1)(x-1)$

Criticals: $x = -1, 0, 1$

a) Inc: $[-1, 0] \cup [1, \infty)$

Dec: $(-\infty, -1] \cup [0, 1]$

b) Min: $f(-1) = 2$

Min: $f(1) = 2$

Max: $f(0) = 3$

f	dec	inc	dec	inc
f'	$-$	$+$	$-$	$+$
	-2	$-\frac{1}{2}$	$\frac{1}{2}$	2
	-1	0	1	

$f''(x) = 12x^2 - 4$

$0 = 12x^2 - 4$

$\frac{4}{12} = x^2$

$\pm\sqrt{\frac{1}{3}} = x$

f	up	down	up
f''	$+$	$-$	$+$
	-1	0	1
	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{1}{3}}$	

c) 2 inflection points at $x = \pm\sqrt{\frac{1}{3}}$

*

45. Suppose the derivative of a function f is $f'(x) = (x + 1)^2(x - 3)^5(x - 6)^4$. On what interval is f increasing?

Critical Pts at $x = -1, 3, 6$

inc on $[3, \infty)$

f	dec	dec	inc	inc
f'	$-$	$-$	$+$	$+$
	-2	0	4	7
	-1	3	6	

$\frac{(x+2)(x-2)}{(x+2)(x+4)} = \frac{x-2}{x+4}$ $\frac{-4}{2} = -2$

Use the function $f(x) = \frac{x^2-4}{x^2+6x+8}$ to answer the questions on this page.

hole: $x=-2$ v. asymptote at $x=-4$

1. State the intervals of increasing and decreasing.

$f(x) = \frac{x-2}{x+4}$ $f'(x) = \frac{(x+4)-(x-2)}{(x+4)^2} = \frac{6}{(x+4)^2}$

Critical Pts $x=-4$

f: inc, inc
f': +, +
-5, -4, 0

2. State the Local Extremes (Min/Max) on the interval [-10, 10].

always increasing

NO min/max

3. Find the absolute minimum on the interval [-10, 10].

up: $(-\infty, -4)$
down: $(-4, \infty)$

f: up, down
f'': +, −
-5, -4, 0

4. State the intervals of concavity on the interval [-10. 10].

$f'(x) = \frac{6}{(x+4)^2}$ $f''(x) = \frac{(x+4)^2(0) - 6 \cdot 2(x+4)}{(x+4)^4} = \frac{-12x-48}{(x+4)^4}$

Potential Infl. $x=-4$

5. State the location of any inflection point(s). Explain your answer.

$x=-4$

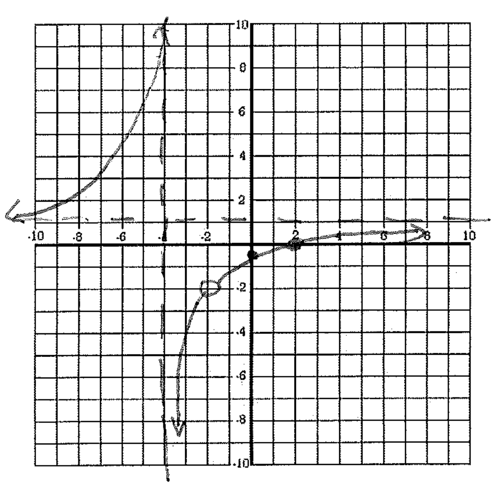
6. Describe the end behavior of f(x) at ±infinity. Find any horizontal asymptotes.

$\lim_{x\to\infty} \frac{x-2}{x+4} = 1$ $\lim_{x\to-\infty} = 1$ horiz asymp. at y = 1

7. Find the y-intercept, x-intercept(s), and any vertical asymptotes.

$\frac{0-2}{0+4} = -\frac{1}{2}$ $0 = \frac{x-2}{x+4}$ $x=2$ $x=-4$

8. Sketch the curve using your solutions from 1-7.



x	y
0	$-\frac{1}{2}$
2	0
-4	v asym.
-2	-2 hole

Use the function $g(x) = (1/3)x^3 - (1/2)x^2 - 6x$ to answer the questions on this page.

1. State the intervals of increasing and decreasing.

$g'(x) = x^2 - x - 6$

$0 = (x-3)(x+2)$

critical pts $x = 3$ & -2

f inc dec inc

f' + − +

−3 −2 0 3 4

Inc: $(-\infty, -2] \cup [3, \infty)$

Dec: $[-2, 3]$

2. State the Local Extremes (Min/Max) on the interval [-10, 10].

max $x = -2$ $y = 7.\overline{3}$

min $x = 3$ $y = -13.5$

3. ~~Find the absolute minimum on the interval [-10, 10].~~

4. State the intervals of concavity on the interval [-10. 10].

$g''(x) = 2x - 1$

$0 = 2(x) - 1$ $x = \frac{1}{2}$

f down up

f'' − +

0 $\frac{1}{2}$ 1

down: $(-\infty, \frac{1}{2})$

up: $(\frac{1}{2}, \infty)$

5. State the location of any inflection point(s). Explain your answer.

at $x = \frac{1}{2}$ $y = -3.08\overline{3}$

6. Describe the end behavior of f(x) at ±infinity. Find any horizontal asymptotes.

$\lim_{x \to \infty} = \infty$ $\lim_{x \to -\infty} = -\infty$ no

7. Find the y-intercept, x-intercept(s), and any vertical asymptotes.

$x = 0$

$y = 0$

$0 = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x$

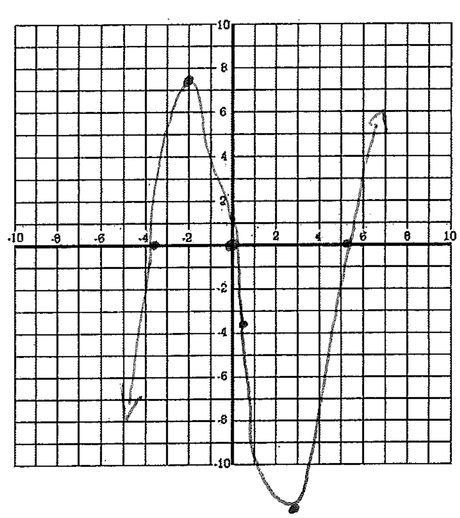
$0 = 2x^3 - 3x^2 - 36x$

$0 = x(2x^2 - 3x - 36)$

no

$\frac{3 \pm \sqrt{(-3)^2 - 4(2)(-36)}}{2(2)} = -3.56$, 5.06

8. Sketch the curve using your solutions from 1-7.



x	y
−2	$7.\overline{3}$ max
3	−13.5 min
$\frac{1}{2}$	$-3.08\overline{3}$
0	0
−3.56	0
5.06	0

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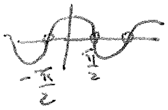
1. State the intervals of increasing and decreasing. 

$h'(x) = 1 \cdot \tan x + x \sec^2(x)$

$0 = \tan x + \frac{x}{\cos^2 x}$ $\quad x = 0$ is critical point

h dec inc

h' − +

−.01 0 .01

min

2. State the Local Extremes (Min/Max)

3. Find the absolute minimum and maximum

$x(\sec x)^2$

$\tan x + x \cdot \sec^2(x)$

4. State the intervals of concavity $h''(x) = \sec^2 x + \sec^2(x) + x\, 2\sec x \cdot \sec x \tan x$

$\sec^2 x(2 + 2x\tan x)$ $\quad 2 + 2x\tan x = 0$

$x = 0$ $\quad x =$ none

5. State the location of any inflection point(s). Explain your answer.

h up up

h'' + +

−.01 0 .01

no infl. points

6. Find the y-intercept, x-intercept(s), and any vertical asymptotes.

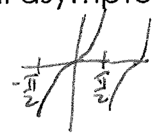
$x = 0$

$0 \cdot \tan 0 = 0$

$(0, 0)$

$0 = x \cdot \tan x$

$x = 0 \quad x = 0$



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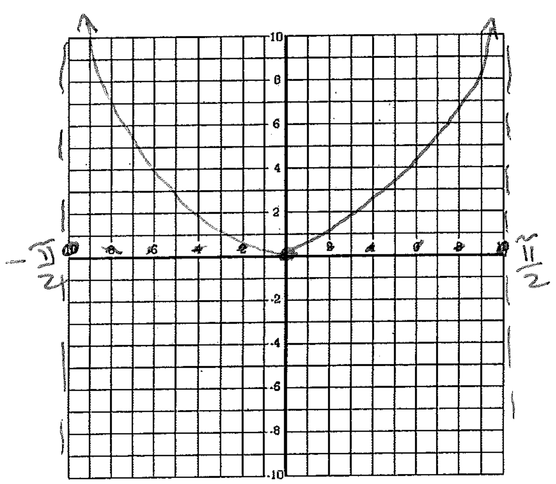

X	Y
0	0