Calculus Test 4 Prep

Review for Quiz 12 (Sec 5-5 to 5-10)

Topic: General Log and Exponential (AP Book section 5-5, Sinclair Book section 6-4*)

Differentiate the following:

31. $f(x) = \log_2(1 - 3x)$

38. $y = \left(\sqrt{x}\right)^x$

40. $y = (\sin x)^{\ln x}$

50. $y = 4^x 5^{\sin x}$

1. Consider the equation $f'(x) = \log(x^3 - x + 1)$ for x≥0

When is f(x) increasing and decreasing

Find the inflection points for f(x)

2. The slope of tangent line to the graph $\ln(x) + 2e^y = x$ at (1, -.693) is

3. Let *f* be the function given by $f(x) = 3e^{\frac{2}{x}}$ For what value of *x* is the slope of the line tangent to the graph of *f* at *(x, f(x))* equal to -2.3?

A. -1.168 B. 1.276 C. 2.318 D. 2.438 E. 3.551

4. Find the inflection point on $f(x) = e^x - x^2$ write your answer in perfect form (no decimal approximation)

Topic: Growth and Decay / Differential Equations (AP Book section 5-6 and 5-7, Sinclair Book section 6-5)

14. A curve passes through the point $(0, 5)$ and has the property that the slope of the curve at every point P is twice the y-coordinate of P. What is the equation of the curve?

19. The rate of change of atmospheric pressure P with respect to altitude h is proportional to P, provided that the temperature is constant. At 15°C the pressure is 101.3 kPa at sea level and 87.14 kPa at $h = 1000$ m.

(a) What is the pressure at an altitude of 3000 m?

(b) What is the pressure at the top of Mount McKinley, at an altitude of 6187 m?

5. At time t = 0 minutes, the temperature of a hot pocket is 200 degrees Fahrenheit and the room temperature is 72 degrees Fahrenheit. The hot pocket cools according to the differential equation $\frac{dT}{dt} = \frac{-1}{3}T + 24$

a) Find an expression for T(t) using the initial condition T(0) = 200.

b) Find the limit as $t \to \infty$ for T(t) and explain what this limit means in the context of the problem.

c) How hot is the hot pocket at the time when the temperature is decreasing at the rate of 10º F per minute?

d) At what time t is the temperature of the hot pocket equal your answer from part c?

Topic: Inverse Trig (AP Book section 5-8 and 5-9, Sinclair Book section 6-6)

39. Find y' if $\tan^{-1}(x) = x + y^3$

64. $\int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x}\, dx$

Now find the average value of the function on the interval

67. $\int \frac{t^2}{\sqrt{1 - t^6}}\, dt$

Topic: Hyperbolic Functions (AP Book section 5-10, Sinclair Book section 6-7)

Find the derivative:

30. $f(x) = e^x \cosh x$

31. $f(x) = \tanh \sqrt{x}$

37. $y = e^{\cosh 3x}$

Calculus Test 4 Prep

Key

Review for Quiz 12 (Sec 5-5 to 5-10)*

Topic: General Log and Exponential (AP Book section 5-5, Sinclair Book section 6-4*)

*Differentiate:

31. $f(x) = \log_2(1 - 3x)$

$$f'(x) = \frac{-3}{(1-3x)\ln 2}$$

38. $y = (\sqrt{x})^x$

$$\ln y = \ln(\sqrt{x}^x)$$

$$\ln y = x \ln \sqrt{x}$$

$$\frac{y'}{y} = 1 \cdot \ln\sqrt{x} + x \frac{\frac{1}{2}x^{-1/2}}{\sqrt{x}}$$

$$y' = y\left(\ln\sqrt{x} + \frac{x}{2\sqrt{x}\sqrt{x}}\right) \qquad y' = (\sqrt{x})^x\left(\frac{\ln\sqrt{x}}{\text{[illegible]}} + \frac{1}{2}\right)$$

40. $y = (\sin x)^{\ln x}$

$$\ln y = \ln((\sin x)^{\ln x})$$

$$\ln y = \ln x \cdot \ln(\sin x)$$

$$\frac{y'}{y} = \frac{1}{x}\ln(\sin x) + \ln x \frac{\cos x}{\sin x}$$

$$y' = (\sin x)^{\ln x}\left(\frac{1}{x}(\sin x) + \ln x \cdot \cot x\right)$$

50. $y = 4^x 5^{\sin x}$

$$y' = (4^x \cdot 1 \cdot \ln 4)5^{\sin x} + 4^x(5^{\sin x} \cdot \cos x \cdot \ln 5)$$

* Graphing Calculator Section

1. Consider the equation $f'(x) = \log(x^3 - x + 1)$ for x≥0

When is f(x) increasing and decreasing

f dec inc

f' − + (0, 1)

f is dec on $0 \le x \le 1$

f is inc on $x \ge 1$

Find the inflection points for f(x)

f down up

f'' − + (½)

inf. pt at $x = \frac{1}{2}$ because f'' changes from − to +

2. The slope of tangent line to the graph $\ln(x) + 2e^y = x$ at $(\overset{x}{1}, \overset{y}{-.693})$ is

$$\frac{1}{x} + 2e^y \cdot y' = 1$$

$$\frac{1}{1} + 2e^{-.693} y' = 1$$

$$2e^{-.693} y' = 0 \qquad y' = 0$$

3. Let f be the function given by $f(x) = 3e^{\frac{2}{x}}$ For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to -2.3?

A. -1.168 B. 1.276 C. 2.318 (D) 2.438 E. 3.551

4. Find the inflection point on $f(x) = e^x - x^2$ write your answer in perfect form (no decimal approximation)

$$f'(x) = e^x - 2x$$

$$f''(x) = e^x - 2$$

$$0 = e^x - 2$$

$$2 = e^x$$

$$\ln 2 = x$$

f down up

f'' − + (.5, ln 2, 1)

$x = \ln(2)$ is an infl. pt. because f'' changes from − to + so f changes from conc. down to up.

Topic: Growth and Decay / Differential Equations (AP Book section 5-6 and 5-7, Sinclair Book section 6-5)

14. A curve passes through the point (0, 5) and has the property that the slope of the curve at every point P is twice the y-coordinate of P. What is the equation of the curve?

$\ln(g(x)) = \frac{g'(x)}{g(x)}$

$y' = 2y$

$\int \frac{y'}{y} = \int 2$

$\ln|y| = 2x$

$y = Ce^{2x}$

use $(0, 5)$ (x, y)

$5 = Ce^{2(0)}$

$5 = C$

$\boxed{y = 5e^{2x}}$

* **19.** The rate of change of atmospheric pressure P with respect to altitude h is proportional to P, provided that the temperature is constant. At 15°C the pressure is 101.3 kPa at sea level and 87.14 kPa at $h = 1000$ m.

(a) What is the pressure at an altitude of 3000 m?

(b) What is the pressure at the top of Mount McKinley, at an altitude of 6187 m?

(h, P)

$(0, 101.3)$

$(1000, 87.14)$

$\frac{dP}{dh} = kP$

$P' = kP$

$\int \frac{P'}{P} = \int k$

$\ln|P| = kh$

$P = Ce^{kh}$

use $(0, 101.3)$

$101.3 = Ce^{k \cdot 0}$

$101.3 = C$

$P = 101.3e^{kh}$

use $(1000, 87.14)$

$87.14 = 101.3e^{k(1000)}$

$.86 = e^{1000k}$

$\ln .86 = 1000k$

$k = \frac{\ln .86}{1000} = -.00015$

$\boxed{P = 101.3e^{-.00015h}}$

5. At time t = 0 minutes, the temperature of a hot pocket is 200 degrees Fahrenheit and the room temperature is 72 degrees Fahrenheit. The hot pocket cools according to the differential equation $\frac{dT}{dt} = \frac{-1}{3}T + 24$

a) Find an expression for T(t) using the initial condition T(0) = 200.

$dT = -\frac{1}{3}(T - 72)dt$

$\int \frac{1}{T-72} dT = \int -\frac{1}{3} dt$

$\ln|T - 72| = -\frac{1}{3}t$

$T - 72 = e^{-\frac{1}{3}t}$

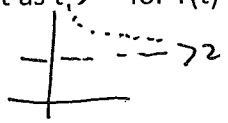
$T = Ce^{-t/3} + 72$

$200 = Ce^0 + 72$

$128 = C$

$T(t) = 128e^{-t/3} + 72$

b) Find the limit as $t \to \infty$ for T(t) and explain what this limit means in the context of the problem.



$\lim_{t \to \infty} T(t) = 72°F$

c) How hot is the hot pocket at the time when the temperature is decreasing at the rate of 10º F per minute?

$T' = -\frac{1}{3}T + 24$

$-10 = -\frac{1}{3}T + 24$

$-34 = -\frac{1}{3}T$

$Temp = 102°F$

d) At what time t is the temperature of the hot pocket equal your answer from part c?

$102 = 128e^{-t/3} + 72$

$30 = 128e^{-t/3}$

$.234 = e^{-t/3}$

$-1.45 = -t/3$

$t = 4.35$ min

Topic: Inverse Trig (AP Book section 5-8 and 5-9, Sinclair Book section 6-6)

* **39. Find y' if $\tan^{-1}(x$) = x + y^3**

$$\frac{1}{1+x^2} = 1 + 3y^2 y'$$

$$\frac{1}{1+x^2} - 1 = 3y^2 y'$$

$$\frac{1}{3y^2}\left(\frac{1}{1+x^2} - 1\right) = y'$$

* **64.** $\int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x}\, dx$

* Also, what is the average value of the function?

$$-\arctan(\cos x)\Big|_0^{\pi/2}$$

$$-\arctan\left(\cos\tfrac{\pi}{2}\right) + \arctan(\cos 0) = .7853$$

67. $\int \frac{t^2}{\sqrt{1 - t^6}}\, dt$

$$\int \frac{3t^2}{\sqrt{1-(t^3)^2}} \quad \frac{1}{3}\arcsin(t^3) + C$$

Topic: Hyperbolic Functions (AP Book section 5-10, Sinclair Book section 6-7)

Find the derivative:

30. $f(x) = e^x \cosh x$

$$f'(x) = e^x \cosh x + e^x \sinh x$$

31. $f(x) = \tanh \sqrt{x}$

$$f'(x) = 1 - \tanh^2(\sqrt{x})\left(\tfrac{1}{2}x^{-1/2}\right)$$

37. $y = e^{\cosh 3x}$

$$y' = e^{\cosh 3x}(\sinh(3x))\,3$$