Calculus Test 3 Prep

Review for Quiz 10 (Sec 4-4 to 4-5)

Evaluate the Integrals:

9. $\int (1 - 2x)^9 \, dx$

10. $\int \sin t \sqrt{1 + \cos t} \, dt$

17. $\int \sec^2\theta \tan^3\theta \, d\theta$

22. $\int \frac{\cos(\pi/x)}{x^2} \, dx$

35. $\int_0^1 \cos(\pi t/2) \, dt$

47. $\int_1^2 x\sqrt{x - 1} \, dx$

1. Find f(x) if $f''(x) = 12x^2 + 6$ given that $f'(1) = 6$ and $f(1) = -2$

2. Consider $f(x) = x^2 + \sin x$ on $[0, \pi]$

   a. find the rate of change at $x = \pi$

   b. find the average value of f(x) on $[0, \pi]$

3. Find the value of $\int_0^6 f(x)dx$ given that $\int_3^0 f(x)dx = 4$ and $\int_3^6 2 \cdot f(x)dx = 20$

Calculus Test 3 Prep

Review for Quiz 10 (Sec 4-4 to 4-5)

Evaluate the Integrals:

9. $\int (1-2x)^9\,dx$

$u = 1-2x$
$du = -2dx$
$-\frac{1}{2}du = dx$

$\int u^9\left(-\frac{1}{2}du\right)$
$\frac{1}{10}u^{10}\left(-\frac{1}{2}\right)$
$-\frac{1}{20}(1-2x)^{10}+C$

10. $\int \sin t\sqrt{1+\cos t}\,dt$

$u = 1+\cos t$
$du = -\sin t\,dt$
$-du = \sin t\,dt$

$\int u^{1/2}(-du)$
$-\frac{2}{3}u^{3/2}$
$-\frac{2}{3}(1+\cos t)^{3/2}+C$

$(\tan\theta)^3$

17. $\int \sec^2\theta\,\tan^3\theta\,d\theta$

$u = \tan\theta$
$du = \sec^2\theta\,d\theta$

$\int u^3\,du$
$\frac{1}{4}u^4$
$\frac{1}{4}\tan^4\theta + C$

* **22.** $\int \frac{\cos(\pi/x)}{x^2}\,dx$

$u = \pi x^{-1}$
$du = -\pi x^{-2}\,dx$
$\frac{du}{-\pi} = \frac{1}{x^2}dx$

$\int \cos(u)\left(\frac{du}{-\pi}\right)$
$-\frac{1}{\pi}\int \cos(u)\,du$
$-\frac{1}{\pi}\sin(u)$
$-\frac{1}{\pi}\sin\left(\frac{\pi}{x}\right)+C$

35. $\int_0^1 \cos(\pi t/2)\,dt$

$u = \frac{\pi}{2}t$
$du = \frac{\pi}{2}dt$
$\frac{2}{\pi}du = dt$

$\int \cos(u)\left(\frac{2}{\pi}du\right)$
$\frac{2}{\pi}\int \cos(u)\,du$
$\frac{2}{\pi}\sin(u)$
$\frac{2}{\pi}\sin\left(\frac{\pi}{2}t\right)\Big|_0^1$
$\frac{2}{\pi}\sin\left(\frac{\pi}{2}\cdot 1\right) - \frac{2}{\pi}\sin\left(\frac{\pi}{2}\cdot 0\right)$
$.637$
or $\frac{2}{\pi}$

47. $\int_1^2 x\sqrt{x-1}\,dx$

$u = x-1$
$du = dx$

$\int_0^1 (u+1)u^{1/2}\,du$
$\int_0^1 \left(u^{3/2}+u^{1/2}\right)du$
$\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{3/2}\Big|_0^1$

$u = x-1$
$u+1 = x$

$\frac{2}{5}(1)^{\frac{5}{2}} + \frac{2}{3}(1)^{3/2} - \left[\frac{2}{5}(0)^{\frac{5}{2}} + \frac{2}{3}(0)^{3/2}\right]$
$1.0\overline{6}$
or $\frac{16}{15}$

Lower
$u = x-1$
$u = 1-1 = 0$

Upper
$u = x-1$
$u = 2-1 = 1$

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1. Find f(x) if $f''(x) = 12x^2 + 6$ given that $f'(1) = 6$ and $f(1) = -2$

$f'(x) = \frac{12}{3}x^3 + 6x + C$

$6 = 4(1)^3 + 6(1) + C$

$6 = 10 + C$

$-10 \quad -10$

$-4 = C$

$f'(x) = 4x^3 + 6x - 4$

$f(x) = \frac{4}{4}x^4 + \frac{6}{2}x^2 - 4x + C$

$-2 = (1)^4 + 3(1)^2 - 4(1) + C$

$-2 = C$

$\boxed{f(x) = x^4 + 3x^2 - 4x - 2}$

*

2. Consider $f(x) = x^2 + \sin x$ on $[0, \pi]$

a. find the rate of change at $x = \pi$

$f'(x) = 2x + \cos x$

$f'(\pi) = 2(\pi) + \cos(\pi)$

$= 2\pi - 1$

b. find the average value of f(x) on $[0, \pi]$

$\frac{\text{Area}}{\text{width}}$

$\frac{1}{\pi}\int_0^{\pi}(x^2 + \sin x)\,dx$

$\frac{1}{\pi}\left[\frac{1}{3}x^3 - \cos x \Big|_0^{\pi}\right] = \frac{1}{\pi}\left[\left(\frac{1}{3}(\pi)^3 - \cos(\pi)\right) - \left(\frac{1}{3}(0)^3 - \cos(0)\right)\right]$

$- 0 + 1$

$\frac{1}{\pi}\left[\frac{\pi^3}{3} + 1 + 1\right]$

$\frac{\pi^3}{3\pi} + \frac{2}{\pi} = \frac{\pi^2}{3} + \frac{2}{\pi}$

$= 3.926$

3. Find the value of $\int_0^6 f(x)dx$ given that $\int_3^0 f(x)dx = 4$ and $\int_3^6 2 \cdot f(x)dx = 20$

$\int_0^6 = \int_0^3 + \int_3^6$

$= -4 + 10$

$= 6$

$-\int_0^3 f(x)dx = 4$

$\int_0^3 f(x)dx = -4$

$2\int_3^6 f(x)dx = 20$

$\int_3^6 f(x)dx = 10$