# Calculus Test 3 Prep

***Review for Quiz 8 (Sec 3-5 to 4-1)***

**Part 1 – No Calculator**

Q1 A supermarket employee wants to construct an open-top box from a 14 by 30 inch piece of cardboard. The employee cuts out squares of equal size from the four corners so the four sides can be bent upwards.

a. Find a formula for the volume of the box based on x, the side length of the squares cut from the corners.

b. Find a formula for the surface area of the box based on x, the side length of the squares cut from the corners.

c. Find a formula for the waste from the original piece of cardboard based on x, the side length of the squares cut from the corners.

Q2 A rectangle has a perimeter of 10. Maximize the area.

x =

y =

area =

Q3 If $y - 2x = 6$, then find the maximum value for $x^2 - y^2$

x =

y =

value =

Q4 If $y = x - 1$, then minimize the function $g(x) = xy + 5x$

x =

y =

value =

Q4 Use differential equations to find the approximate value of f(3.01) given that f(3) = -1 and f'(3) = 2

Q5 Use differential equations to find the approximate value of g(1.9) given that g(2) = 6 and g'(2) = -4

**Part 2 – Calculator Allowed**

Q6 Which point on the graph of $y = \sqrt{x}$ is closest to the point (7, 0)?

x =

y =

distance =

Q7 Two vertical poles, on 16 feet high and the other 24 feet high, stand 30 feet apart on a flat field. A worker wants to support both poles by running rope from the ground to the top of each post. If the worker wants to stake both ropes in the ground at the same point, where should the stake be placed to use the least amount of rope?

Distance from 16 foot pole =

Distance from 24 foot pole =

Length of rope =

Q8 Use the tangent line at x = -1 to approximate the value of g(-1.1) along $g(x) = x^3 + x^2 - 2x$

Q9 Use the tangent line at x = 2 to approximate the value x = 2.04 along the curve $f(x) = x^2 - 4x$

**Part 3 – Challenge Questions**

Q10 Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side length 4. One side of the rectangle lies on the base of the triangle.

Length =

Width =

Area =

Q11 Boat A leaves a dock at 2:00 PM and travels due south at a speed of 20 mph. Boat B has been heading due east at 15 mph and reaches the same dock at 3:00 PM. At what time were the two boats closest together?

Time =

Distance =

Boat A position =

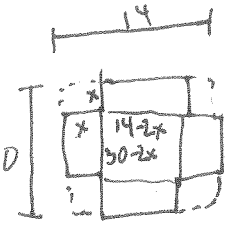
Boat B position =

# Calculus Test 3 Prep

Key

**Review for Quiz 8 (Sec 3-5 to 4-1)**

## Part 1 – No Calculator




Q1 A supermarket employee wants to construct an open-top box from a 14 by 30 inch piece of cardboard. The employee cuts out squares of equal size from the four corners so the four sides can be bent upwards.

a. Find a formula for the volume of the box based on x, the side length of the squares cut from the corners.

$V(x) = x(14-2x)(30-2x)$

b. Find a formula for the surface area of the box based on x, the side length of the squares cut from the corners.

$SA(x) = (14-2x)(30-2x) + 2(x)(14-2x) + 2(x)(30-2x)$

c. Find a formula for the waste from the original piece of cardboard based on x, the side length of the squares cut from the corners.

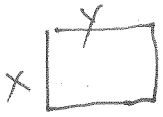
$W(x) = 4x^2$

Q2 A rectangle has a perimeter of 10. Maximize the area.

x = 2.5

y = 2.5

area = 6.25




$P = 2x + 2y$

$10 = 2x + 2y$

$5 - x = y$

x·y

$A(x) = x(5-x)$

$A(x) = 5x - x^2$

$A'(x) = 5 - 2x$

$0 = 5 - 2x$

$x = 2.5$

A(x) inc dec

A'(x) + | −

1 2.5 3

MAX

Q3 If $y - 2x = 6$, then find the maximum value for $x^2 - y^2$

x = -4

y = -2

value = 12

$y = 6 + 2x$

$y = -2$

$(-4)^2 - (-2)^2$

$16 - 4 = 12$

$f(x) = x^2 - y^2$

$f(x) = x^2 - (6+2x)^2$

$f(x) = x^2 - 36 - 24x - 4x^2$

$f(x) = -3x^2 - 36 - 24x$

$f'(x) = -6x - 24$

$0 = -6x - 24$

$x = -4$

f(x) inc dec

f'(x) + | −

-5 -4 -3

MAX

Q4 If $y = x - 1$, then minimize the function $g(x) = xy + 5x$.

x = -2

y = -3

value = -4

$(-2)(-3) + 5(-2)$

$6 - 10$

$-4$

$g(x) = x(x-1) + 5x$

$g(x) = x^2 - x + 5x$

$g(x) = x^2 + 4x$

$g'(x) = 2x + 4$

$0 = 2x + 4$

$x = -2$

g(x) dec inc

g'(x) − | +

-3 -2 -1

MIN

Q4 Use differential equations to find the approximate value of f(3.01) given that f(3) = -1 and f'(3) = 2

Make tang. line at 3 $y + 1 = 2(x-3)$

Use it on $3.01 = x$

$y + 1 = 2(3.01 - 3)$ $y + 1 = .02$ $\boxed{y = -.98}$

Q5 Use differential equations to find the approximate value of g(1.9) given that g(2) = 6 and g'(2) = -4

Make tan line at 2 $y - 6 = -4(x-2)$

Use it on $x = 1.9$

$y - 6 = -4(1.9 - 2)$

$y - 6 = -.4$

$\boxed{y = 5.6}$

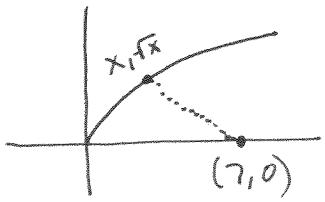
## Part 2 – Calculator Allowed

Q6 Which point on the graph of $y = \sqrt{x}$ is closest to the point (7, 0)?

x = 6.5

y = 2.549

distance = 2.598



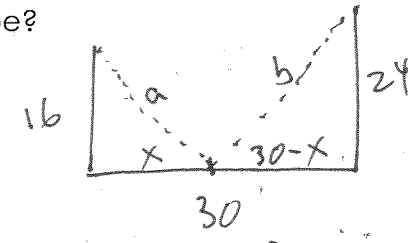

$f(x) = \sqrt{(x-7)^2 + (\sqrt{x}-0)^2}$

Q7 Two vertical poles, on 16 feet high and the other 24 feet high, stand 30 feet apart on a flat field. A worker wants to support both poles by running rope from the ground to the top of each post. If the worker wants to stake both ropes in the ground at the same point, where should the stake be placed to use the least amount of rope?

Distance from 16 foot pole = 12

Distance from 24 foot pole = 18

Length of rope = 50




$g(x) = a + b$

$g(x) = \sqrt{x^2+16^2} + \sqrt{24^2+(30-x)^2}$

$x^2 + 16^2 = a^2$

$a = \sqrt{x^2+16^2}$

$24^2 + (30-x)^2 = b^2$

$b = \sqrt{24^2+(30-x)^2}$

Q8 Use the tangent line at x = -1 to approximate the value of g(-1.1) along $g(x) = x^3 + x^2 - 2x$

$y - 2 = -1(x+1)$

$y - 2 = -1(-1.1+1)$

$y - 2 = -1(-.1)$

$y - 2 = .1$

$y = 2.1$

$g(-1) = 2$

$g'(-1) = -1$

Q9 Use the tangent line at x = 2 to approximate the value x = 2.04 along the curve $f(x) = x^2 - 4x$

$y + 4 = 0(x-2)$

$y + 4 = 0(2.04-2)$

$y + 4 = 0$

$y = -4$

$f(2) = -4$

$f'(2) = 0$

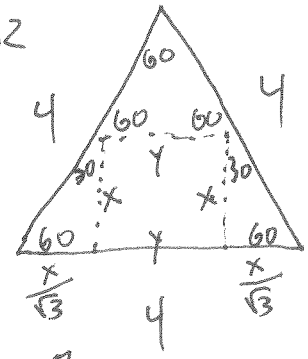
## Part 3 – Challenge Questions

Q10 Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side length 4. One side of the rectangle lies on the base of the triangle.

Length = 1.732

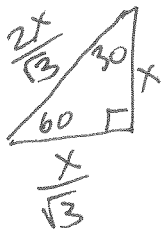
Width = 2

Area = 3.464




$A(x) = x\,y$

$A(x) = x\left(4 - \frac{2}{\sqrt{3}}x\right)$




$y + \frac{x}{\sqrt{3}} + \frac{x}{\sqrt{3}} = 4$

$y = 4 - \frac{2x}{\sqrt{3}}$

Q11 Boat A leaves a dock at 2:00 PM and travels due south at a speed of 20 mph. Boat B has been heading due east at 15 mph and reaches the same dock at 3:00 PM. At what time were the two boats closest together?

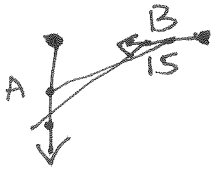
Time = .36 hours

Distance = 12

Boat A position = (0, 7.2)

Boat B position = (9.6, 0)

At 2:00 PM




$D(x) = \sqrt{(15-15x-0)^2 + (0-20x)^2}$

x = Hours

A's position

(0, 20x)

B's position

(15 − 15x, 0)