Calculus Test 2 Prep

*** Review for Quiz 5 ***

Find the derivatives:

36. $y = \dfrac{\sqrt{x}}{2 + x}$

37. $y = x^2 \sin x + 7x - 1$

38. $y = (\tan x)^{\frac{3}{4}} + \frac{1}{x}$

13. $f(\theta) = \cos(\theta^2)$

16. $f(t) = t \sin \pi t$

36. $y = x \sin \dfrac{1}{x}$

23. $y = \sqrt{\dfrac{x}{x + 1}}$

24. $y = \sin^3(\sqrt{x})$

59. Find all points on the graph of the function $f(x) = 2 \sin x + \sin^2 x$ at which the tangent line is horizontal.

53. (a) The curve $y = 1/(1 + x^2)$ is called a **witch of Maria Agnesi**. Find an equation of the tangent line to this curve at the point $\left(-1, \frac{1}{2}\right)$.

(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

skip b

69. Suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$. Find the following values.

(a) $(fg)'(5)$ (b) $(f/g)'(5)$ (c) $(g/f)'(5)$

25. (a) Find an equation of the tangent line to the curve $y = 2x \sin x$ at the point $(\pi/2, \pi)$.

Calculus Test 2 Prep

*** Review for Quiz 5 ***

Find the derivatives:

36. $y = \dfrac{\sqrt{x}}{2+x}$ $\rightarrow x^{1/2}$

$y' = \dfrac{(2+x)\frac{1}{2}x^{-\frac{1}{2}} - \sqrt{x}}{(2+x)^2}$

37. $y = x^2 \sin x + 7x - 1$

$\dfrac{dy}{dx} = 2x\sin x + x^2\cos x + 7$

38. $y = (\tan x)^{\frac{3}{4}} + \frac{1}{x}$ $\nearrow x^{-1}$

$y' = \frac{3}{4}(\tan x)^{-1/4}\sec^2 x - 1x^{-2}$

$= \dfrac{3\sec^2 x}{4(\tan x)^{1/4}} - \dfrac{1}{x^2}$

13. $f(\theta) = \cos(\theta^2)$

$f'(\theta) = -\sin(\theta^2)\cdot 2\theta$

16. $f(t) = t\sin \pi t$

$f'(t) = 1\cdot\sin(\pi t) + t\cos(\pi t)\pi$

36. $y = x\sin\dfrac{1}{x}$ $\quad y = x\sin(x^{-1})$

$\dfrac{dy}{dx} = 1\cdot\sin(x^{-1}) + x\cos(x^{-1})(-1x^{-2})$

23. $y = \sqrt{\dfrac{x}{x+1}}$ $\quad \left(\dfrac{x}{x+1}\right)^{\frac{1}{2}}$

$y' = \frac{1}{2}\left(\dfrac{x}{x+1}\right)^{-1/2}\dfrac{(x+1) - x}{(x+1)^2}$

$= \dfrac{\sqrt{x+1}}{2\sqrt{x}\,(x+1)^2}$

24. $y = \sin^3(\sqrt{x})$ $\quad y = \left(\sin(x^{\frac{1}{2}})\right)^3$

$y' = 3\left(\sin(x^{\frac{1}{2}})\right)^2\cos(x^{\frac{1}{2}})\,\frac{1}{2}x^{-1/2}$

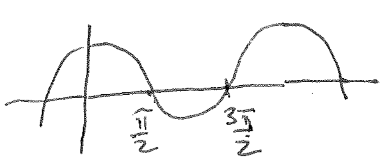
59. Find all points on the graph of the function
$f(x) = 2\sin x + \sin^2 x$ at which the tangent line is horizontal.

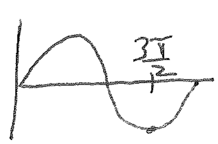
$f'(x) = 2\cos x + 2(\sin x)'\cos x$

$0 = 2\cos x\cdot(1 + \sin x)$

$\downarrow$ $\qquad 1 + \sin x = 0$

$\qquad \sin x = -1$





$\frac{\pi}{2} + k\pi \;\big|\; k \in \mathbb{Z}$

53. (a) The curve $y = 1/(1 + x^2)$ is called a **witch of Maria Agnesi**. Find an equation of the tangent line to this curve at the point $\left(-1, \frac{1}{2}\right)$.

~~(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.~~

$y - \frac{1}{2} = \frac{1}{2}(x+1)$

$y = \frac{1}{1+x^2}$

$y' = \frac{(1+x^2)\cdot 0 - 1\cdot 2x}{(1+x^2)^2}$

$y' = \frac{-2x}{(1+x^2)^2}$

$x=-1 \quad \frac{-2(-1)}{(1+(-1)^2)^2} = \frac{2}{4}$

x	5
f	1
f'	6
g	-3
g'	2

69. Suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$. Find the following values.

(a) $(fg)'(5)$

$h(x) = f(x)\cdot g(x)$

Find $h'(5)$

$h'(x) = f'(x)g(x) + f(x)g'(x)$

$h'(5) = 6(-3) + 1\cdot 2$

$-18 + 2$

$h'(5) = -16$

(b) $(f/g)'(5)$

$h(x) = \frac{f(x)}{g(x)}$

Find $h'(5)$

$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

$h'(5) = \frac{-3(6) - 1(2)}{9}$

$h'(5) = \frac{-18-2}{9} = \frac{-20}{9}$

(c) $(g/f)'(5)$

$h(x) = \frac{g(x)}{f(x)}$

Find $h'(5)$

$h'(x) = \frac{f(x)\cdot g'(x) - g(x)f'(x)}{(f(x))^2}$

$h'(5) = \frac{1\cdot 2 - -3(6)}{1^2}$

$h'(5) = 2 + 18 = 20$

(d) $h(x) = [f(x)]^3 \cdot g(x)$

Find $h'(5)$

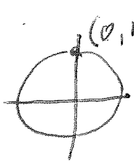
$h'(x) = 3[f(x)]^2 \cdot f'(x)\cdot g(x) + [f(x)]^3 \cdot g'(x)$

$h'(5) = 3\cdot 1^2 \cdot 6\cdot(-3) + 1^3 \cdot 2$

$= -54 + 2$

$h'(5) = -52$

25. (a) Find an equation of the tangent line to the curve $y = 2x \sin x$ at the point $(\pi/2, \pi)$.



$\frac{dy}{dx} = 2\sin x + 2x\cos x$

$2\sin\left(\frac{\pi}{2}\right) + \underbrace{2\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right)}$

$2\cdot 1 + 0$

$\frac{dy}{dx} = 2$

$y - \pi = 2\left(x - \frac{\pi}{2}\right)$