Calculus Review for Sinclair Exam 1

*** Quiz 2 Material ***** Sec 1.4

Problem Type 3: Continuity

47. The *signum* (or sign) *function*, denoted by sgn, is defined by

$$\operatorname{sgn} x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

(a) Sketch the graph of this function.

(b) Find each of the following limits or explain why it does not exist.

(i) $\lim_{x \to 0^+} \operatorname{sgn} x$ (ii) $\lim_{x \to 0^-} \operatorname{sgn} x$

(iii) $\lim_{x \to 0} \operatorname{sgn} x$ (iv) $\lim_{x \to 0} |\operatorname{sgn} x|$

11–14 Use the definition of continuity and the properties of limits to show that the function is continuous at the given number a.

11. $f(x) = (x + 2x^3)^4, \quad a = -1$

For #20, explain why the function is discontinuous at point a. Sketch the graph.

20. $f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases} \qquad a = 1$

23–24 How would you "remove the discontinuity" of f? In other words, how would you define $f(2)$ in order to make f continuous at 2?

23. $f(x) = \dfrac{x^2 - x - 2}{x - 2}$

45. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$$

Calculus Review for Sinclair Exam 1

Key

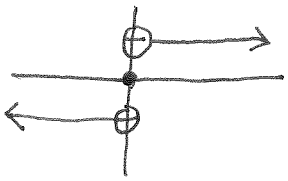
*** Quiz 2 Material ***

Problem Type 3: Continuity

Lim f(x) exists
x→c
f(c) exists
f(x) = lim f(x)
x→c

47. The *signum* (or sign) *function*, denoted by sgn, is defined by

$$\operatorname{sgn} x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

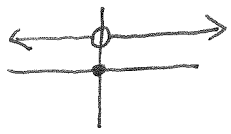


(a) Sketch the graph of this function.

(b) Find each of the following limits or explain why it does not exist.

(i) $\lim_{x \to 0^+} \operatorname{sgn} x = 1$ (ii) $\lim_{x \to 0^-} \operatorname{sgn} x = -1$

(iii) $\lim_{x \to 0} \operatorname{sgn} x$ DNE (iv) $\lim_{x \to 0} |\operatorname{sgn} x| = 1$



11–14 Use the definition of continuity and the properties of limits to show that the function is continuous at the given number a.

11. $f(x) = (x + 2x^3)^4, \quad a = -1$

$f(-1) = (-1+2(-1)^3)^4 = 81$
Limit exist and also equals 81
∴ f(x) is continuous at x = -1

For #20, explain why the function is discontinuous at point a. Sketch the graph.

20. $f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases} \qquad a = 1$

$\frac{x(x-1)}{(x+1)(x-1)} \quad \frac{x}{x+1} \quad \frac{1}{1+1} = \frac{1}{2}$

$\lim_{x \to 1} f(x) = \frac{1}{2} \quad \frac{1}{2} \neq 1$

Not cont. at x = 1

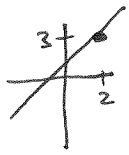
23–24 How would you "remove the discontinuity" of f? In other words, how would you define $f(2)$ in order to make f continuous at 2?

23. $f(x) = \dfrac{x^2 - x - 2}{x - 2}$

$\frac{(x-2)(x+1)}{x-2} = x+1 = 2+1 = 3$

$\lim_{x \to 2} f(x) = 3$ need $f(2) = 3$

$\begin{cases} \frac{x^2-x-2}{x-2} & x \neq 2 \\ 3 & x = 2 \end{cases}$



45. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$

$c4 + 4$
$8 - c2$

$4c + 4 = 8 - 2c$
$6c = 4$
$c = 2/3$