# AP Calculus Syllabus

## Quarter 1

### Summer Work

Students will complete one worksheet per month during June, July, and August that will review the main concepts from Pre-Calculus that are essential to success in Calculus. This work is due the first day of class. Students will take a quiz over these concepts the second day of school.

#### **Primary Text**

Calculus of a Single Variable, 7<sup>th</sup> Edition, 2002. Authors: Larson, Hostetler, and Edwards.

### Unit 1: Limits

| Day   | Topic / Objective   | Book Section<br>& Assignment |
|-------|---|------------------------------|
| 1 – 2 | Basic Limits  | 1-2A                         |
|       | Students will orally describe the concept of a limit.   | 1-2B                         |
|       | Students will analyze/evaluate/compare limits using tables by hand and with a graphing calculator.                          |                              |
|       | Students will analyze/evaluate limits from a given graph.   |                              |
|       | Students will analyze/evaluate limits by creating a graph on the calculator.  |                              |
|       | Students will recognize/evaluate common limits in trigonometry.   |                              |
| 3 – 4 | Applying Algebra to Limits  | 1-3A                         |
|       | Students will analyze limits through direct substitution and algebraic manipulation (analytically).                         | 1-3B                         |
| 5     | Quiz 1  |                              |
| 6 – 7 | <u>Continuity</u>   | 1-4A                         |
|       | Students will orally demonstrate an intuitive understanding of continuity.  | 1-4B                         |
|       | Students will find one-sided limits using a table, a graph, and through algebraic manipulation with and without technology. |                              |
|       | Students will apply the Intermediate Value Theorem and the Extreme Value Theorem.   |                              |
|       | Continued on the next page  |                              |

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| Day   | Topic / Objective  | Book Section<br>& Assignment |
|-------|--|------------------------------|
|       | Students will analyze continuity using the 3-part continuity test (in terms of limits). This will apply at a point as well as along the entire function.   |                              |
|       | Students will recognize discontinuities in functions with and without graphing calculator technology.  |                              |
|       | Students will recognize when a discontinuity is removable.   |                              |
| 8 – 9 | Limits Involving Vertical Asymptotes   | 1-5A                         |
|       | Students will analyze/evaluate limits that involve vertical asymptotes using tables, graphs, and analytic processes.   | 1-5B                         |
| 10    | Quiz 2   |                              |
| 11    | Project 1 – The Proof is in the Pi   |                              |
|       | Students will use the classic method of exhaustion (limits and Archimedes) to prove the value of pi through approximating the area of a circle using an increasing number of triangles inside the unit circle. |                              |
|       | Students should apply multiple methods of investigation that include tables, graphs, and the use of graphing technology.   |                              |
|       | Emphasis will be placed on writing complete sentences to explain the how the limit process applied to this investigation.  |                              |

## **Unit 2: Derivatives**

| Day      | Topic / Objective  | Book Section<br>& Assignment |
|----------|--|------------------------------|
| 1 – 3    | Formal Definition of a Derivative  | 2-1A                         |
|          | Students will visualize and approximate the direction and shape of a tangent line along a given curve. | 2-1B                         |
|          | Students will apply limits to find derivatives using the difference quotient.                          |                              |
|          | Students will approximate the rate of change from graphs and tables.                                   |                              |
|          | Continued on the next page   |                              |
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| Day   | Topic / Objective  | Book Section<br>& Assignment |
|-------|--|------------------------------|
|       | Students will find the slope of a curve at a point. This includes points at which the tangent is vertical or does not exist.   |                              |
|       | Students will evaluate instantaneous rate of change as a limit of the average rate of change.  |                              |
|       | Students will make connections between differentiability and continuity. Students will know that differentiability implies continuity but that the reverse is not necessarily true.  |                              |
|       | Students will verify the value of a derivative at a point using graphing calculator technology.  |                              |
|       | Students will recognize characteristics in graphs that would make<br>them not differentiable at those values and explain them orally.  |                              |
| 4 – 6 | Power Rule<br>Students will use the power rule to find derivatives   | 2-2A                         |
|       |  | 2-2B                         |
|       | Students will find the derivative of the sine and cosine functions.  | 2-2C                         |
|       | Students will use graphing calculator technology to find and verify<br>the value of derivative at a point as well as to graph the entire<br>derivative function.   | 2-2D                         |
|       | Students will be able to state the equation of a tangent line at a point with and without graphing calculator technology.  |                              |
|       | Students will rewrite functions in order to apply the power rule.  |                              |
|       | Students will be able to apply derivatives to displacement functions<br>in order to calculate velocity, acceleration, and instantaneous rate of<br>change.   |                              |
| 7 – 9 | Project 2 – Catapults<br>Students will construct a catapult out of a mousetrap. Students will<br>record data for multiple trials involving launching a ping-pong ball<br>from the catapult (time in air in seconds and max height reached).<br>Students will use graphing calculator technology to find a quadratic<br>regression equation for the displacement of the ball over time. |                              |
|       | Students will apply ideas of derivatives to find velocity and acceleration functions for the ball over time. These functions will be used to answer a series of questions. Students will be expected to graph the functions and use complete sentences to describe what the parts of the graph represent.  |                              |

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| Day     | Topic / Objective   | Book Section<br>& Assignment |
|---------|---|------------------------------|
| 10      | Quiz 3  |                              |
| 11 – 13 | Product and Quotient Rules  | 2-3A                         |
|         | Students will apply the Product Rule and Quotient Rule in order to find derivative functions  | 2-3B                         |
|         | Students will find the derivative of the tangent secant cosecant  | 2-3C                         |
|         | and cotangent functions.  | 2-3D                         |
|         | Students will find higher order derivatives.  | 2-3E                         |
|         | Students will find and verify derivative values using graphing calculator technology.   |                              |
| 14 – 16 | Chain Rule  | 2-4A                         |
|         | Students will apply the chain rule to find derivative functions.  | 2-4B                         |
| 17      | Quiz 4  |                              |
| 18 – 20 | Implicit Differentiation  | 2-5A                         |
|         | Students will identify implicit versus explicit functions and employ implicit differentiation (chain rule on y) when appropriate to find first order derivatives as well as higher order derivatives.                 | 2-5B                         |
|         | Students will apply implicit differentiation to find slope at an order pair along the original implicit function. Students should be able to use this information to state the equation of tangent line at the point. |                              |
| 21 – 22 | Related Rates   | 2-6A                         |
|         | Students will identify variables and quantities necessary to solve related rates and will understand the meaning of differentiating with  | 2-6B                         |
|         | respect to time.  | 2-6C                         |
|         | Students will apply implicit differentiation to find derivatives with respect to time and will use this derivative function to answer related rate questions.   |                              |

| Day | Topic / Objective  | Book Section |
|-----|--|--------------|
|     |  | & Assignment |
| 23  | Project 3 – Related Rates  |              |
|     | Students will research applications of related rates on the internet<br>and reconstruct an application or experiment. Students will create a<br>presentation to the class about what they learned with emphasis on<br>the calculus of related rates. |              |
|     | Students will be expected to clearly communicate the calculus involved in their project orally and through written expressions.  |              |
|     | Students should employ graphing calculator technology to enhance their presentation.   |              |
|     | Potential Alternate: Tootsie Pop Experiment.   |              |

## **Quarter 1 Assessment**

This Assessment will cover the content in Unit 1 and Unit 2.

# **Unit 3: Applications of Derivatives**

| Day    | Topic / Objective   | Book Section<br>& Assignment |
|--------|---|------------------------------|
| 1 – 2  | Extreme Values and Critical Numbers   | 3-1A                         |
|        | Investigation: Students will analyze the graphs of multiple functions<br>and their derivatives by identifying extrema, roots, and undefined<br>values. Students should use the information to make<br>generalizations about finding extrema through derivatives.<br>Students will be able to find critical numbers and locate extreme | 3-1B                         |
|        | values (local/relative and global/absolute).  |                              |
| 3 – 4  | Mean Value Theorem  | 3-2A                         |
|        | Students will be able to use the Mean Value Theorem to show that a specific rate of change must exist on an interval.   | 3-2B                         |
| 5      | Quiz 5  |                              |
| 6 – 8  | Increasing, Decreasing, and the First Derivative Test   | 3-3A                         |
|        | Students will be able to use the First Derivative Test to locate intervals of increasing and decreasing along a function and analyze observatoristics of f and f?   | 3-3B                         |
|        |   | 3-30                         |
|        | Students will be able to verify and locate intervals of increasing and decreasing using graphing calculator technology. Students will be able to identify monotonic functions.  |                              |
| 9 – 12 | Concavity and the Second Derivative Test  | 3-4A                         |
|        | Students will be able to identify points of inflection as places where concavity changes.   | 3-4B                         |
|        | Chudente will be able to use the Casend Derivetive Test to leasts   | 3-4C                         |
|        | intervals concavity along a function and relate the concavity to the sign of f".  | 3-4D                         |
|        | Students will be able to verify and locate intervals of concavity using graphing calculator technology.   |                              |
|        | Students will describe characteristics of f, f', and f".  |                              |

| Day     | Topic / Objective  | Book Section<br>& Assignment |
|---------|--|------------------------------|
| 13      | <u>Quiz 6</u>  |                              |
| 14 – 15 | Limits at Infinity and End Behavior  | 3-5A                         |
|         | Students will identify the end behavior of rational functions with and without technology.   |                              |
|         | Students will state the equation of horizontal asymptotes of a rational function with and without technology.  |                              |
|         | Students will compare relative magnitudes of functions and their rates of change.  |                              |
|         | Students should connect the Family of Functions and basic transformations of function to the end behavior models.  |                              |
| 16 – 18 | Project 4 - Curve Sketching  | 3-6A                         |
|         | Students will be given a series of functions that must be accurately sketched with attention paid to roots, extrema, critical points, inflection points, and end behavior. |                              |
|         | Students will draw conclusions about the functions and describe them using complete sentences.   |                              |
| 19      | Quiz 7   |                              |
| 20 – 24 | Optimization   | 3-7A                         |
|         | Students will construct equations for given situations and apply optimization skills (derivatives) to find meaningful extreme values                                       | 3-7B                         |
|         | Students will use graphing calculator technology to verify results   | 3-7C                         |
|         | and assist in constructing graphs.   |                              |
|         | Students must clearly communicate the meaning of the answers using complete sentences.   |                              |
| 25 – 29 | Differentials.   | 3-9A                         |
|         | Students will be able to find differential equations and use them to answer application questions, including motion along a line.  | 3-9B                         |
|         | Students will know the difference between delta y and dy   |                              |

| Day | Topic / Objective | Book Section<br>& Assignment |
|-----|-------------------|------------------------------|
| 30  | Quiz 8            |                              |

## **Quarter 2 Assessment**

This Assessment will cover the content in Unit 3 with some review of Unit 1 and Unit 2.

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# Unit 4: Integration

| Day    | Topic / Objective  | Book Section<br>& Assignment |
|--------|--|------------------------------|
| 1 – 2  | The Antiderivative and the Indefinite Integral   | 4-1A                         |
|        | Investigation: Students will analyze power rule functions and their derivatives to discover rules/properties of integration.     | 4-1B                         |
|        | Students will be able to find antiderivatives and indefinite integrals.  |                              |
|        | Students can rewrite functions using algebraic manipulation to assist in finding antiderivatives.                                |                              |
|        | Students will be able to find the initial condition for an integral.   |                              |
|        | Students will see the relationship between differentiating and integrating.  |                              |
| 3 – 6  | Approximating Area Under a Curve   | 4-2A                         |
|        | Investigation: Students will approximate the area under curves without instruction to see what methods they can generate.        | 4-2B                         |
|        | Students will use sigma notation to approximate the area under a curve using right hand, left hand, midpoint, and trap. methods. |                              |
|        | Students will be able to use graphing calculator technology to approximate the area under curves.                                |                              |
| 7      | Quiz 9   |                              |
| 8 – 10 | Reimann Sums and Definite Integrals  | 4-3A                         |
|        | Students will be able to use algebra, graphs, and tables with the Reimann Sum method to approximate the area under a curve.      | 4-3B                         |
|        | Students will be apply limits to Reimann Sums to create definite integrals.  |                              |
|        | Students will be able to use definite integrals to find the area under curves.   |                              |
|        | Students will know that continuity implies integrability.  |                              |

| Day     | Topic / Objective   | Book Section<br>& Assignment |
|---------|---|------------------------------|
| 11 – 15 | Fundamental Theorem of Calculus   | 4-4A                         |
|         | Students will be able to evaluate a definite integral using the fundamental theorem of calculus.  | 4-4B                         |
|         | Students will be able to find area under a curve using the fundamental theorem of calculus.   | 4-4C                         |
|         | Students will be able to find the area under a curve with the use of graphing calculator technology.                                    |                              |
|         | Students will be able to find the Average Value of a Function.  |                              |
|         | Students will be able to apply part 2 of the fundamental theorem of calculus to see the area under the curve as a function of x.        |                              |
| 16      | <u>Quiz 10</u>  |                              |
| 17 – 20 | U Substitution.   | 4-5A                         |
|         | Investigation: students will find a pattern when integrating functions within other functions. This will lead to an understanding of u- | 4-5B                         |
|         | substitution.   | 4-5C                         |
|         | Students will identify a correct value for u, the inside function.  |                              |
|         | Students will correctly change the variables and the interval of integration.   |                              |
|         | Students will be able to find the area under a curve with the use of graphing calculator technology.                                    |                              |
| 21      | Quiz 11   |                              |

# **Unit 5: Differentiation and Integration of other Functions**

| Day     | Topic / Objective  | Book Section<br>& Assignment |
|---------|--|------------------------------|
| 1 – 4   | Logarithms   | 5-1A                         |
|         | Students will find and evaluate the derivative of a natural log.   | 5-2A                         |
|         | Students will find and evaluate the derivative of a common log.  | 5-2B                         |
|         | Students will integrate functions with natural logs and common logs.   | 5-2C                         |
|         | Students will differentiate and integrate with graphing calculator technology.   |                              |
| 5 – 6   | Inverse Functions  | 5-3A                         |
|         | Students will find and evaluate the derivative of an inverse function.   |                              |
| 7       | Quiz 12  |                              |
| 8 – 9   | Natural Exponential Functions  | 5-4A                         |
|         | Students will find and evaluate the derivative of a natural exponential function.  | 5-4B                         |
|         | Students will integrate functions with natural exponentials.   | 5-4C                         |
|         | Students will differentiate and integrate with graphing calculator technology.   |                              |
| 10 – 11 | Exponential Functions  | 5-5A                         |
|         | Students will find and evaluate the derivative of an exponential function.   |                              |
|         | Students will integrate functions with exponentials.   |                              |
|         | Students will solve application questions involving differentiation and integration of exponentials, such as exponential growth. |                              |
|         | Students will differentiate and integrate with graphing calculator technology.   |                              |
| 12      | <u>Quiz 13</u>   |                              |

| Day     | Topic / Objective   | Book Section<br>& Assignment                |
|---------|---|---|
| 13 – 15 | Growth and Decay  | 5-6A  |
|         | Students will solve growth and decay models using calculus.   | 5-6B  |
| 16      | Project 5 – Growth and Decay in Finance   |   |
|         | Students will study formulas of financial growth and decay and apply theories of calculus to answer questions.  |   |
|         | Students will be expected to clearly communicate the calculus involved in their project and should employ graphing calculator technology to enhance their product.      |   |
| 17 – 19 | Slope Fields  | Slope Field                                 |
|         | Students will use graphing calculator technology to create, investigate and explore slope fields.   | Supplement A<br>Slope Field<br>Supplement B |
|         | Students will be able to interpret slope fields when given slope field graphs.  |   |
| 20 – 22 | Differential Equations: Separation of Variables   | 5-7A  |
|         | Students will interpret the family of curves known as solution curves.  | 5-7B  |
|         | Students will be able to solve problems using differential equations.   |   |
|         | Students will be able to separate variables when necessary.   |   |
|         | Students will see the relationship between slope fields and solution curves for differential equations.   |   |
| 23      | Inverse Trigonometry  | 5-8A  |
|         | Students will find and evaluate the derivative of an inverse trig. functions. Students will use implicit differentiation to find the derivative of an inverse function. |   |
|         | Students will integrate functions with inverse trig. Functions.   |   |
|         | Students will differentiate inverse trig. functions with graphing calculator technology.  |   |
| 24      | Quiz 14   |   |

## **Quarter 3 Assessment**

This Assessment will cover the content in Unit 4 and Unit 5 with some review of Unit 1 through Unit 3.

## **Unit 6: Volumes and Slope Fields**

| Day   | Topic / Objective   | Book Section<br>& Assignment |
|-------|---|------------------------------|
| 1 – 2 | The Area between two curves   | 6-1A                         |
|       | Students will be able to setup definite integrals to calculate the area between two curves.   | 6-1B                         |
|       | Students will be able to verify and solve problems involving the area between two curves using graphing calculator technology.      |                              |
| 3 – 6 | Volume of Revolution: Disk Method   | 6-2A                         |
|       | Students will be able to setup definite integrals to calculate the volume of a solid that is rotated about an axis or a line. These | 6-2B                         |
|       | volumes of revolution may create disks or washers.  | 6-2C                         |
|       | Students will be able to verify and solve problems involving the volume of disks and washers using graphing calculator technology.  |                              |
| 7     | Quiz 15   |                              |

### **AP Exam Preparation**

At this point, the remaining time will be spent reviewing for the AP Exam.

## After the AP Exam

Students will work on capstone projects that ask them to apply the skills learned in High School to a real world problem or study. This project will be a culminating experience for the students.

### **Quarter 4 Assessment**

Students who take the AP Exam are exempt from this assessment. For all other students, this assessment will cover the content in Unit 6 with some review of Unit 1 through Unit 5.