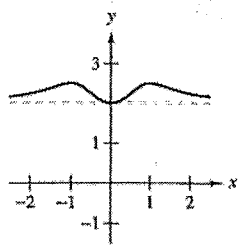
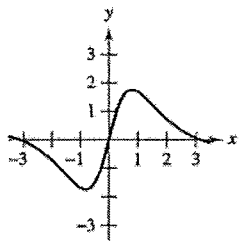
Calculus **Worksheet 3-5A** **Name**_____ **Pd**_____

In Exercises 1–6, match the function with one of the graphs [(a), (b), (c), (d), (e), or (f)] using horizontal asymptotes as an aid.

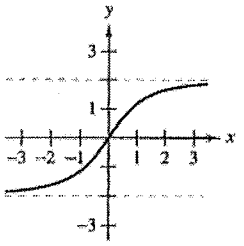
(a)



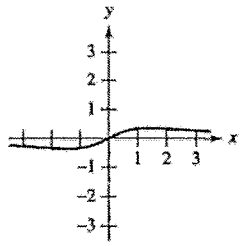
(b)



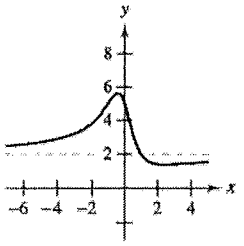
(c)



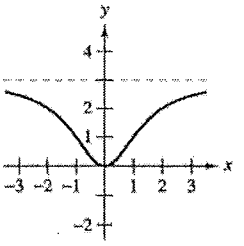
(d)



(e)



(f)



1. $f(x) = \dfrac{3x^2}{x^2 + 2}$

2. $f(x) = \dfrac{2x}{\sqrt{x^2 + 2}}$

3. $f(x) = \dfrac{x}{x^2 + 2}$

4. $f(x) = 2 + \dfrac{x^2}{x^4 + 1}$

5. $f(x) = \dfrac{4 \sin x}{x^2 + 1}$

6. $f(x) = \dfrac{2x^2 - 3x + 5}{x^2 + 1}$

In Exercises 19–32, find the limit.

19. $\lim\limits_{x \to \infty} \dfrac{2x - 1}{3x + 2}$

21. $\lim\limits_{x \to \infty} \dfrac{x}{x^2 - 1}$

23. $\lim\limits_{x \to -\infty} \dfrac{5x^2}{x + 3}$

25. $\lim\limits_{x \to -\infty} \dfrac{x}{\sqrt{x^2 - x}}$

27. $\lim\limits_{x \to -\infty} \dfrac{2x + 1}{\sqrt{x^2 - x}}$

29. $\lim\limits_{x \to \infty} \dfrac{\sin 2x}{x}$

31. $\lim\limits_{x \to \infty} \dfrac{1}{2x + \sin x}$

Note: The AP Questions for 3-5A and B are from the actual 1988 exam. Different from the current exam.

2. What is the domain of the function f given by $f(x) = \frac{\sqrt{x^2 - 4}}{x - 3}$?

(A) $\{x: x \neq 3\}$ (B) $\{x: |x| \leq 2\}$ (C) $\{x: |x| \geq 2\}$

(D) $\{x: |x| \geq 2 \text{ and } x \neq 3\}$ (E) $\{x: x \geq 2 \text{ and } x \neq 3\}$

22. If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$

(A) $\frac{1}{e^2}$ (B) $\frac{1}{e}$ (C) e (D) $2e$ (E) e^2

Calculator for 29

29. The $\lim_{h \to 0} \frac{\tan 3(x + h) - \tan(3x)}{h}$ is

(A) 0 (B) $3 \sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3 \cot(3x)$ (E) nonexistent

31. If $f(x) = \frac{x}{x + 1}$, then the inverse function, f^{-1}, is given by $f^{-1}(x) =$

(A) $\frac{x - 1}{x}$ (B) $\frac{x + 1}{x}$ (C) $\frac{x}{1 - x}$ (D) $\frac{x}{x + 1}$ (E) λ

1. Let f be the function given by $f(x) = \sqrt{x^4 - 16x^2}$.

(a) Find the domain of f.

(b) Describe the symmetry, if any, of the graph of f.

Calculator for 4

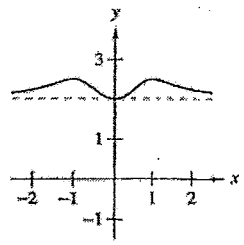
4. Let f be the function defined by $f(x) = 2xe^{-x}$ for all real numbers x.

(a) Write an equation of the horizontal asymptote for the graph of f.

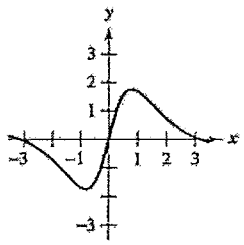
Calculus Worksheet 3-5A Name Key Pd

In Exercises 1–6, match the function with one of the graphs [(a), (b), (c), (d), (e), or (f)] using horizontal asymptotes as an aid.

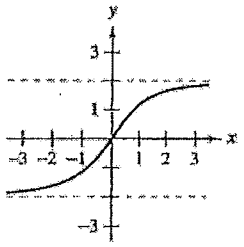
(a)



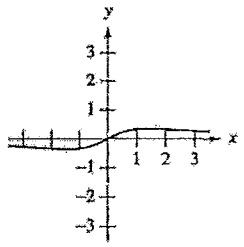
(b)



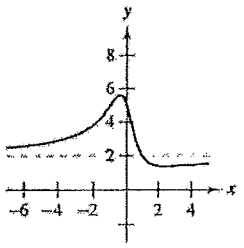
(c)



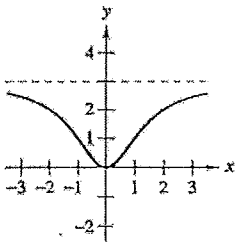
(d)



(e)



(f)



1. $f(x) = \frac{3x^2}{x^2+2}$
2. $f(x) = \frac{2x}{\sqrt{x^2+2}}$
3. $f(x) = \frac{x}{x^2+2}$
4. $f(x) = 2 + \frac{x^2}{x^4+1}$
5. $f(x) = \frac{4\sin x}{x^2+1}$
6. $f(x) = \frac{2x^2-3x+5}{x^2+1}$

	$\lim_{x\to\infty}$	$\lim_{x\to-\infty}$	Graph
1	3	3	F
2	2	−2	C
3	0	0	$f(0)=0$ $f(1)=1/3$ D
4	2	2	$f(0)=2$ A
5	0	0	$f(1)=1.6$ B
6	2	2	$f(0)=5$ E

In Exercises 19–32, find the limit.

19. $\lim_{x\to\infty} \frac{2x-1}{3x+2}$ $\frac{2}{3}$
21. $\lim_{x\to\infty} \frac{x}{x^2-1}$ 0
23. $\lim_{x\to-\infty} \frac{5x^2}{x+3}$ try −10 $\frac{5(-10)^2}{-10+3} = \frac{500}{-7}$ $-\infty$
25. $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2-x}} = \frac{-\infty}{\infty} = -1$
27. $\lim_{x\to-\infty} \frac{2x+1}{\sqrt{x^2-x}}$ $\frac{-2\infty}{\infty} = -2$
29. $\lim_{x\to\infty} \frac{\sin 2x}{x}$ 0
31. $\lim_{x\to\infty} \frac{1}{2x+\sin x}$ 0

Note: The AP Questions for 3-5A and B are from the actual 1988 exam. Different from the current exam.

2. What is the domain of the function f given by $f(x) = \frac{\sqrt{x^2-4}}{x-3}$?

$x \geq 2 \quad x \leq -2$

$x \neq 3$

(A) $\{x: x \neq 3\}$ (B) $\{x: |x| \leq 2\}$ (C) $\{x: |x| \geq 2\}$

(D) $\{x: |x| \geq 2 \text{ and } x \neq 3\}$ (E) $\{x: x \geq 2 \text{ and } x \neq 3\}$

22. If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$

$\ln x - \ln\left(\frac{1}{x}\right) = e^2$

$e^{\ln x - \ln(\frac{1}{x})} = e^2$

$x e^{\ln((\frac{1}{x})^{-1})} = e^2$

$x \cdot x = e^2$

$x^2 = e^2$

$x = \sqrt{e^2} = e^{2/2} = e$

(A) $\frac{1}{e^2}$ (B) $\frac{1}{e}$ (C) e (D) $2e$ (E) e^2

Calculator for 29

29. The $\lim_{h \to 0} \frac{\tan 3(x+h) - \tan(3x)}{h}$ is

$Y = \tan(3x)$

$Y' = \sec^2(3x) \cdot 3$

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

$f(2) = \frac{2}{3}$ find which has $f^{-1}\left(\frac{2}{3}\right) = 2$

31. If $f(x) = \frac{x}{x+1}$, then the inverse function, f^{-1}, is given by $f^{-1}(x) =$

$x = \frac{y}{y+1}$

$x(y+1) = y$

$\frac{y+1}{y} = \frac{1}{x}$

$1 + \frac{1}{y} = \frac{1}{x}$

$\frac{1}{y} = \frac{1}{x} - 1$

$\frac{1}{y} = \frac{1-x}{x}$

$x = y(1-x)$

$\frac{x}{1-x} = y$

(A) $\frac{x-1}{x}$ (B) $\frac{x+1}{x}$ (C) $\frac{x}{1-x}$ (D) $\frac{x}{x+1}$ (E) $\frac{x}{1-x}$

$\frac{\frac{2}{3}}{1-\frac{2}{3}} = \frac{\frac{2}{3}}{\frac{1}{3}} = \frac{2}{3} \cdot \frac{3}{1} = 2$

1. Let f be the function given by $f(x) = \sqrt{x^4 - 16x^2}$.

(a) Find the domain of f.

D: $x \leq -4 \cup x \geq 4$

$x^4 - 16x^2 = 0$

$x^2(x^2 - 16) = 0$

$x^2(x+4)(x-4) = 0$

$x = 0, \pm 4$

+ − − +

-4 0 4

(b) Describe the symmetry, if any, of the graph of f.

-4 4

Calculator for 4

4. Let f be the function defined by $f(x) = 2xe^{-x}$ for all real numbers x.

(a) Write an equation of the horizontal asymptote for the graph of f.

end behavior

$\lim_{x \to \infty} 2\infty \cdot e^{-\infty} = 0$ $\lim_{x \to \infty} 2(-\infty)e^{\infty} = -\infty$

$x = 0$ is the hor. asymp.

Calculus **Worksheet 3-5B** **Name**_____ **Pd**_____

In Exercises 13 and 14, find $\lim_{x\to\infty} h(x)$, if possible.

13. $f(x) = 5x^3 - 3x^2 + 10$

(a) $h(x) = \dfrac{f(x)}{x^2}$ (b) $h(x) = \dfrac{f(x)}{x^3}$

(c) $h(x) = \dfrac{f(x)}{x^4}$

14. $f(x) = 5x^2 - 3x + 7$

(a) $h(x) = \dfrac{f(x)}{x}$ (b) $h(x) = \dfrac{f(x)}{x^2}$

(c) $h(x) = \dfrac{f(x)}{x^3}$

In Exercises 15–18, find each of the limits, if possible.

15. (a) $\lim_{x\to\infty} \dfrac{x^2+2}{x^3-1}$

(b) $\lim_{x\to\infty} \dfrac{x^2+2}{x^2-1}$

(c) $\lim_{x\to\infty} \dfrac{x^2+2}{x-1}$

16. (a) $\lim_{x\to\infty} \dfrac{3-2x}{3x^3-1}$

(b) $\lim_{x\to\infty} \dfrac{3-2x}{3x-1}$

(c) $\lim_{x\to\infty} \dfrac{3-2x^2}{3x-1}$

17. (a) $\lim_{x\to\infty} \dfrac{5-2x^{3/2}}{3x^2-4}$

(b) $\lim_{x\to\infty} \dfrac{5-2x^{3/2}}{3x^{3/2}-4}$

(c) $\lim_{x\to\infty} \dfrac{5-2x^{3/2}}{3x-4}$

18. (a) $\lim_{x\to\infty} \dfrac{5x^{3/2}}{4x^2+1}$

(b) $\lim_{x\to\infty} \dfrac{5x^{3/2}}{4x^{3/2}+1}$

(c) $\lim_{x\to\infty} \dfrac{5x^{3/2}}{4\sqrt{x}+1}$

Find each limit:

20. $\lim_{x\to\infty} \dfrac{3x^3+2}{9x^3-2x^2+7}$

22. $\lim_{x\to\infty} \left(4+\dfrac{3}{x}\right)$

24. $\lim_{x\to-\infty} \left(\dfrac{1}{2}x - \dfrac{4}{x^2}\right)$

26. $\lim_{x\to-\infty} \dfrac{x}{\sqrt{x^2+1}}$

28. $\lim_{x\to-\infty} \dfrac{-3x+1}{\sqrt{x^2+x}}$

30. $\lim_{x\to\infty} \dfrac{x-\cos x}{x}$

32. $\lim_{x\to\infty} \cos\dfrac{1}{x}$

32. Which of the following does NOT have a period of π ?

(A) $f(x) = \sin\left(\frac{1}{2}x\right)$ (B) $f(x) = |\sin x|$ (C) $f(x) = \sin^2 x$
(D) $f(x) = \tan x$ (E) $f(x) = \tan^2 x$

Calculator for 33 and 37

33. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

(A) 4 (B) 2 (C) 1 (D) 0 (E) -2

37. If $f(x) = e^x \sin x$, then the number of zeros of f on the closed interval $[0, 2\pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

42. The graph of which of the following equations has $y = 1$ as an asymptote?

(A) $y = \ln x$ (B) $y = \sin x$ (C) $y = \frac{x}{x+1}$ (D) $y = \frac{x^2}{x-1}$ (E) $y = e^{-x}$

44. Let f and g be odd functions. If p, r, and s are nonzero functions defined as follows, which must be odd?

I. $p(x) = f(g(x))$
II. $r(x) = f(x) + g(x)$
III. $s(x) = f(x)g(x)$

(A) I only (B) II only (C) I and II only
(D) II and III only (E) I, II, and III

Calculus Worksheet 3-5B Name Key Pd

In Exercises 13 and 14, find $\lim_{x\to\infty} h(x)$, if possible.

13. $f(x) = 5x^3 - 3x^2 + 10$

(a) $h(x) = \frac{f(x)}{x^2}$ ∞ (b) $h(x) = \frac{f(x)}{x^3} = 5$

(c) $h(x) = \frac{f(x)}{x^4} = 0$

14. $f(x) = 5x^2 - 3x + 7$

(a) $h(x) = \frac{f(x)}{x} = \infty$ (b) $h(x) = \frac{f(x)}{x^2} = 5$

(c) $h(x) = \frac{f(x)}{x^3} = 0$

In Exercises 15–18, find each of the limits, if possible.

15. (a) $\lim_{x\to\infty} \frac{x^2+2}{x^3-1} = 0$

(b) $\lim_{x\to\infty} \frac{x^2+2}{x^2-1} = 1$

(c) $\lim_{x\to\infty} \frac{x^2+2}{x-1} = \infty$

16. (a) $\lim_{x\to\infty} \frac{3-2x}{3x^3-1} = 0$

(b) $\lim_{x\to\infty} \frac{3-2x}{3x-1} = -\frac{2}{3}$

(c) $\lim_{x\to\infty} \frac{3-2x^2}{3x-1} = \frac{-}{+} = -\infty$

17. (a) $\lim_{x\to\infty} \frac{5-2x^{3/2}}{3x^2-4}$ 0

(b) $\lim_{x\to\infty} \frac{5-2x^{3/2}}{3x^{3/2}-4} = -\frac{2}{3}$

(c) $\lim_{x\to\infty} \frac{5-2x^{3/2}}{3x-4} = -\infty$

18. (a) $\lim_{x\to\infty} \frac{5x^{3/2}}{4x^2+1} = 0$

(b) $\lim_{x\to\infty} \frac{5x^{3/2}}{4x^{3/2}+1} = \frac{5}{4}$

(c) $\lim_{x\to\infty} \frac{5x^{3/2}}{4\sqrt{x}+1} = \infty$

Find each limit:

20. $\lim_{x\to\infty} \frac{3x^3+2}{9x^3-2x^2+7}$ $\frac{1}{3}$

22. $\lim_{x\to\infty} \left(4+\frac{3}{x}\right)$ $4+0=4$

24. $\lim_{x\to-\infty} \left(\frac{1}{2}x - \frac{4}{x^2}\right)$ $-\infty - 0 = -\infty$

26. $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+1}} = \frac{-1}{+1} = -1$

28. $\lim_{x\to-\infty} \frac{-3x+1}{\sqrt{x^2+x}} = \frac{3}{1} = 3$

30. $\lim_{x\to\infty} \frac{x-\cos x}{x}$ $1 - \frac{\cos x}{x} = 1 - 0 = 1$

32. $\lim_{x\to\infty} \cos\frac{1}{x}$ $\cos\frac{1}{\infty} = \cos 0 = 1$

per = $\frac{2\pi}{K}$ $y = \sin(Kx)$

32. Which of the following does NOT have a period of π?

$\frac{2\pi}{\frac{1}{2}} = 4\pi$

(A) $f(x) = \sin\left(\frac{1}{2}x\right)$ (B) $f(x) = |\sin x|$ (C) $f(x) = \sin^2 x$

(D) $f(x) = \tan x$ (E) $f(x) = \tan^2 x$

Calculator for 33 and 37

33. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

(A) 4 (B) 2 (C) 1 (D) 0 (E) -2

include edges

37. If $f(x) = e^x \sin x$, then the number of zeros of f on the closed interval $[0, 2\pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

horizontal asymp. End behavior $\lim_{x\to\infty} f(x) = 1$

42. The graph of which of the following equations has $y = 1$ as an asymptote?

(A) $y = \ln x$ (B) $y = \sin x$ (C) $y = \frac{x}{x+1}$ (D) $y = \frac{x^2}{x-1}$ (E) $y = e^{-x}$

$\frac{+}{+} = 1$

44. Let f and g be odd functions. If p, r, and s are nonzero functions defined as follows, which must be odd?

think $f = x^3$ $g = x^5$

I. $p(x) = f(g(x))$ $(x^5)^3 = x^{15}$ Yes odd

II. $r(x) = f(x) + g(x)$ $x^3 + x^5$ NO Not odd

III. $s(x) = f(x)g(x)$ $x^3 x^5 = x^8$ NO

(A) I only (B) II only (C) I and II only

(D) II and III only (E) I, II, and III