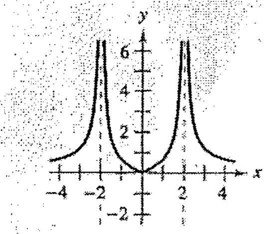
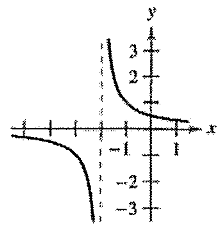
**Calculus** **Worksheet 1-5A** **Name**____________________________________ **Pd**______

**In Exercises 1–4, determine whether $f(x)$ approaches $\infty$ or $-\infty$ as $x$ approaches $-2$ from the left and from the right.**

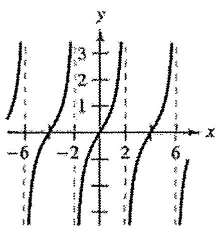
**1.** $f(x) = 2\left|\dfrac{x}{x^2 - 4}\right|$



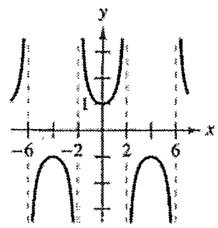

**2.** $f(x) = \dfrac{1}{x + 2}$




**3.** $f(x) = \tan \dfrac{\pi x}{4}$




**4.** $f(x) = \sec \dfrac{\pi x}{4}$




**Use Calculator for 5-8**

***Numerical and Graphical Analysis*** **In Exercises 5–8, determine whether $f(x)$ approaches $\infty$ or $-\infty$ as $x$ approaches $-3$ from the left and from the right by completing the table. Use a graphing utility to graph the function and confirm your answer.**

| $x$ | $-3.5$ | $-3.1$ | $-3.01$ | $-3.001$ |
|---|---|---|---|---|
| $f(x)$ | | | | |

| $x$ | $-2.999$ | $-2.99$ | $-2.9$ | $-2.5$ |
|---|---|---|---|---|
| $f(x)$ | | | | |

**5.** $f(x) = \dfrac{1}{x^2 - 9}$

**6.** $f(x) = \dfrac{x}{x^2 - 9}$

**7.** $f(x) = \dfrac{x^2}{x^2 - 9}$

**8.** $f(x) = \sec \dfrac{\pi x}{6}$

| x→ | -3.01 | -3.001 | Left Limit | -2.99 | -2.999 | Right Limit | Final Limit |
|---|---|---|---|---|---|---|---|
| #5 | | | | | | | |
| #6 | | | | | | | |
| #7 | | | | | | | |
| #8 | | | | | | | |

**In Exercises 9–28, find the vertical asymptotes (if any) of the function.**

**9.** $f(x) = \dfrac{1}{x^2}$

**10.** $f(x) = \dfrac{4}{(x - 2)^3}$

**11.** $h(x) = \dfrac{x^2 - 2}{x^2 - x - 2}$

**12.** $g(x) = \dfrac{2 + x}{x^2(1 - x)}$

**13.** $f(x) = \dfrac{x^2}{x^2 - 4}$

**14.** $f(x) = \dfrac{-4x}{x^2 + 4}$

**15.** $g(t) = \dfrac{t - 1}{t^2 + 1}$

**16.** $h(s) = \dfrac{2s - 3}{s^2 - 25}$

4. If the function $f$ is continuous for all real numbers and if $f(x) = \dfrac{x^2 - 7x + 12}{x - 4}$ when $x \neq 4$, then $f(4) =$

(A) 1 (B) $\frac{8}{\text{[illegible]}}$ (C) $-1$ (D) 0 (E) undefined

C

31. $\displaystyle \lim_{h \to 0} \frac{\tan\left(\frac{\pi}{6} + h\right) - \tan\left(\frac{\pi}{6}\right)}{h} =$

(A) $\frac{\sqrt{3}}{3}$ (B) $\frac{4}{3}$ (C) $\sqrt{3}$ (D) 0 (E) $\frac{3}{4}$

20. The function $f$ is given by $f(x) = \begin{cases} \ln 2x, & 0 < x < 2 \\ 2\ln x, & x \geq 2 \end{cases}$

The limit $\displaystyle \lim_{x \to 2} f(x)$ is

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) $2\ln 2$

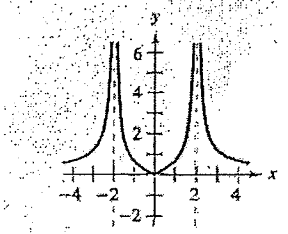
(E) nonexistent

Hint: log(4^3) = 3log(4)

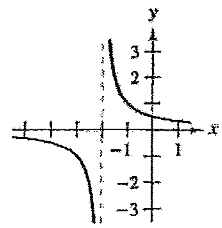
Calculus Worksheet 1-5A Name Key Pd

In Exercises 1–4, determine whether $f(x)$ approaches $\infty$ or $-\infty$ as $x$ approaches $-2$ from the left and from the right.

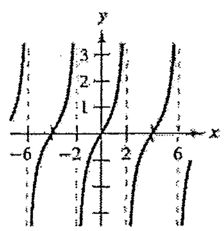
1. $f(x) = 2\left|\dfrac{x}{x^2-4}\right|$



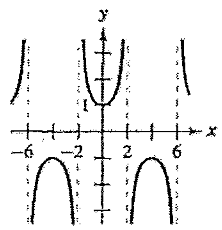
2. $f(x) = \dfrac{1}{x+2}$



3. $f(x) = \tan\dfrac{\pi x}{4}$



4. $f(x) = \sec\dfrac{\pi x}{4}$



| # | L | R | Lim |
|---|---|---|---|
| 1 | $\infty$ | $\infty$ | $\infty$ |
| 2 | $-\infty$ | $\infty$ | und. |
| 3 | $\infty$ | $-\infty$ | und. |
| 4 | $-\infty$ | $\infty$ | und. |

**Use Calculator for 5-8**

*Numerical and Graphical Analysis* In Exercises 5–8, determine whether $f(x)$ approaches $\infty$ or $-\infty$ as $x$ approaches $-3$ from the left and from the right by completing the table. Use a graphing utility to graph the function and confirm your answer.

| $x$ | $-3.5$ | $-3.1$ | $-3.01$ | $-3.001$ |
|---|---|---|---|---|
| $f(x)$ | | | | |

| $x$ | $-2.999$ | $-2.99$ | $-2.9$ | $-2.5$ |
|---|---|---|---|---|
| $f(x)$ | | | | |

5. $f(x) = \dfrac{1}{x^2-9}$ 6. $f(x) = \dfrac{x}{x^2-9}$

7. $f(x) = \dfrac{x^2}{x^2-9}$ 8. $f(x) = \sec\dfrac{\pi x}{6}$ $\dfrac{1}{\cos(\pi x/6)}$

| x→ | -3.01 | -3.001 | Left Limit | -2.99 | -2.999 | Right Limit | Final Limit |
|---|---|---|---|---|---|---|---|
| #5 | | | $\infty$ | | | $-\infty$ | und. |
| #6 | | | $-\infty$ | | | $\infty$ | und. |
| #7 | | | $\infty$ | | | $-\infty$ | und |
| #8 | | | $-\infty$ | | | $\infty$ | und. |

In Exercises 9–28, find the vertical asymptotes (if any) of the function.

9. $f(x) = \dfrac{1}{x^2}$ $x=0$

10. $f(x) = \dfrac{4}{(x-2)^3}$ $x=2$

11. $h(x) = \dfrac{x^2-2}{x^2-x-2}$ $(x-2)(x+1)$ $x=2$ & $-1$

12. $g(x) = \dfrac{2+x}{x^2(1-x)}$ $x=0$ & $1$

13. $f(x) = \dfrac{x^2}{x^2-4}$ $x=2$ & $-2$

14. $f(x) = \dfrac{-4x}{x^2+4}$ None

15. $g(t) = \dfrac{t-1}{t^2+1}$ None

16. $h(s) = \dfrac{2s-3}{s^2-25}$ $5$ & $-5$

$(x-4)(x-3)$ $\quad x-3 \quad 4-3=1$

4. If the function $f$ is continuous for all real numbers and if $f(x)=\dfrac{x^2-7x+12}{x-4}$ when $x \neq 4$, then $f(4)=$

(A) 1 (B) $\frac{8}{7}$ (C) $-1$ (D) 0 (E) undefined

C

31. $\displaystyle\lim_{h\to 0}\frac{\tan\left(\frac{\pi}{6}+h\right)-\tan\left(\frac{\pi}{6}\right)}{h}=$

(A) $\frac{\sqrt{3}}{3}$ (B) $\frac{4}{3}$ (C) $\sqrt{3}$ (D) 0 (E) $\frac{3}{4}$

$\ln 2 \cdot 2 = \ln 4$

$2\ln 2 = \ln 2^2 = \ln 4$

20. The function $f$ is given by $f(x)=\begin{cases}\ln 2x, & 0<x<2\\ 2\ln x, & x\geq 2\end{cases}$

The limit $\displaystyle\lim_{x\to 2} f(x)$ is

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) $2\ln 2$

(E) nonexistent

Hint: log(4^3) = 3log(4)

**Calculus** **Worksheet 1-5B** **Name**______________________________ **Pd**_____

**In Exercises 9–28, find the vertical asymptotes (if any) of the function.**

**17.** $f(x) = \tan 2x$

**18.** $f(x) = \sec \pi x$

**19.** $T(t) = 1 - \dfrac{4}{t^2}$

**20.** $g(x) = \dfrac{\frac{1}{2}x^3 - x^2 - 4x}{3x^2 - 6x - 24}$

**21.** $f(x) = \dfrac{x}{x^2 + x - 2}$

**22.** $f(x) = \dfrac{4x^2 + 4x - 24}{x^4 - 2x^3 - 9x^2 + 18x}$

**23.** $g(x) = \dfrac{x^3 + 1}{x + 1}$

**24.** $h(x) = \dfrac{x^2 - 4}{x^3 + 2x^2 + x + 2}$

**25.** $f(x) = \dfrac{x^2 - 2x - 15}{x^3 - 5x^2 + x - 5}$

**26.** $h(t) = \dfrac{t^2 - 2t}{t^4 - 16}$

**27.** $s(t) = \dfrac{t}{\sin t}$

**28.** $g(\theta) = \dfrac{\tan \theta}{\theta}$

**Use Calculator for 29-32 and 49-52**

**In Exercises 29–32, determine whether the function has a vertical asymptote or a removable discontinuity at $x = -1$. Graph the function using a graphing utility to confirm your answer.**

**29.** $f(x) = \dfrac{x^2 - 1}{x + 1}$

**30.** $f(x) = \dfrac{x^2 - 6x - 7}{x + 1}$

**31.** $f(x) = \dfrac{x^2 + 1}{x + 1}$

**32.** $f(x) = \dfrac{\sin(x + 1)}{x + 1}$

**In Exercises 49–52, use a graphing utility to graph the function and determine the one-sided limit.**

**49.** $f(x) = \dfrac{x^2 + x + 1}{x^3 - 1}$

$\lim_{x \to 1^+} f(x)$

**50.** $f(x) = \dfrac{x^3 - 1}{x^2 + x + 1}$

$\lim_{x \to 1^-} f(x)$

**51.** $f(x) = \dfrac{1}{x^2 - 25}$

$\lim_{x \to 5^-} f(x)$

**52.** $f(x) = \sec \dfrac{\pi x}{6}$

$\lim_{x \to 3^+} f(x)$

## In Exercises 33–48, find the limit.

**33.** $\lim_{x \to 2^+} \dfrac{x - 3}{x - 2}$

**35.** $\lim_{x \to 3^+} \dfrac{x^2}{x^2 - 9}$

**37.** $\lim_{x \to -3^-} \dfrac{x^2 + 2x - 3}{x^2 + x - 6}$

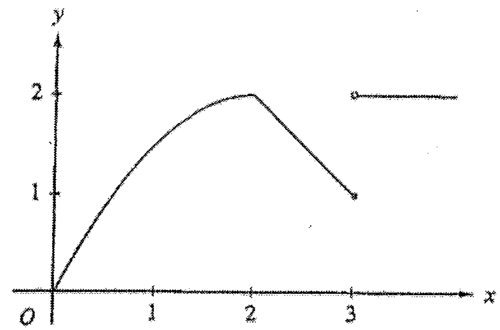
**39.** $\lim_{x \to 1} \dfrac{x^2 - x}{(x^2 + 1)(x - 1)}$

**41.** $\lim_{x \to 0^-} \left(1 + \dfrac{1}{x}\right)$

**43.** $\lim_{x \to 0^+} \dfrac{2}{\sin x}$

**45.** $\lim_{x \to \pi} \dfrac{\sqrt{x}}{\csc x}$

**47.** $\lim_{x \to 1/2} x \sec \pi x$




7. The graph of the function $f$ is shown in the figure above. Which of the following statements about $f$ is false?

(A) $f(x)$ is continuous at $x = 2$.

(B) $\lim_{x \to 2} f(x) = \lim_{x \to 3^+} f(x)$

(C) $f(3)$ is not defined.

(D) $f(x)$ is not differentiable at $x = 2$.

(E) $f'(3)$ does not exist.

7. Find $k$ so that $f(x) = \begin{cases} \dfrac{x^2 - 16}{x - 4} & ; x \neq 4 \\ k & ; x = 4 \end{cases}$ is continuous for all $x$.

(A) All real values of $k$ make $f(x)$ continuous for all $x$.

(B) 0

(C) 16

(D) 8

(E) There is no real value of $k$ that makes $f(x)$ continuous for all $x$.

**Calculus** **Worksheet 1-5B** Name Key Pd______

* Calc

In Exercises 9–28, find the vertical asymptotes (if any) of the function.

C 17. $f(x) = \tan 2x$ $x = \pm\frac{\pi}{4}$

C 18. $f(x) = \sec \pi x$ $x = \pm 1$

19. $T(t) = 1 - \frac{4}{t^2}$ $t = 0$

20. $g(x) = \frac{\frac{1}{2}x^3 - x^2 - 4x}{3x^2 - 6x - 24}$ None

21. $f(x) = \frac{x}{x^2 + x - 2}$ $x = 1 \,\&\, -2$

22. $f(x) = \frac{4x^2 + 4x - 24}{x^4 - 2x^3 - 9x^2 + 18x}$ $x = 0 \,\&\, 3$

23. $g(x) = \frac{x^3 + 1}{x + 1}$ None

24. $h(x) = \frac{x^2 - 4}{x^3 + 2x^2 + x + 2}$ None

25. $f(x) = \frac{x^2 - 2x - 15}{x^3 - 5x^2 + x - 5}$ None

26. $h(t) = \frac{t^2 - 2t}{t^4 - 16}$ $x = -2$

27. $s(t) = \frac{t}{\sin t}$ $t = \pm\pi$

28. $g(\theta) = \frac{\tan \theta}{\theta}$ $\theta = \pm\frac{\pi}{2}$

**Use Calculator for 29-32 and 49-52**

In Exercises 29–32, determine whether the function has a vertical asymptote or a removable discontinuity at $x = -1$. Graph the function using a graphing utility to confirm your answer.

29. $f(x) = \frac{x^2 - 1}{x + 1}$ $\frac{(x+1)(x-1)}{\cancel{x+1}}$ hole at $(-1,-2)$

30. $f(x) = \frac{x^2 - 6x - 7}{x + 1}$ Hole at $(-1,-8)$

31. $f(x) = \frac{x^2 + 1}{x + 1}$ V.A. at $x = -1$

32. $f(x) = \frac{\sin(x + 1)}{x + 1}$ Hole at $(-1, 1)$

AP In Exercises 49–52, use a graphing utility to graph the function and determine the one-sided limit.

49. $f(x) = \frac{x^2 + x + 1}{x^3 - 1}$

$\lim_{x\to 1^+} f(x)$ $\infty$

50. $f(x) = \frac{x^3 - 1}{x^2 + x + 1}$

$\lim_{x\to 1^-} f(x) = 0$

51. $f(x) = \frac{1}{x^2 - 25}$

$\lim_{x\to 5^-} f(x) = -\infty$

52. $f(x) = \sec \frac{\pi x}{6}$

$\lim_{x\to 3^+} f(x) = -\infty$

## In Exercises 33–48, find the limit.

33. $\lim_{x\to 2^+} \frac{x - 3}{x - 2}$ $\frac{2.01 - 3}{2.01 - 2} = \frac{-.99}{.01} = -99$ $-\infty$

35. $\lim_{x\to 3^+} \frac{x^2}{x^2 - 9}$ $\frac{(3.01)^2}{(3.01)^2 - 9} = \frac{9.0601}{9.0601 - 9} = \frac{9.0601}{.0601} = 150...$ $\infty$

37. $\lim_{x\to -3^-} \frac{x^2 + 2x - 3}{x^2 + x - 6}$ ~~[illegible]~~

39. $\lim_{x\to 1} \frac{x^2 - x}{(x^2 + 1)(x - 1)}$ $\frac{(x+3)(x-1)}{(x+3)(x-2)} = \frac{x-1}{x-2}$ $\frac{-3-1}{-3-2} = \frac{-4}{-5}$ = Hole at $(-3, 4/5)$

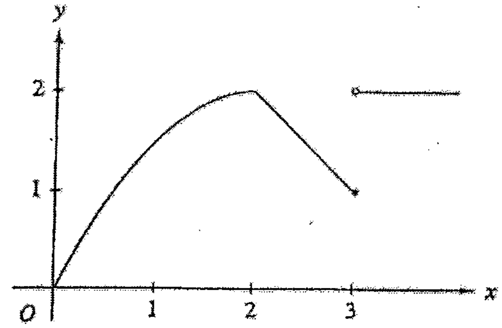
41. $\lim_{x\to 0^-} \left(1 + \frac{1}{x}\right)$ $x = -.001$ $1 + \frac{1}{-.001} = 1 + -1000 = -999$ $-\infty$

43. $\lim_{x\to 0^+} \frac{2}{\sin x}$ $x = .01$ $\frac{2}{\sin(.01)} = \frac{2}{.001} = 200$ $\infty$

small positive

C 45. $\lim_{x\to \pi} \frac{\sqrt{x}}{\csc x}$ $= \sqrt{x}\sin x = 0$

C 47. $\lim_{x\to 1/2} x \sec \pi x$ undef




7. The graph of the function $f$ is shown in the figure above. Which of the following statements about $f$ is false?
   (A) $f(x)$ is continuous at $x = 2$.
   (B) $\lim_{x \to 2} f(x) = \lim_{x \to 3^+} f(x)$
   (C) $f(3)$ is not defined.
   (D) $f(x)$ is not differentiable at $x = 2$.
   (E) $f'(3)$ does not exist.

$\frac{(x+4)(x-4)}{x-4}$ $\quad x+4$

$4+4=8$

7. Find $k$ so that $f(x) = \begin{cases} \frac{x^2-16}{x-4} & ; x \neq 4 \\ k & ; x = 4 \end{cases}$ is continuous for all $x$.

8

   (A) All real values of $k$ make $f(x)$ continuous for all $x$.
   (B) 0
   (C) 16
   (D) 8
   (E) There is no real value of $k$ that makes $f(x)$ continuous for all $x$.