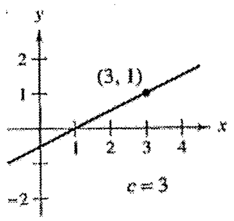
Calculus **Worksheet 1-4A** **Name**____ **Pd**______

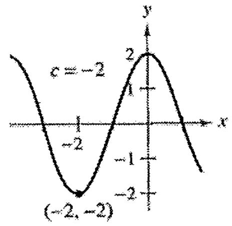
In Exercises 1–6, use the graph to determine the limit, and discuss the continuity of the function.

(a) $\lim\limits_{x\to c^+} f(x)$ (b) $\lim\limits_{x\to c^-} f(x)$ (c) $\lim\limits_{x\to c} f(x)$

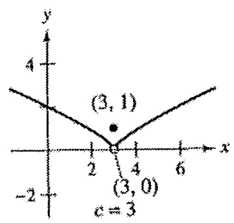
1.



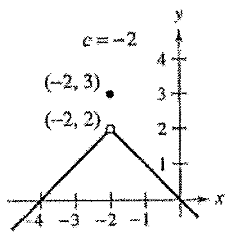
2.



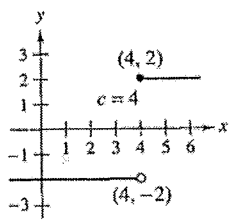
3.



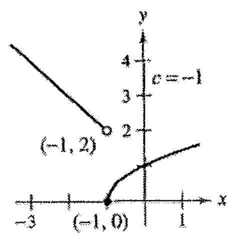
4.



5.



6.



Use calculator for 29-32

In Exercises 29–32, discuss the continuity of the function on the closed interval.

29. $g(x) = \sqrt{25 - x^2}, \quad [-5, 5]$

30. $f(t) = 3 - \sqrt{9 - t^2}, \quad [-3, 3]$

31. $f(x) = \begin{cases} 3 - x, & x \le 0 \\ 3 + \frac{1}{2}x, & x > 0 \end{cases}, \quad [-1, 4]$

32. $g(x) = \dfrac{1}{x^2 - 4}, \quad [-1, 2]$

In Exercises 7–24, find the limit (if it exists). If it does not exist, explain why.

7. $\lim\limits_{x\to 5^+} \dfrac{x - 5}{x^2 - 25}$

8. $\lim\limits_{x\to 2^+} \dfrac{2 - x}{x^2 - 4}$

9. $\lim\limits_{x\to -3^-} \dfrac{x}{\sqrt{x^2 - 9}}$

10. $\lim\limits_{x\to 4^-} \dfrac{\sqrt{x} - 2}{x - 4}$

11. $\lim\limits_{x\to 0^-} \dfrac{|x|}{x}$

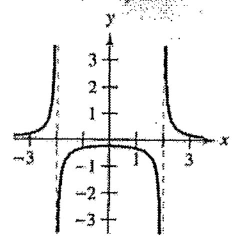
12. $\lim\limits_{x\to 2^+} \dfrac{|x - 2|}{x - 2}$

13. $\lim\limits_{\Delta x\to 0^-} \dfrac{\dfrac{1}{x + \Delta x} - \dfrac{1}{x}}{\Delta x}$

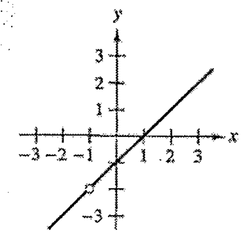
14. $\lim\limits_{\Delta x\to 0^+} \dfrac{(x + \Delta x)^2 + x + \Delta x - (x^2 + x)}{\Delta x}$

In Exercises 25–28, discuss the continuity of each function.

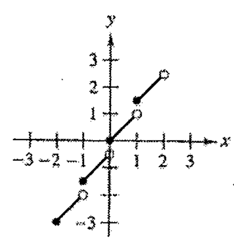
25. $f(x) = \dfrac{1}{x^2 - 4}$



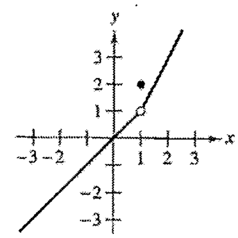
26. $f(x) = \dfrac{x^2 - 1}{x + 1}$

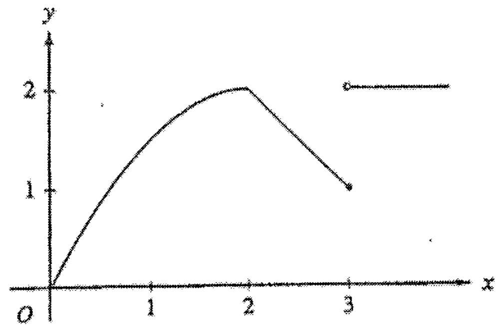


27. $f(x) = \frac{1}{2}[\![x]\!] + x$



28. $f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x - 1, & x > 1 \end{cases}$





7. The graph of the function f is shown in the figure above. Which of the following statements about f is false?

(A) $f(x)$ is continuous at $x = 2$.

(B) $\lim_{x \to 2} f(x) = \lim_{x \to 3^+} f(x)$

(C) $f(3)$ is not defined.

(D) $f(x)$ is not differentiable at $x = 2$.

(E) $f'(3)$ does not exist.

4. If the function f is continuous for all real numbers and if $f(x) = \dfrac{x^2 - 7x + 12}{x - 4}$ when $x \neq 4$, then $f(4) =$

(A) 1 (B) $\frac{8}{1}$ (C) –1 (D) 0 (E) undefined

3. $\lim_{x \to 5} \dfrac{x^2 - 25}{x - 5}$ is

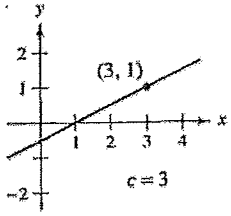
(A) 0
(B) 10
(C) –10
(D) 5
(E) The limit does not exist.

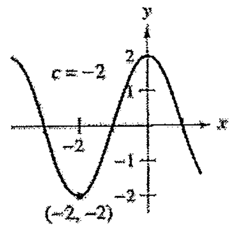
Calculus Worksheet 1-4A Name Key Pd____

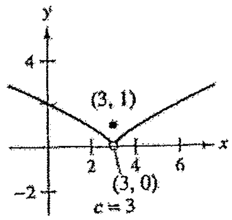
In Exercises 1–6, use the graph to determine the limit, and discuss the continuity of the function.

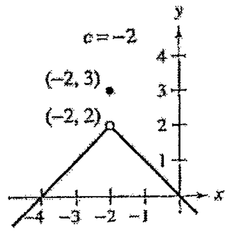
(a) $\lim_{x\to c^+} f(x)$ (b) $\lim_{x\to c^-} f(x)$ (c) $\lim_{x\to c} f(x)$

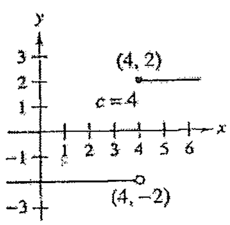
#	f	lim	cont
1	1	1	Y
2	-2	-2	Y
3	1	0	N
4	3	2	N
5	4	N	N
6	-1	N	N

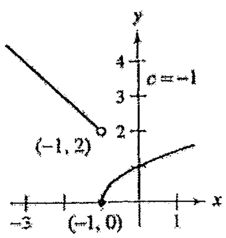
1.


2.


3.


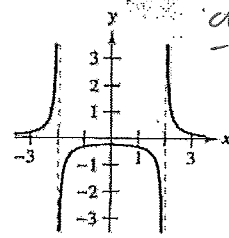
4.


5.


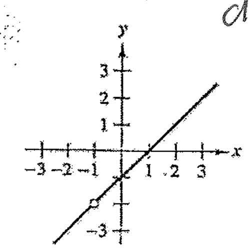
6.


In Exercises 25–28, discuss the continuity of each function.

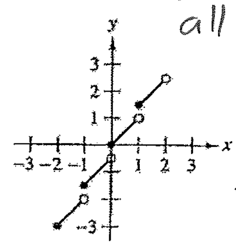
25. $f(x) = \dfrac{1}{x^2-4}$ Disc. at −2 & 2



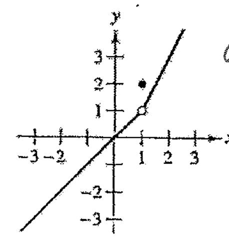
26. $f(x) = \dfrac{x^2-1}{x+1}$ disc. at $x=-1$



27. $f(x) = \frac{1}{2}[\![x]\!] + x$ disc. at all integers



28. $f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x-1, & x > 1 \end{cases}$ disc. at $x=1$



Use calculator for 29-32

In Exercises 29–32, discuss the continuity of the function on the closed interval.

29. $g(x) = \sqrt{25-x^2}$, $[-5, 5]$ circle cont. on interval

30. $f(t) = 3 - \sqrt{9-t^2}$, $[-3, 3]$ ✓ cont on interval

31. $f(x) = \begin{cases} 3-x, & x \le 0 \\ 3+\frac{1}{2}x, & x > 0 \end{cases}$, $[-1, 4]$ try 0 in both $3-0=3$, $3+\frac{1}{2}\cdot 0=3$ cont. on interval

32. $g(x) = \dfrac{1}{x^2-4}$, $[-1, 2]$ cont. on int. but undef at $x=2$

In Exercises 7–24, find the limit (if it exists). If it does not exist, explain why.

7. $\lim_{x\to 5^+} \dfrac{x-5}{x^2-25}$

8. $\lim_{x\to 2^+} \dfrac{2-x}{x^2-4}$

9. $\lim_{x\to -3^-} \dfrac{x}{\sqrt{x^2-9}}$

10. $\lim_{x\to 4^-} \dfrac{\sqrt{x}-2}{x-4}$

11. $\lim_{x\to 0^-} \dfrac{|x|}{x}$

12. $\lim_{x\to 2^+} \dfrac{|x-2|}{x-2}$

13. $\lim_{\Delta x\to 0^+} \dfrac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x}$

14. $\lim_{\Delta x\to 0^+} \dfrac{(x+\Delta x)^2 + x + \Delta x - (x^2+x)}{\Delta x}$

7. $\dfrac{x-5}{(x-5)(x+5)} = \dfrac{1}{x+5}$ $\dfrac{1}{5+5} = \dfrac{1}{10}$

8. $\dfrac{2-x}{(x+2)(x-2)} = \dfrac{-1(x-2)}{(x+2)(x-2)} = \dfrac{-1}{x+2} = \dfrac{-1}{2+2} = \dfrac{-1}{4}$

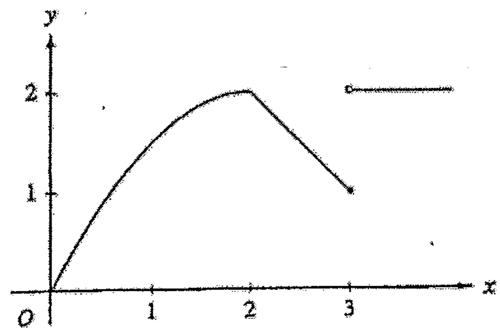
$\boxed{\text{See other paper for 13-14}}$

9. $\dfrac{x}{\sqrt{}}$ ← +/- ← always + as $x\to -3^-$ try −3.001 $\dfrac{-3.001}{+} = -\infty$ not hole, asym.

10. $\dfrac{\sqrt{x}-2}{x-4}$ $\dfrac{\sqrt{x}-2}{(\sqrt{x}+2)(\sqrt{x}-2)} = \dfrac{1}{\sqrt{x}+2}$ at 4 $\dfrac{1}{\sqrt{4}+2} = \dfrac{1}{4}$

11. try −.001 $\dfrac{|-.001|}{-.001} = -1$

12. try 2.001 $\dfrac{|2.001-2|}{2.001-2} = \dfrac{.001}{.001} = 1$



7. The graph of the function f is shown in the figure above. Which of the following statements about f is false?

(A) $f(x)$ is continuous at $x = 2$.

(B) $\lim_{x \to 2} f(x) = \lim_{x \to 3^+} f(x)$

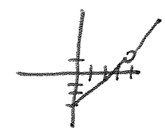
(C) $f(3)$ is not defined.

(D) $f(x)$ is not differentiable at $x = 2$.

(E) $f'(3)$ does not exist.

$\frac{(x-3)(x-4)}{x-4} = x - 3$

$4 - 3 = 1$



4. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2 - 7x + 12}{x - 4}$ when $x \neq 4$, then $f(4) =$ 1

(A) 1 (B) $\frac{8}{-}$ (C) -1 (D) 0 (E) undefined

3. $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$ is

$\frac{(x+5)(x-5)}{x-5} = x + 5$

$5 + 5 = 10$

(A) 0
(B) 10
(C) -10
(D) 5
(E) The limit does not exist.

Calc 1-4A

(13)

$$\lim_{\Delta x \to 0^-} \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\frac{\Delta x}{1}} \qquad \frac{\Delta x}{x+\Delta x} - \frac{\Delta x}{x}$$

$$\lim_{x\to 0} \quad \frac{0}{x+0} - \frac{0}{x} = 0$$

$$\frac{-.01}{x-.01} + \frac{.01}{x}$$

As $\Delta x \to 0$ top goes to zero

Concrete $$\frac{-.01x + .01(x-.01)}{x(x-.01)} = \frac{.001}{x(x-.01)} = 0$$

On BACK

$$\frac{\Delta x(x) - \Delta x(x+\Delta x)}{x(x+\Delta x)} = \frac{-(\Delta x)^2}{x(x+\Delta x)}$$

$$\lim_{x\to 0} = \frac{-0^2}{x(x+0)} = \frac{0}{x^2} = 0$$

---

(14)

$$\lim_{\Delta x\to 0^-} \frac{(x+\Delta x)^2 + x + \Delta x - (x^2+x)}{\Delta x} = \frac{x^2 + 2x\Delta x + (\Delta x)^2 + x + \Delta x - x^2 - x}{\Delta x}$$

$$= \frac{2x\Delta x + (\Delta x)^2 + \Delta x}{\Delta x}$$

$$= \frac{\Delta x}{\Delta x}\left(\frac{2x + \Delta x + 1}{1}\right) = 2x + \Delta x + 1$$

$$\lim_{\Delta x \to 0} = 2x + 1$$

(13)

$$\lim_{\Delta x \to 0^-} \frac{\frac{1}{x+\Delta x} + \frac{-1}{x}}{\frac{\Delta x}{1}}$$

$$\left[\frac{1}{x+\Delta x} + \frac{-1}{x}\right]\frac{1}{\Delta x}$$

$$\left[\frac{1}{(x+\Delta x)}\cdot\frac{x}{x} + \frac{-1}{x}\cdot\frac{(x+\Delta x)}{(x+\Delta x)}\right]\frac{1}{\Delta x}$$

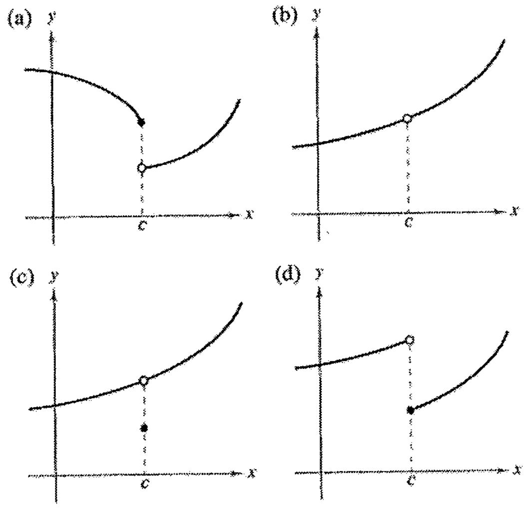
$$\left[\frac{x - x - \Delta x}{x(x+\Delta x)}\right]\frac{1}{\Delta x}$$

$$\left[\frac{-1\cdot \cancel{\Delta x}}{x(x+\Delta x)}\right]\frac{\Delta x}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{-1}{x(x+\Delta x)} = \frac{-1}{x(x+0)} = \frac{-1}{x^2}$$

Calculus **Worksheet 1-4B** **Name**______________________________ **Pd**_____

87. State how continuity is destroyed at $x = c$ for each of the following.



In Exercises 7–24, find the limit (if it exists). If it does not exist, explain why.

15. $\lim\limits_{x\to 3^-} f(x)$, where $f(x) = \begin{cases} \dfrac{x+2}{2}, & x \le 3 \\ \dfrac{12-2x}{3}, & x > 3 \end{cases}$

16. $\lim\limits_{x\to 2} f(x)$, where $f(x) = \begin{cases} x^2 - 4x + 6, & x < 2 \\ -x^2 + 4x - 2, & x \ge 2 \end{cases}$

17. $\lim\limits_{x\to 1} f(x)$, where $f(x) = \begin{cases} x^3 + 1, & x < 1 \\ x + 1, & x \ge 1 \end{cases}$

18. $\lim\limits_{x\to 1^+} f(x)$, where $f(x) = \begin{cases} x, & x \le 1 \\ 1 - x, & x > 1 \end{cases}$

In Exercises 33–54, find the x-values (if any) at which f is not continuous. Which of the discontinuities are removable?

33. $f(x) = x^2 - 2x + 1$

35. $f(x) = 3x - \cos x$

37. $f(x) = \dfrac{x}{x^2 - x}$

39. $f(x) = \dfrac{x}{x^2 + 1}$

41. $f(x) = \dfrac{x+2}{x^2 - 3x - 10}$

43. $f(x) = \dfrac{|x+2|}{x+2}$

45. $f(x) = \begin{cases} x, & x \le 1 \\ x^2, & x > 1 \end{cases}$

47. $f(x) = \begin{cases} \frac{1}{2}x + 1, & x \le 2 \\ 3 - x, & x > 2 \end{cases}$

7. Find k so that $f(x)=\begin{cases}\dfrac{x^2-16}{x-4}\,; & x\neq 4\\ k\,; & x=4\end{cases}$ is continuous for all x.

(A) All real values of k make $f(x)$ continuous for all x.

(B) 0

(C) 16

(D) 8

(E) There is no real value of k that makes $f(x)$ continuous for all x.

15. If $f(x)=\dfrac{x^2+5x-24}{x^2+10x+16}$, then $\lim\limits_{x\to-8} f(x)$ is

(A) 0 (B) 1 (C) $-\dfrac{3}{2}$ (D) $\dfrac{11}{6}$ (E) Nonexistent

20. The function f is given by $f(x)=\begin{cases}\ln 2x, & 0<x<2\\ 2\ln x, & x\geq 2\end{cases}$.

The limit $\lim\limits_{x\to 2} f(x)$ is

(A) 0

(B) $\dfrac{1}{2}$

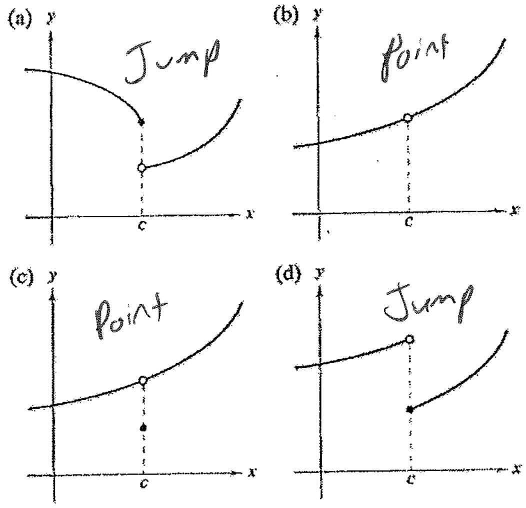
(C) 1

(D) $2\ln 2$

(E) nonexistent

Calculus **Worksheet 1-4B** Name Key Pd____

87. State how continuity is destroyed at $x = c$ for each of the following.



In Exercises 7–24, find the limit (if it exists). If it does not exist, explain why.

15. $\lim_{x\to 3^-} f(x)$, where $f(x) = \begin{cases} \dfrac{x+2}{2}, & x \le 3 \quad L \\ \dfrac{12-2x}{3}, & x > 3 \quad R \end{cases}$ L $\frac{3+2}{2} = \frac{5}{2} = 2.5$

16. $\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} x^2 - 4x + 6, & x < 2 \to 4-8+6 = 2 \\ -x^2 + 4x - 2, & x \ge 2 \to -4+8-2 = 2 \end{cases}$ limit = 2

17. $\lim_{x\to 1} f(x)$, where $f(x) = \begin{cases} x^3 + 1, & x < 1 \to 1+1=2 \\ x + 1, & x \ge 1 \to 1+1=2 \end{cases}$ limit = 2

18. $\lim_{x\to 1} f(x)$, where $f(x) = \begin{cases} x, & x \le 1 \to 1 \\ 1 - x, & x > 1 \to 0 \end{cases}$ Limit does not exist

In Exercises 33–54, find the x-values (if any) at which f is not continuous. Which of the discontinuities are removable?

33. $f(x) = x^2 - 2x + 1$ Cont.

35. $f(x) = 3x - \cos x$ cont

37. $f(x) = \dfrac{x}{x^2 - x}$ $\dfrac{x}{x(x-1)}$ hole at $x=0$, asymp at $x=1$

39. $f(x) = \dfrac{x}{x^2 + 1}$ cont.

41. $f(x) = \dfrac{x+2}{x^2 - 3x - 10}$ $\dfrac{x+2}{(x+2)(x-5)} =$ hole at $x=-2$, asymp at $x=5$

43. $f(x) = \dfrac{|x+2|}{x+2}$ jump at $x = -2$

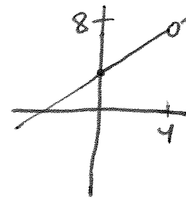
45. $f(x) = \begin{cases} x, & x \le 1 \quad \text{at } x=1 \ y=1 \\ x^2, & x > 1 \quad \text{at } x=1 \ y=1 \end{cases}$ cont.

47. $f(x) = \begin{cases} \frac{1}{2}x + 1, & x \le 2 \quad \text{at } x=2 \quad \frac{1}{2}(2)+1 = 2 \\ 3 - x, & x > 2 \quad \text{at } x=2 \quad 3-2=1 \end{cases}$ jump at $x=2$

7. Find k so that $f(x)=\begin{cases}\dfrac{x^2-16}{x-4} & ; x\neq 4\\ k & ; x=4\end{cases}$ is continuous for all x.

$\dfrac{(x+4)(x-4)}{x-4} = x+4$

$4+4=8$



(A) All real values of k make $f(x)$ continuous for all x.
(B) 0
(C) 16
(D) 8
(E) There is no real value of k that makes $f(x)$ continuous for all x.

15. If $f(x)=\dfrac{x^2+5x-24}{x^2+10x+16}$, then $\lim\limits_{x\to -8} f(x)$ is

$\dfrac{(x+8)(x-3)}{(x+8)(x+2)}$ $\dfrac{x-3}{x+2}$ at $x=-8$ $\dfrac{-8-3}{-8+2} = \dfrac{-11}{-6}$

$\dfrac{11}{6}$

(A) 0 (B) 1 (C) $-\dfrac{3}{2}$ (D) $\dfrac{11}{6}$ (E) Nonexistent

20. The function f is given by $f(x)=\begin{cases}\ln 2x, & 0<x<2\\ 2\ln x, & x\geq 2\end{cases}$.

The limit $\lim\limits_{x\to 2} f(x)$ is

(A) 0
(B) $\dfrac{1}{2}$
(C) 1
(D) $2\ln 2$
(E) nonexistent

Calculus **Worksheet 1-4C** **Name**______________________________ **Pd**_____

In Exercises 33–54, find the x-values (if any) at which f is not continuous. Which of the discontinuities are removable?

34. $f(x) = \dfrac{1}{x^2 + 1}$

36. $f(x) = \cos \dfrac{\pi x}{2}$

38. $f(x) = \dfrac{x}{x^2 - 1}$

40. $f(x) = \dfrac{x - 3}{x^2 - 9}$

42. $f(x) = \dfrac{x - 1}{x^2 + x - 2}$

44. $f(x) = \dfrac{|x - 3|}{x - 3}$

46. $f(x) = \begin{cases} -2x + 3, & x < 1 \\ x^2, & x \geq 1 \end{cases}$

Writing **In Exercises 75–78, explain why the function has a zero in the specified interval.**

75. $f(x) = \frac{1}{16}x^4 - x^3 + 3, \quad [1, 2]$

76. $f(x) = x^3 + 3x - 2, \quad [0, 1]$

77. $f(x) = x^2 - 2 - \cos x, \quad [0, \pi]$

78. $f(x) = -\dfrac{4}{x} + \tan\left(\dfrac{\pi x}{8}\right), \quad [1, 3]$

Use Calculator for 79-82

In Exercises 79–82, use the Intermediate Value Theorem and a graphing utility to approximate the zero of the function in the interval [0, 1]. Repeatedly "zoom in" on the graph of the function to approximate the zero accurate to two decimal places. Use the root-finding capabilities of the graphing utility to approximate the zero accurate to four decimal places.

79. $f(x) = x^3 + x - 1$

80. $f(x) = x^3 + 3x - 2$

81. $g(t) = 2 \cos t - 3t$

82. $h(\theta) = 1 + \theta - 3 \tan \theta$

23. $\lim_{x \to 0} 4 \dfrac{\sin x \cos x - \sin x}{x^2} =$

(A) 2　　(B) $\dfrac{40}{3}$　　(C) ∞　　(D) 0　　(E) undefined

24. $\lim_{x \to 0} \dfrac{\tan(3x) + 3x}{\sin(5x)} =$

(A) 0　　(B) $\dfrac{3}{5}$　　(C) 1　　(D) $\dfrac{6}{5}$　　(E) Nonexistent

15. If $f(x) = \dfrac{x^2 + 5x - 24}{x^2 + 10x + 16}$, then $\lim_{x \to -8} f(x)$ is

(A) 0　　(B) 1　　(C) $-\dfrac{3}{2}$　　(D) $\dfrac{11}{6}$　　(E) Nonexistent

24. $\lim_{x \to 0} \dfrac{\tan(3x) + 3x}{\sin(5x)} =$

(A) 0　　(B) $\dfrac{3}{5}$　　(C) 1　　(D) $\dfrac{6}{5}$　　(E) Nonexistent

Calculus Worksheet 1-4C Name Key Pd_____

In Exercises 33–54, find the x-values (if any) at which f is not continuous. Which of the discontinuities are removable?

34. $f(x) = \dfrac{1}{x^2+1}$ cont.

36. $f(x) = \cos\dfrac{\pi x}{2}$ cont.

38. $f(x) = \dfrac{x}{x^2-1}$ $x^2-1=0$ $(x+1)(x-1)=0$ $x \neq \pm 1$ Disc. at $x=\pm 1$ Not Removable

40. $f(x) = \dfrac{x-3}{x^2-9}$ $\dfrac{x-3}{(x-3)(x+3)} = \dfrac{1}{x+3}$ Disc. at $x=\pm 3$ $x=3$ is Removable

42. $f(x) = \dfrac{x-1}{x^2+x-2}$ $\dfrac{x-1}{(x-1)(x+2)} = \dfrac{1}{x+2}$ Disc at $x=1,-2$ $x=1$ is Removable

44. $f(x) = \dfrac{|x-3|}{x-3}$ Disc at $x=3$, Not Remov.

46. $f(x) = \begin{cases} -2x+3, & x < 1 \\ x^2, & x \geq 1 \end{cases}$ $-2(1)+3=1$ $1^2=1$ Cont.

Writing In Exercises 75–78, explain why the function has a zero in the specified interval.

75. $f(x) = \frac{1}{16}x^4 - x^3 + 3$, $[1, 2]$ $f(1)=2.0625$ $f(2)=-4$
76. $f(x) = x^3 + 3x - 2$, $[0, 1]$ $f(0)=-2$ $f(1)=2$
77. $f(x) = x^2 - 2 - \cos x$, $[0, \pi]$ $f(0)=-3$ $f(\pi)=8.87$
78. $f(x) = -\dfrac{4}{x} + \tan\left(\dfrac{\pi x}{8}\right)$, $[1, 3]$ $f(1)=-3.58$ $f(3)=1.08$

Function is continuous on the interval and changes from − to + OR + to − $\therefore$ a zero must exist by the I.V.T.

Use Calculator for 79-82

In Exercises 79–82, use the Intermediate Value Theorem and a graphing utility to approximate the zero of the function in the interval [0, 1]. Repeatedly "zoom in" on the graph of the function to approximate the zero accurate to two decimal places. Use the root-finding capabilities of the graphing utility to approximate the zero accurate to four decimal places.

* Put 0 in Y_2

79. $f(x) = x^3 + x - 1$ .682
80. $f(x) = x^3 + 3x - 2$ .596
81. $g(t) = 2\cos t - 3t$ .563
82. $h(\theta) = 1 + \theta - 3\tan\theta$ .450

zero

$$\frac{\sin x}{x}\left(\frac{\cos x - 1}{x}\right)$$

23. $\lim\limits_{x\to 0} 4\,\dfrac{\sin x\cos x - \sin x}{x^2} =$

(A) 2 (B) $\frac{40}{3}$ (C) ∞ (D) 0 (E) undefined

$$\frac{\frac{\tan(3x)}{x} + \frac{3x}{x}}{\frac{\sin 5x}{x}} = \frac{3+3}{5}$$

24. $\lim\limits_{x\to 0} \dfrac{\tan(3x) + 3x}{\sin(5x)} =$

(A) 0 (B) $\frac{3}{5}$ (C) 1 (D) $\frac{6}{5}$ (E) Nonexistent

15. If $f(x) = \dfrac{x^2 + 5x - 24}{x^2 + 10x + 16}$, then $\lim\limits_{x\to -8} f(x)$ is

$$\frac{(x+8)(x-3)}{(x+8)(x+2)} = \frac{x-3}{x+2} \quad \frac{-8-3}{-8+2} = \frac{-11}{-6}$$

(A) 0 (B) 1 (C) $-\frac{3}{2}$ (D) $\frac{11}{6}$ (E) Nonexistent

24. $\lim\limits_{x\to 0} \dfrac{\tan(3x) + 3x}{\sin(5x)} =$

(A) 0 (B) $\frac{3}{5}$ (C) 1 (D) $\frac{6}{5}$ (E) Nonexistent