**Calculus** **Worksheet 1-3A** **Name**________________________________________ **Pd**______

**In Exercises 5–22, find the limit.**

5. $\lim_{x \to 2} x^4$

7. $\lim_{x \to 0} (2x - 1)$

9. $\lim_{x \to -3} (x^2 + 3x)$

11. $\lim_{x \to -3} (2x^2 + 4x + 1)$

13. $\lim_{x \to 2} \frac{1}{x}$

15. $\lim_{x \to 1} \frac{x - 3}{x^2 + 4}$

17. $\lim_{x \to 7} \frac{5x}{\sqrt{x + 2}}$

19. $\lim_{x \to 3} \sqrt{x + 1}$

21. $\lim_{x \to -4} (x + 3)^2$

**In Exercises 23–26, find the limits.**

23. $f(x) = 5 - x,\ g(x) = x^3$
 (a) $\lim_{x \to 1} f(x)$ (b) $\lim_{x \to 4} g(x)$ (c) $\lim_{x \to 1} g(f(x))$

24. $f(x) = x + 7,\ g(x) = x^2$
 (a) $\lim_{x \to -3} f(x)$ (b) $\lim_{x \to 4} g(x)$ (c) $\lim_{x \to -3} g(f(x))$

25. $f(x) = 4 - x^2,\ g(x) = \sqrt{x + 1}$
 (a) $\lim_{x \to 1} f(x)$ (b) $\lim_{x \to 3} g(x)$ (c) $\lim_{x \to 1} g(f(x))$

26. $f(x) = 2x^2 - 3x + 1,\ g(x) = \sqrt[3]{x + 6}$
 (a) $\lim_{x \to 4} f(x)$ (b) $\lim_{x \to 21} g(x)$ (c) $\lim_{x \to 4} g(f(x))$

**In Exercises 45–48, find the limit of the function (if it exists). Write a simpler function that agrees with the given function at all but one point. Use a graphing utility to confirm your result.**

45. $\lim_{x \to -1} \frac{x^2 - 1}{x + 1}$

46. $\lim_{x \to -1} \frac{2x^2 - x - 3}{x + 1}$

47. $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$

48. $\lim_{x \to -1} \frac{x^3 + 1}{x + 1}$

3. $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$ is

(A) 0
(B) 10
(C) –10
(D) 5
(E) The limit does not exist.

23. $\lim_{x \to 0} 4 \frac{\sin x \cos x - \sin x}{x^2} =$

(A) 2 (B) $\frac{40}{3}$ (C) $\infty$ (D) 0 (E) undefined

20. The function $f$ is given by $f(x) = \begin{cases} \ln 2x, & 0 < x < 2 \\ 2 \ln x, & x \geq 2 \end{cases}$.
The limit $\lim_{x \to 2} f(x)$ is

(A) 0
(B) $\frac{1}{2}$
(C) 1
(D) $2 \ln 2$
(E) nonexistent

Calculus Worksheet 1-3A Name Key Pd____

**In Exercises 5–22, find the limit.**

5. $\lim_{x\to 2} x^4$ 16

7. $\lim_{x\to 0} (2x-1)$ $-1$

9. $\lim_{x\to -3} (x^2+3x)$ 15

11. $\lim_{x\to -3} (2x^2+4x+1)$ 7

13. $\lim_{x\to 2} \frac{1}{x} = \frac{1}{2}$

15. $\lim_{x\to 1} \frac{x-3}{x^2+4} = \frac{-2}{5}$

17. $\lim_{x\to 7} \frac{5x}{\sqrt{x+2}} = \frac{35}{3}$

19. $\lim_{x\to 3} \sqrt{x+1} = 2$

21. $\lim_{x\to -4} (x+3)^2 = 1$

**In Exercises 23–26, find the limits.**

23. $f(x) = 5-x,\ g(x) = x^3$

(a) $\lim_{x\to 1} f(x)$ 4 (b) $\lim_{x\to 4} g(x)$ 64 (c) $\lim_{x\to 1} g(f(x))$ 64 $\leftarrow (5-x)^3$

24. $f(x) = x+7,\ g(x) = x^2$

(a) $\lim_{x\to -3} f(x)$ 4 (b) $\lim_{x\to 4} g(x)$ 16 (c) $\lim_{x\to -3} g(f(x))$ $(\underset{-3}{x}+7)^2 = 16$

25. $f(x) = 4-x^2,\ g(x) = \sqrt{x+1}$

(a) $\lim_{x\to 1} f(x)$ 3 (b) $\lim_{x\to 3} g(x) = 2$ (c) $\lim_{x\to 1} g(f(x))$ $\sqrt{(4-\underset{1}{x^2})+1} = 2$

26. $f(x) = 2x^2-3x+1,\ g(x) = \sqrt[3]{x+6}$

(a) $\lim_{x\to 4} f(x) = \cancel{45}$ 21 (b) $\lim_{x\to 21} g(x) = 3$ (c) $\lim_{x\to 4} g(f(x))$ $\sqrt[3]{(2x^2-3x+1)+6} = 3$

**In Exercises 45–48, find the limit of the function (if it exists). Write a simpler function that agrees with the given function at all but one point. Use a graphing utility to confirm your result.**

45. $\lim_{x\to -1} \frac{x^2-1}{x+1}$ $\frac{(x+1)(x-1)}{x+1}$ $x-1$ $-1-1=-2$

46. $\lim_{x\to -1} \frac{2x^2-x-3}{x+1}$ $\frac{(2x-3)(x+1)}{x+1}$

47. $\lim_{x\to 2} \frac{x^3-8}{x-2}$

48. $\lim_{x\to -1} \frac{x^3+1}{x+1}$

$$
\begin{array}{r|l}
 & x^2+2x+4 \\
x-2 & x^3+0x^2+0x-8 \\
 & -x^3+2x^2 \\
 & 2x^2+0x \\
 & -2x^2+4x \\
 & 4x-8 \\
 & -4x+8 \\
 & 0
\end{array}
$$

$\frac{(x^2+2x+4)(x-2)}{(x-2)} = x^2+2x+4$

at $x=2$

$4+4+4=12$

$$
\begin{array}{r|l}
 & x^2-x+1 \\
x+1 & x^3+0x^2+0x+1 \\
 & -x^3+-x^2 \\
 & -1x^2+0x \\
 & +x^2+x \\
 & 1x+1 \\
 & -x+1 \\
 & 0
\end{array}
$$

$\frac{(x^2-x+1)(x+1)}{(x+1)}$

$x^2-x+1$

at $x=-1$

$1+1+1=3$

3. $\lim\limits_{x\to 5}\dfrac{x^2-25}{x-5}$ is

$$\frac{(x+5)(x-5)}{x-5} = x+5$$

$$5+5=10$$

(A) 0

(B) 10

(C) −10

(D) 5

(E) The limit does not exist.

$$-1(1-\cos x)$$

$$4\cdot\frac{\sin x}{x}\cdot\frac{\cos x-1}{x} = 4\cdot 1\cdot -1\cdot 0 = 0$$

23. $\lim\limits_{x\to 0} 4\,\dfrac{\sin x\cos x-\sin x}{x^2} =$

(A) 2 (B) $\dfrac{40}{3}$ (C) $\infty$ (D) 0 (E) undefined

20. The function $f$ is given by $f(x)=\begin{cases}\ln 2x, & 0<x<2 \\ 2\ln x, & x\ge 2\end{cases}$ at 2 $\ln 2\cdot 2 = \ln 4$; at 2 $2\ln 2 = \ln 2^2 = \ln 4$

The limit $\lim\limits_{x\to 2} f(x)$ is

(A) 0

(B) $\dfrac{1}{2}$

(C) 1

(D) $2\ln 2$ $= \ln 4$

(E) nonexistent

**Calculus** **Worksheet 1-3B** **Name**________________________________ **Pd**_____

Find the limit:

**6.** $\lim_{x \to -2} x^3$

**8.** $\lim_{x \to -3} (3x + 2)$

**10.** $\lim_{x \to 1} (-x^2 + 1)$

**12.** $\lim_{x \to 1} (3x^3 - 2x^2 + 4)$

**14.** $\lim_{x \to -3} \dfrac{2}{x + 2}$

**16.** $\lim_{x \to 3} \dfrac{2x - 3}{x + 5}$

**18.** $\lim_{x \to 3} \dfrac{\sqrt{x + 1}}{x - 4}$

**20.** $\lim_{x \to 4} \sqrt[3]{x + 4}$

**22.** $\lim_{x \to 0} (2x - 1)^3$

**In Exercises 27–36, find the limit of the trigonometric function.**

**27.** $\lim_{x \to \pi/2} \sin x$

**28.** $\lim_{x \to \pi} \tan x$

**29.** $\lim_{x \to 2} \cos \dfrac{\pi x}{3}$

**30.** $\lim_{x \to 1} \sin \dfrac{\pi x}{2}$

**31.** $\lim_{x \to 0} \sec 2x$

**32.** $\lim_{x \to \pi} \cos 3x$

**33.** $\lim_{x \to 5\pi/6} \sin x$

**34.** $\lim_{x \to 5\pi/3} \cos x$

**In Exercises 49–62, find the limit (if it exists).**

**49.** $\lim_{x \to 5} \dfrac{x - 5}{x^2 - 25}$

**50.** $\lim_{x \to 2} \dfrac{2 - x}{x^2 - 4}$

**51.** $\lim_{x \to -3} \dfrac{x^2 + x - 6}{x^2 - 9}$

**52.** $\lim_{x \to 4} \dfrac{x^2 - 5x + 4}{x^2 - 2x - 8}$

**53.** $\lim_{x \to 0} \dfrac{\sqrt{x + 5} - \sqrt{5}}{x}$

**54.** $\lim_{x \to 0} \dfrac{\sqrt{2 + x} - \sqrt{2}}{x}$

2. The domain of the function $f(x) = \sqrt{4-x^2}$ is

(A) $x < -2$ or $x > 2$
(B) $x \leq -2$ or $x \geq 2$
(C) $-2 < x < 2$
(D) $-2 \leq x \leq 2$
(E) $x \leq 2$

15. If $f(x) = \dfrac{x^2+5x-24}{x^2+10x+16}$, then $\lim\limits_{x \to -8} f(x)$ is

(A) 0 (B) 1 (C) $-\dfrac{3}{2}$ (D) $\dfrac{11}{6}$ (E) Nonexistent

24. $\lim\limits_{x \to 0} \dfrac{\tan(3x)+3x}{\sin(5x)} =$

(A) 0 (B) $\dfrac{3}{5}$ (C) 1 (D) $\dfrac{6}{5}$ (E) Nonexistent

C

31. $\lim\limits_{h \to 0} \dfrac{\tan\left(\dfrac{\pi}{6}+h\right)-\tan\left(\dfrac{\pi}{6}\right)}{h} =$

(A) $\dfrac{\sqrt{3}}{3}$ (B) $\dfrac{4}{3}$ (C) $\sqrt{3}$ (D) 0 (E) $\dfrac{3}{4}$

**Calculus** **Worksheet 1-3B** Name________________ Pd_____

Find the limit:

6. $\lim_{x\to -2} x^3$ $-8$

8. $\lim_{x\to -3} (3x+2)$ $-7$

10. $\lim_{x\to 1} (-x^2+1) = 0$

12. $\lim_{x\to 1} (3x^3 - 2x^2 + 4) = 5$

14. $\lim_{x\to -3} \frac{2}{x+2} = -2$

16. $\lim_{x\to 3} \frac{2x-3}{x+5} = \frac{3}{8}$

18. $\lim_{x\to 3} \frac{\sqrt{x+1}}{x-4}$ $\frac{2}{-1} = -2$

20. $\lim_{x\to 4} \sqrt[3]{x+4} = 2$

22. $\lim_{x\to 0} (2x-1)^3 = -1$

**In Exercises 27–36, find the limit of the trigonometric function.**

27. $\lim_{x\to \pi/2} \sin x = 1$

28. $\lim_{x\to \pi} \tan x = 0$

29. $\lim_{x\to 2} \cos \frac{\pi x}{3} = -\frac{1}{2}$

30. $\lim_{x\to 1} \sin \frac{\pi x}{2} = 1$

31. $\lim_{x\to 0} \sec 2x = 1$

32. $\lim_{x\to \pi} \cos 3x = -1$

33. $\lim_{x\to 5\pi/6} \sin x = \frac{1}{2}$

34. $\lim_{x\to 5\pi/3} \cos x = \frac{1}{2}$

**In Exercises 49–62, find the limit (if it exists).**

49. $\lim_{x\to 5} \frac{x-5}{x^2-25}$

50. $\lim_{x\to 2} \frac{2-x}{x^2-4}$

51. $\lim_{x\to -3} \frac{x^2+x-6}{x^2-9}$

52. $\lim_{x\to 4} \frac{x^2-5x+4}{x^2-2x-8}$

53. $\lim_{x\to 0} \frac{\sqrt{x+5}-\sqrt{5}}{x}$

54. $\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x}$

49. $\frac{x-5}{(x+5)(x-5)} = \frac{1}{x+5} = \frac{1}{5+5} = \frac{1}{10}$

50. $\frac{-1(x-2)}{(x-2)(x+2)} = \frac{-1}{x+2} = \frac{-1}{4}$

51. $\frac{(x+3)(x-2)}{(x+3)(x-3)} = \frac{x-2}{x-3} = \frac{-5}{-6} = \frac{5}{6}$

52. $\frac{(x-4)(x-1)}{(x-4)(x+2)} = \frac{x-1}{x+2} = \frac{3}{6} = \frac{1}{2}$

53. } see other page
54. }

2. The domain of the function $f(x)=\sqrt{4-x^2}$ is

circle radius 2

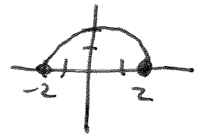
(A) $x<-2$ or $x>2$

(B) $x\leq -2$ or $x\geq 2$

(C) $-2<x<2$

(D) $-2\leq x\leq 2$

(E) $x\leq 2$




15. If $f(x)=\dfrac{x^2+5x-24}{x^2+10x+16}$, then $\lim\limits_{x\to -8} f(x)$ is

$$\frac{(x+8)(x-3)}{(x+8)(x+2)} = \frac{x-3}{x+2} \qquad \frac{-11}{-6} = \frac{11}{6}$$

(A) 0 (B) 1 (C) $-\dfrac{3}{2}$ (D) $\dfrac{11}{6}$ (E) Nonexistent

24. $\lim\limits_{x\to 0} \dfrac{\tan(3x)+3x}{\sin(5x)} =$

need x under the sin & tan

$$\frac{\frac{\tan(3x)}{1}+\frac{3x}{1}}{\frac{\sin(5x)}{1}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \frac{\frac{\tan 3x}{x}+\frac{3x}{x}}{\frac{\sin 5x}{x}} = \frac{3+3}{5}$$

(A) 0 (B) $\dfrac{3}{5}$ (C) 1 (D) $\dfrac{6}{5}$ (E) Nonexistent

C

31. $\lim\limits_{h\to 0} \dfrac{\tan\left(\frac{\pi}{6}+h\right)-\tan\left(\frac{\pi}{6}\right)}{h} =$

calc. at $x=.0000001$

$x=1.\overline{3}$

(A) $\dfrac{\sqrt{3}}{3}$ (B) $\dfrac{4}{3}$ (C) $\sqrt{3}$ (D) 0 (E) $\dfrac{3}{4}$

1-3B

53.
$$\lim_{x\to 0} \frac{\left(\sqrt{x+5}-\sqrt{5}\right)}{x}\left(\frac{\sqrt{x+5}+\sqrt{5}}{\sqrt{x+5}+\sqrt{5}}\right) = \frac{x+5+\sqrt{x+5}\sqrt{5}-\sqrt{x+5}\sqrt{5}-5}{x\left(\sqrt{x+5}+\sqrt{5}\right)}$$

$$= \frac{x}{x}\cdot\frac{1}{\sqrt{x+5}+\sqrt{5}} =$$

$$\lim_{x\to 0}\frac{1}{\sqrt{0+5}+\sqrt{5}} = \frac{1}{2\sqrt{5}}$$

54.
$$\lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x}\left(\frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}\right) = \frac{2+x-2}{x\left(\sqrt{2+x}+\sqrt{2}\right)}$$

$$= \frac{x}{x}\cdot\frac{1}{\sqrt{2+x}+\sqrt{2}}$$

$$\lim_{x\to 0}\frac{1}{\sqrt{0+2}+\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

$$\cancel{\frac{\frac{\sqrt{2+x}-\sqrt{2}}{1}}{\frac{x}{1}}\,\frac{\left(\frac{\sin x}{x}\right)}{\left(\frac{\sin x}{x}\right)} = \frac{\left(\frac{\sqrt{2+x}-\sqrt{2}}{1}\right)\left(\frac{\sin x}{x}\right)}{\sin x}}$$