Derivative Reference Sheet

Formal definition of a derivative: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ or $f'(x) = \lim_{x_2 - x_1 \to 0} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

Function Name	f(x)	f'(x)
Constant function	$f(x) = 5$	$f'(x) = 0$
Power Rule	$f(x) = ax^n$	$f'(x) = anx^{n-1}$
Product Rule	$f(x) = g(x) \bullet h(x)$	$f'(x) = g'(x) \bullet h(x) + g(x) \bullet h'(x)$
Quotient Rule	$f(x) = \frac{g(x)}{h(x)}$	$f'(x) = \frac{h(x) \bullet g'(x) - h'(x) \bullet g(x)}{[h(x)]^2}$
Sine	$f(x) = \sin(g(x))$	$f'(x) = \cos(g(x)) \bullet g'(x)$
Cosine	$f(x) = \cos(g(x))$	$f'(x) = -\sin(g(x)) \bullet g'(x)$
Tangent	$f(x) = \tan(g(x))$	$f'(x) = \sec^2(g(x)) \bullet g'(x)$
Cosecant	$f(x) = \csc(g(x))$	$f'(x) = -\csc(g(x))\cot(g(x)) \bullet g'(x)$
Secant	$f(x) = \sec(g(x))$	$f'(x) = \sec(g(x))\tan(g(x)) \bullet g'(x)$
Cotangent	$f(x) = \cot(g(x))$	$f'(x) = -\csc^2(g(x)) \bullet g'(x)$
Chain Rule	$f(x) = g(h(x))$	$f'(x) = g'(h(x)) \bullet h'(x)$
Absolute value	$f(x) = \lvert g(x) \rvert$	$f'(x) = \frac{g(x) \bullet g'(x)}{\lvert g(x) \rvert}$
Natural Log	$f(x) = \ln(g(x))$	$f'(x) = \frac{g'(x)}{g(x)}$
e	$f(x) = e^{g(x)}$	$f(x) = g'(x) \bullet e^{g(x)}$
Logarithm	$f(x) = \log_a g(x)$	$f'(x) = \frac{g'(x)}{g(x) \bullet \ln a}$
Exponential	$f(x) = a^{g(x)}$	$f'(x) = g'(x) \bullet a^{g(x)} \bullet \ln a$
Inverses	f(x) and g(x) are inverse functions where $f^{-1}(x) = g(x)$ and $g^{-1}(x) = f(x)$	$f'(x) = \frac{1}{g'(f(x))}$ where the bottom is not zero
Arcsine	$f(x) = \arcsin(g(x))$	$f'(x) = \frac{g'(x)}{\sqrt{1-g(x)^2}}$
Arccosine	$f(x) = \arccos(g(x))$	$f'(x) = \frac{-g'(x)}{\sqrt{1-g(x)^2}}$
Arctangent	$f(x) = \arctan(g(x))$	$f'(x) = \frac{g'(x)}{1+g(x)^2}$

Application of derivatives:

- Original function = displacement
- First derivative = velocity (absolute value of velocity = speed)
- Second derivative = acceleration

Calculator Reference

- Numerical derivative: NDER(2x-1,x,3) means what is the numerical value of the function 2x-1 with respect to x at the value x=3. To find NDER press MATH and 8.
- To graph a derivative: type the function in Y_1, then type NDER(Y_1,X,X) in Y_2. To find Y_1, press VARS, RIGHT ARROW, ENTER, ENTER.
- To graph a tangent line, graph the equation and trace to the point of tangency. Press 2^{ND}, DRAW, 5, ENTER. To remove the line, press 2^{ND}, DRAW, 1.

Rules

- A function f is differentiable at a point a if f'(a) exists.
- If f is differentiable at a, then f is continuous at a.
- Three ways a function is not differentiable: corner, discontinuity, vertical tangent line

Implicit differentiation example:

Find y' if $x^3 - y^2 = 7$

Step 1 $3x^2 - 2y \cdot y' = 0$

Step 2 $y' = \frac{3x^2}{2y}$