Algebra One Notes for Chapter Two

Section 2-2 – solving with + and -

Math Terms

Additive Identity = 0

Additive Inverses = numbers that add to 0

The Main Rules of Equation Solving

Whatever you do to one side of the equation, you must also do to the other side.

The goal is to get the x alone on one side.

Equation solving works backwards through the order of operations.

Making Zero – use this when a number is added or subtracted to x.

Example

Example	x + 4 = 7	4 is added to x, cancel it out with something that adds to 0 with 4
	-4 -4	remember to do this to both sides of the equation
	x + 0 = 3	so x = 3
	x – 8 = 5	8 is subtracted from x, cancel it out with something that adds to 0 with -8
	+8 +8	remember to add the 8 to both sides.
	x + 0 = 13	so x = 13
	x + 2/3 = 1	2/3 is added to x, cancel it out with something that adds to 0 with 2/3
	-2/3 -2/3	remember to do this to both sides of the equation
	x + 0 = 1/3	so x = 1/3
	8 = x – (-5)	rewrite the "minus a negative" as a plus sign (super plus)
	8 = x + 5	5 is added to x, cancel it out with something that adds to 0 with 5
	-5 -5	remember to do this to both sides of the equation
	3 = x	so x = 3

Words

When given in words, the "is" or "is equal to" mean = sign.
Example: A number increased by seven is 12. Find the number.

x + 7 = 12
- 7 - 7
x = 5

Section ~~3-2~~ 2-2 – solving with • and ÷

Math Terms

Multiplicative Identity = 1

Multiplicative Inverses = numbers that multiply to 1

The Main Rules of Equation Solving

*Whatever you do to one side of the equation, you must also do to the other side.

*The goal is to get the x alone on one side.

*Equation solving works backwards through the order of operations.

Making One – use this when a number is multiplied or divided into x.

Method 1 – multiply by the reciprocal

Example 5x = 30 5 is multiplied by x, cancel it out with something that multiplies to 1 with 5

(1/5) 5x = (1/5) 30 the 1/5 times 5 creates a 1, 1/5 times 30 is 6

1x = 6 so x = 6

(3/4) x = 6 ¾ is multiplied by x, cancel it out by multiplying by 4/3

(4/3)(3/4) x = (4/3) 6 (4/3) time (3/4) creates 1, (4/3) times 6 is 8

1x = 8 so x = 8

Method 2 – do the opposite: multiply when x is divided and divide when x is multiplied.

Example 5x = 30 5 is multiplied by x, cancel it out by dividing by 5

$\frac{5x}{5} = \frac{30}{5}$ 5/5 is 1 and 30/5 is 6

1x = 6 so x = 6

(¾) x = 6 ¾ is multiplied by x, cancel it out by dividing by ¾

$\frac{(¾)\,x}{¾} = \frac{6}{¾}$ ¾ divided by ¾ is 1 and 3 divided by ¾ is 8

1x = 8 so x = 8

$\frac{x}{4} = 7$ x is divided by 4, cancel it out by multiplying by 4

$4 \bullet \frac{x}{4} = 4 \bullet 7$ 4 / 4 is 1 and 4 • 7 is 28

1x = 28 so x = 28

Section 2-3 – 2 step equation solving

Math Terms

Order of Operations: Parenthesis, Exponents, Multiply and Divide, Add and Subtract

The Main Rules of Equation Solving

Step 1 – Simplify [distributive property, like terms, cancel fractions]

Step 2 – Get x by itself on one side.

Whatever you do to one side of the equation, you must also do to the other side.

*Equation solving works backwards through the order of operations.

First cancel the add/sub. number (make 0), then cancel the mult./div. number (make 1)

Example	$5x + 3 = 18$ $-3 \quad -3$	5 is multiplied by x and 3 is added, cancel the 3 by subtracting 3 3 – 3 is 0, 18 – 3 is 15
	$\frac{5x}{5} = \frac{15}{5}$	5 is multiplied by x, cancel it by dividing by 5 (or multiply by 1/5) 5/5 is 1 and 15/5 is 3
	$1x = 3$	so x = 3
	$\frac{1}{2}x - 4 = 6$ $+4 \quad +4$	½ is multiplied by x and 4 is subtracted, cancel the 4 by adding 4 -4 + 4 is 0, 6 + 4 is 10
	$\frac{\frac{1}{2}x}{\frac{1}{2}} = \frac{10}{\frac{1}{2}}$	5 is multiplied by x, cancel it by dividing by 5 (or multiply by 1/5) ½ ÷ ½ is 1 and 10/ ½ is 20
	$1x = 20$	so x = 20
	$\frac{x}{4} - 5 = 7$	x is divided by 4 and 5 is subtracted, cancel the 5 by adding 5
	$\frac{x}{4} - 5 + 5 = 7 + 5$	-5 + 5 is 0, 7 + 5 is 12
	$\frac{x}{4} = 12$	x is divided by 4, cancel it out by multiplying by 4
	$4 \bullet \frac{x}{4} = 4 \bullet 12$	4 / 4 = 1 and 4 • 12 is 48
	$1x = 48$	so x = 48

Section 2-4 – equation solving with more than 1 x

The Main Rules of Equation Solving

Step 1 – Simplify [distributive property, like terms, cancel fractions]

Step 2 – Get all the x's on the same side

*If there is an x on opposite sides of the equal sign, then you must cancel one out (make 0) so there is only x on one side of the equation. Whatever you do to one side of the equation, you must also do to the other side.

Step 3 – Get x by itself on one side.

Whatever you do to one side of the equation, you must also do to the other side.

Equation solving works backwards through the order of operations.

Cancel the add/sub. number (make 0), then cancel the mult./div. number (make 1)

Example	$5x + 3x - 1 = 15$	5x and 3x are on the same side. Add the like terms 5x + 3x = 8x
	$8x - 1 = 15$	Now add 1 to both sides. -1 to 1 is 0 and 15 + 1 is 16
	$8x = 16$	Now divide by 8. 8/8 is 1 and 16/8 is 2
	$1x = 8$	so x = 8

$3x - 2 = 5x + 9$ $-3x \quad\quad -3x$	3x and 5x are on opposite sides. Cancel the 3x by subtracting 3x. 3x – 3x is 0 and 5x – 3x is 2x. Now x is only on the right side.
$-2 = 2x + 9$ $-9 \quad\quad -9$	2 is multiplied by x and 9 is added. Add -9 to both sides. -2 + -9 is -11 and 9 + -9 is 0.
$\frac{-11}{2} = \frac{2x}{2}$	divide by 2. -11 / 2 is -5.5 abd 2 / 2 is 1.
$-5.5 = 1x$	so x = -5.5
	Note: the same answer is reached by subtracting 5x at step 1.

$4(x - 4) + 2 = \frac{1}{2}(4x - 2) + x$	distribute 4 and ½
$4x - 16 + 2 = 2x - 1 + x$	Like terms: -16 and 2 on the left, 2x and x on the right
$4x - 14 = 3x - 1$ $-3x \quad\quad -3x$	4x and 3x are on opposite sides. Add -3x to both sides 4x – 3x = 1x and 3x – 3x = 0
$1x - 14 = -1$	add 14 to both sides. -14 to 14 is 0 and -1 + -14 is -15.
$1x = -15$	so x = -15

Special Case 1: subtracting x

$4 - x = -3$	-1 is multiplied by x and 4 is added, cancel the 4 by subtracting 4
$-4 \qquad -4$	4 – 4 is 0 and -3 – 4 is -7
$\frac{-1x}{-1} = \frac{-7}{-1}$	-1 is multiplied by x, cancel it by dividing by -1 -1/-1 is 1 and -7/-1 is 7
$1x = 7$	so x = 7

Special Case 2: simplify before you solve

$3(2x - 4) = 12$	First distribute the 3. 3•2x is 6x and 3•-4 is -12
$6x - 12 = 12$	Now add 12. -12 + 12 is 0 and 12 + 12 is 24.
$6x = 24$	Now divide by 6. 6/6 is 1 and 24 / 6 is 4.
$1x = 4$	so x = 4.

Special Case 3: simplify before you solve

$2x - 4 + x - 8 = 13$	First combine the like terms on the left. 2x + x = 3x and -3 + -8 = -11
$3x - 11 = 13$	Now add 11. -11 + 11 is 0 and 13 + 11 is 24.
$3x = 24$	Now divide by 3. 3/3 is 1 and 24 / 3 is 8.
$1x = 8$	so x = 8.

Special Case 4: fractions

$\frac{x-5}{4} = -3$	5 is subtracted from x and 4 is divided. We CANNOT add 5.
	We must cancel the 4 by multiplying by 4 (or split into 2 fractions)
$4 \bullet \frac{x-5}{4} = 4 \bullet (-3)$	4 / 4 is 1 and 4 • (-3) is -12
x – 5 = -12 +5 +5	5 is subtracted from x, cancel it by adding 5 -5 + 5 is 0 and -12 + 5 is -7
x = -7	so x = -7

Alternate Method for the above example

$\frac{x-5}{4} = -3$	the (x – 5) over 4 can be split into x/4 – 5/4
$\frac{x}{4} - \frac{5}{4} = -3$	cancel the -5/4 by adding 5/4
$\frac{x}{4} - \frac{5}{4} + \frac{5}{4} = -3 + \frac{5}{4}$	-5/4 + 5/4 is 0 and -3 + 5/4 is -1.75
$\frac{x}{4} = -1.75$	x is divided by 4, cancel it out by multiplying by 4
$4 \bullet \frac{x}{4} = 4(-1.75)$	4/4 is 1 and 4(-1.75) is -7
1x = -7	so x = -7

Special Case 5: fractions

$\frac{2}{3}x + \frac{3}{2} = -3$	One option is to eliminate all the fractions by multiplying by each denominator (bottom of the fraction). In this equation, •3 and •2 First, multiply by 3 to eliminate the fraction with 3 in bottom. This must be done to each term. The 3 terms are 2/3 x, 3/2, and -3
$3 \bullet \frac{2}{3}x + 3 \bullet \frac{3}{2} = -3 \bullet 3$	$3 \bullet \frac{2}{3}x\ creates\ 2x \qquad 3 \bullet \frac{3}{2}\ creates\ \frac{9}{2}\ \ and\ -3 \bullet 3\ \ creates - 9$
$2x + \frac{9}{2} = -9$	Now multiply by 2 to eliminate the fraction with 2 in the bottom.
$2 \bullet 2x + 2 \bullet \frac{9}{2} = -9 \bullet 2$	2•2x = 4x $\quad 2 \bullet \frac{9}{2} = 9$ $\quad$ -9•2 = -18
4x + 9 = -18 -9 -9	Now add -9 to both sides
4x = -27 ÷4 ÷4	Now divide by 4 on both sides
1x = -6.75	so x = -6.75

Special Case 6: *variables disappear.*

5x + 1 = 5x + 5	5x and 5x are on opposite sides. Subtract 5x from both sides.
-5x -5x	5x – 5x = 0 on both sides. All variables disappear.
1 = 5	This is a FALSE statement. So there is NO SOLUTION.
	This means that no number can plug in for x and create a true statement (Left will never equal the Right).

Check it, try x = 1 5(1) + 1 = 5(1) + 5
6 = 10 NO

Special Case 7: *variables disappear again.*

5x + 5 = 5x + 5	5x and 5x are on opposite sides. Subtract 5x from both sides.
-5x -5x	5x – 5x = 0 on both sides. All variables disappear.
5 = 5	This is a TRUE statement. So the solution is ALL REALS.
	This means that any number can plug in for x and create a true statement (Left will equal the Right).

Check it, try x = 1 5(1) + 5 = 5(1) + 5
10 = 10 YES

Check it, try x = 0 5(0) + 5 = 5(0) + 5
5 = 5 YES

Special Case 8: *solving for a variable in a formula.*

Given the circumference formula C = 2•π•r solve for radius.

C = 2πr	To get r by itself, we need to get the 2π away from r.
	Since the 2π is multiplied by r, we need to divide by 2π (make 1).
C ÷ (2π) = 2πr ÷ (2π)	Now the 2π is away from the r. Answer: $\frac{C}{2\pi} = r$

Special Case 9: *solving for a variable in a formula.*

Given the perimeter formula for a rectangle P = 2L + 2W solve for width.

P = 2L + 2W	To get W by itself, we need to get the 2L and the 2 away from W.
-2L -2L	First cancel the 2L by subtracting 2L from both sides (make 0)
P – 2L = 2W	Now divide by 2 to cancel the 2 multiplied by W (make 1)
$\frac{P-2L}{2} = \frac{2W}{2}$	Answer: $\frac{P-2L}{2} = W$

Section 2-5 – Ratios and Proportions

Key Points:

> We will setup proportions to solve these questions. Cross-multiply and divide to solve.

> Label the two things being compared.

> Start with the two numbers that go with each category to start the proportion.

Example: Jack went to bat 50 times and earned 22 hits. If his rate of hits stays the same, then how many hits would Jack earn in 220 at bats?

The two categories are "hits" and "at bats"   To start, 50 at bats and 22 hits

$$\begin{matrix} \textit{Hits} & & \textit{At Bat} \\ \frac{22}{} & = & \frac{50}{} \end{matrix}$$

Now the 220 will go in the "at bats" column and x will go in the "hits" column

$$\begin{matrix} \textit{Hits} & & \textit{At Bats} \\ \frac{22}{x} & = & \frac{50}{220} \end{matrix}$$

Cross-multiply 22•220 and divide the answer by 50. So x = 96.8 which is 97 hits.

Section 2-5 – Solving equations with absolute value

Absolute value equations can have 0, 1, or 2 solutions.

Example with 0 solutions:

Solve $|x| = -2$ There are no values of x that work because the absolute value will never Be negative. Therefore, this equation has no solution.

Example with 1 solution:

Solve $|x| = 0$ Only zero solves this equation, so the solution is $x = 0$.

Example with 2 solutions (**Most will follow this example**):

Solve $|x| = 3$ Since $|3| = 3$ AND $|-3| = 3$ there are two solutions: $x = -3$ and 3.

Solve $|x + 2| = 3$

Set $x + 2 = 3$ This gives the solution $x = 1$
Check $x = 1$ $|1 + 2| = |3| = 3$ yes

Set $x + 2 = -3$ This give the solution $x = -5$
Check $x = -5$ $|-5 + 2| = |-3| = 3$ yes

There are two solutions because the absolute value turns both 3 and -3 into the positive value 3

More complicated examples – First make sure the absolute value is alone on 1 side.

Solve $2|3x| - 4 = 2$ add 4 to both sides
$2|3x| = 3$ divide both sides by 2
$|3x| = 3$

Set $3x = 3$ This gives the solution $x = 1$
Check $x = 1$ $2|3*1| - 4 = 2|3| - 4 = 6 - 4 = 2$ yes

Set $3x = -3$ This gives the solution $x = -1$
Check $x = -1$ $2|3*-1| - 4 = 2|-3| - 4 = 6 - 4 = 2$ yes