

Calculus Test 3 Prep

****Review for Quiz 9 (Sec 4-2 to 4-4)****

Riemann Sums

9-12 Use the Midpoint Rule with the given value of n to approximate the integral. Round the answer to four decimal places.

9. $\int_0^8 \sin \sqrt{x} \, dx, \quad n = 4$

Note: be ready for Left, Right and Trap Rule as well.

Evaluate the integral by interpreting it in terms of areas.

37. $\int_{-3}^0 (1 + \sqrt{9 - x^2}) \, dx$

Fundamental Theorem of Calculus

19-42 Evaluate the integral.

19. $\int_{-2}^3 (x^2 - 3) dx$

21. $\int_{-2}^0 \left(\frac{1}{2}t^4 + \frac{1}{4}t^3 - t\right) dt$

29. $\int_1^4 \sqrt{\frac{5}{x}} dx$

34. $\int_0^{\pi/3} \frac{\sin \theta + \sin \theta \tan^2 \theta}{\sec^2 \theta} d\theta$

Fundamental Theorem of Calculus part 2

EXAMPLE 2 Find the derivative of the function $g(x) = \int_0^x \sqrt{1+t^2} dt$.

11. $F(x) = \int_x^0 \sqrt{1 + \sec t} dt$ find $F'(x)$

56. $g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}} dt$ Find $g'(x)$

Evaluate the following Integrals:

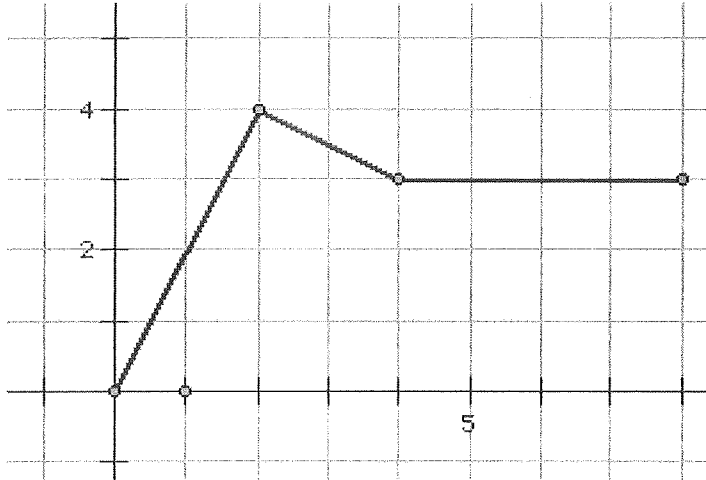
1. $\int (\cos x + x^3) dx$

2. $\int \left(x + \frac{3}{x^4} + 6\sqrt{x} \right) dx$

3. $\int 4 \csc \theta \cot \theta d\theta$ given that $\left(\frac{\pi}{2}, -2 \right)$ is on the original

4. $\int \frac{\sqrt{x+8}}{x^2} dx$ given that $(4, -7)$ is on the original

Car 1 has a velocity that is shown in the graph below where t is in seconds and the rate is in feet per second.



Car 2 has a velocity that is defined by $v(t) = 2\sqrt{t}$

a. Find the velocity of car 1 and car 2 at $t = 5$ seconds. Include units.

b. Find the acceleration of car 1 and car 2 at $t = 1$ second. Include units.

c. Find the total distance travelled by car 1 and the total distance travelled by car 2 over the time interval $0 \leq t \leq 8$. Indicate units.

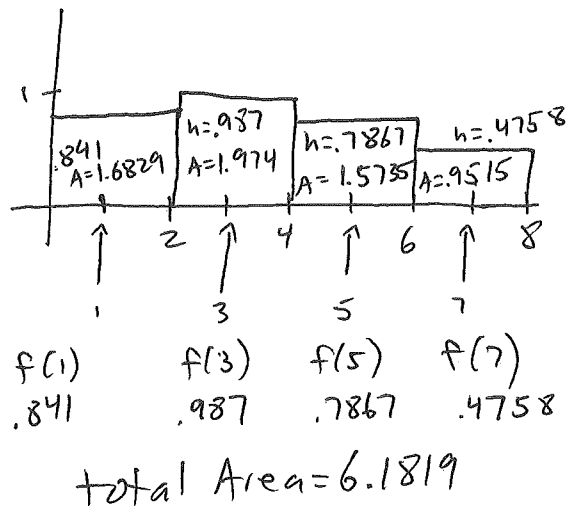
Calculus Test 3 Prep

Review for Quiz 9 (Sec 4-2 to 4-4)

Riemann Sums

9-12 Use the Midpoint Rule with the given value of n to approximate the integral. Round the answer to four decimal places.

9. $\int_0^8 \sin \sqrt{x} dx, n = 4$



Note: be ready for Left, Right and Trap Rule as well.

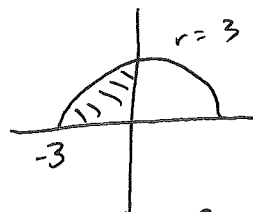
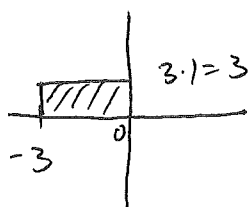
Evaluate the integral by interpreting it in terms of areas.

37. $\int_{-3}^0 (1 + \sqrt{9 - x^2}) dx$

↓
rect.

↓
circle

$$x^2 + y^2 = r^2$$



$$\frac{1}{4} \pi 3^2 = \frac{9}{4} \pi$$

$$3 + \frac{9}{4} \pi = 10.068$$

Fundamental Theorem of Calculus

19-42 Evaluate the integral.

19. $\int_{-2}^3 (x^2 - 3) dx$ $\frac{1}{3}x^3 - 3x \Big|_{-2}^3 = \left[\frac{1}{3}(3)^3 - 3(3) \right] - \left[\frac{1}{3}(-2)^3 - 3(-2) \right]$
 $-3\frac{1}{3}$

* 21. $\int_{-2}^0 (\frac{1}{2}t^4 + \frac{1}{4}t^3 - t) dt$
 $\frac{1}{2} \cdot \frac{1}{5}t^5 + \frac{1}{4} \cdot \frac{1}{4}t^4 - \frac{1}{2}t^2 \Big|_{-2}^0 = [0] - \left[\frac{1}{10}(-2)^5 + \frac{1}{16}(-2)^4 - \frac{1}{2}(-2)^2 \right] = 4.2$

* 29. $\int_1^4 \sqrt{\frac{5}{x}} dx = \sqrt{5} \int_1^4 x^{-1/2} dx = \sqrt{5} \left[2x^{1/2} \right]_1^4 = \sqrt{5} [2\sqrt{4} - 2\sqrt{1}] = 2\sqrt{5} = 4.472$

$\frac{\sin^2 + \cos^2 = 1}{\cos^2} = \sec^2$
 $\tan^2 + 1 = \sec^2$

34. $\int_0^{\pi/3} \frac{\sin \theta + \sin \theta \tan^2 \theta}{\sec^2 \theta} d\theta$
 $\frac{\sin \theta (1 + \tan^2 \theta)}{\sec^2 \theta} = \frac{\sin \theta (\sec^2 \theta)}{\sec^2 \theta} = \sin \theta$

$\int_0^{\pi/3} \sin \theta d\theta = -\cos \theta \Big|_0^{\pi/3} = -\cos \frac{\pi}{3} - (-\cos 0) = .5$

Fundamental Theorem of Calculus part 2

EXAMPLE 2 Find the derivative of the function $g(x) = \int_0^x \sqrt{1+t^2} dt$. $= \sqrt{1+x^2}$
 in terms of t
 constant

11. $F(x) = \int_x^0 \sqrt{1 + \sec t} dt$ find $F'(x)$ $\rightarrow - \int_0^x \sqrt{1 + \sec t} dt$
 $-\sqrt{1 + \sec x}$

56. $g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}} dt$ Find $g'(x)$
 $\rightarrow \text{anti} \Big|_{\tan x}^{x^2} = \text{anti}(x^2) - \text{anti}(\tan x)$
 $g'(x) = \frac{1}{\sqrt{2+(x^2)^4}} (2x) - \frac{1}{\sqrt{2+(\tan x)^4}} \sec^2(x)$
 $\frac{1}{\sqrt{2+x^8}} (2x) - \frac{1}{\sqrt{2+\tan^4 x}} \sec^2(x)$

Evaluate the following Integrals:

$$1. \int (\cos x + x^3) dx = \sin x + \frac{1}{4} x^4 + C$$

$$2. \int \left(x + \frac{3}{x^4} + 6\sqrt{x} \right) dx = \frac{1}{2} x^2 + \frac{3}{-3} x^{-3} + \frac{6 \cdot \frac{2}{3}}{1} x^{\frac{3}{2}} + C$$
$$\int \left(x + 3x^{-4} + 6x^{1/2} \right) dx = \frac{1}{2} x^2 - \frac{1}{x^3} + 4x^{3/2} + C$$

* 3. $\int 4 \csc \theta \cot \theta d\theta$ given that $\left(\frac{\pi}{2}, -2 \right)$ is on the original

* $\csc x$ $y = -4 \csc \theta + C$

$\csc x \cot x$

$$-2 = -4 \csc \left(\frac{\pi}{2} \right) + C$$

$$-2 = -4 + C$$

$$2 = C$$

$$y = -4 \csc \theta + 2$$

4. $\int \frac{\sqrt{x+8}}{x^2} dx$ given that $(4, -7)$ is on the original

$$\int \left(\frac{x^{1/2}}{x^2} + \frac{8}{x^2} \right) dx$$

$$\int \left(x^{-3/2} + 8x^{-2} \right) dx$$

$$y = -2x^{-1/2} - 8x^{-1} + C$$

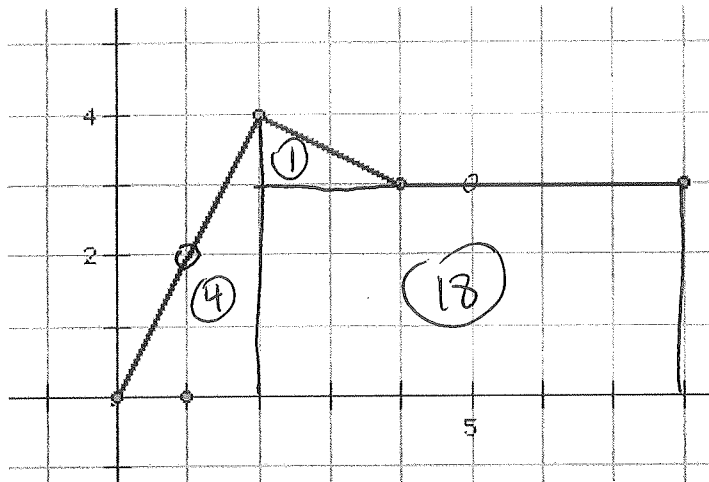
$$-7 = -2(4)^{-1/2} - 8(4)^{-1} + C$$

$$-7 = -3 + C$$

$$-4 = C$$

$$y = \frac{-2}{\sqrt{x}} - \frac{8}{x} - 4$$

Car 1 has a velocity that is shown in the graph below where t is in seconds and the rate is in feet per second.



Car 2 has a velocity that is defined by $v(t) = 2\sqrt{t}$

- a. Find the velocity of car 1 and car 2 at $t = 5$ seconds. Include units.

↓

$$v(5) = 2\sqrt{5} = 4.47 \text{ ft/sec}$$

3 ft/sec

- b. Find the acceleration of car 1 and car 2 at $t = 1$ second. Include units.

↓

$$2 \text{ ft/sec}^2$$

$$v(t) = 2t^{1/2}$$

$$a(t) = t^{-1/2}$$

$$a(1) = 1^{-1/2} = 1 \text{ ft/sec}^2$$

- c. Find the total distance travelled by car 1 and the total distance travelled by car 2 over the time interval $0 \leq t \leq 8$. Indicate units.

$$\int_0^8 v(t) dt = s(8) - s(0) = \text{total displacement}$$

Car 1

$$\int_0^8 v(t) dt = \text{area under the curve.}$$

23 ft

Car 2

$$\int_0^8 2t^{1/2} dt$$

$$\frac{2 \cdot \frac{2}{3} t^{3/2}}{\frac{2}{3}} = \frac{4}{3} t^{3/2} \Big|_0^8$$

$$\frac{4}{3} (8)^{3/2} - \frac{4}{3} (0)^{3/2}$$

30.169 ft