# Calculus Review for Sinclair Exam 1

*** Quiz 3 Material ***

Sec 1.4, extra 1.0, 2.0

## Problem Type 4: IVT

**53–56** Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval.

**53.** $x^4 + x - 3 = 0, \quad (1, 2)$

## Problem Type 5: sigma questions

**14.** Given that $\lim_{x \to 2}(5x - 7) = 3$, illustrate Definition 2 by finding values of $\delta$ that correspond to $\varepsilon = 0.1$, $\varepsilon = 0.05$, and $\varepsilon = 0.01$.

## Problem Type 6: Derivative as slope/rate of change

**13.** If a ball is thrown into the air with a velocity of 40 ft/s, its height (in feet) after $t$ seconds is given by $y = 40t - 16t^2$. Find the velocity when $t = 2$.

**20.** Find an equation of the tangent line to the graph of $y = g(x)$ at $x = 5$ if $g(5) = -3$ and $g'(5) = 4$.

**41.** $\displaystyle \lim_{h \to 0} \frac{\cos(\pi + h) + 1}{h}$

**Problem type 7: Average and instantaneous rate of change**

**EXAMPLE 1** Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.

**EXAMPLE 3** Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, 450 m above the ground. Find the velocity of the ball after 5 seconds.

*Note: distance fallen is modelled by* $s(t) = 4.9t^2$

**EXAMPLE 1** Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.

Key

# Calculus Review for Sinclair Exam 1

*** Quiz 3 Material ***

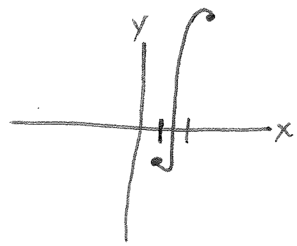
## Problem Type 4: IVT

**53–56** Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval.

**53.** $x^4 + x - 3 = 0, \quad (1, 2)$

$x = 1$: $1^4 + 1 - 3 = -1$, $y = -1$

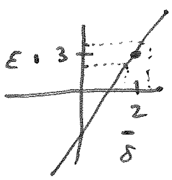
$x = 2$: $2^4 + 2 - 3$, $y = 15$




function $f(x) = x^4 + x - 3$ is cont. on $(1,2)$ and has points $(1,-1)$ and $(2,15)$ there must be a root between $x=1$ and $x=2$ by the I.V.T.

## Problem Type 5: sigma questions

**14.** Given that $\lim_{x \to 2}(5x - 7) = 3$, illustrate Definition 2 by finding values of $\delta$ that correspond to $\varepsilon = 0.1$, $\varepsilon = 0.05$, and $\varepsilon = 0.01$.




$3 + .1 = 3.1$
$5x - 7 = 3.1$
$5x = 10.1$
$x = 2.02$
$\delta = .02$

$3 - .1 = 2.9$
$5x - 7 = 2.9$
$5x = 9.9$
$x = 1.98$
$\delta = .02$

$\delta = .02$

## Problem Type 6: Derivative as slope/rate of change

**13.** If a ball is thrown into the air with a velocity of 40 ft/s, its height (in feet) after $t$ seconds is given by $y = 40t - 16t^2$. Find the velocity when $t = 2$.

L: $t = 1.999$, $y = 16.024$

R: $t = 2.001$, $y = 15.976$

$\dfrac{y - y}{x - x} = \dfrac{15.976 - 16.024}{2.001 - 1.999} = \dfrac{-.048\text{ ft}}{.002\text{ sec}} = -24\text{ ft/sec}$

**20.** Find an equation of the tangent line to the graph of $y = g(x)$ at $x = 5$ if $g(5) = -3$ and $g'(5) = 4$.

$\underset{Y}{-3}$ $\underset{m}{\text{slope}}$

$y - \underset{y}{-3} = \underset{m}{4}\,(x - \underset{x}{5})$

* $y + 3 = 4(x - 5)$

$y + 3 = 4x - 20$

* $y = 4x - 23$

**41.** $\displaystyle\lim_{h \to 0} \frac{\cos(\pi + h) + 1}{h} = 0$

L: $h = -.001$, $y = -.0005$

R: $h = .001$, $y = .0005$

Agree $y = 0$

Key

**Problem type 7: Average and instantaneous rate of change**

**EXAMPLE 1** Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.

x y

| L | R |
|---|---|
| $x = .999$ | $x = 1.001$ |
| $y = .998$ | $y = 1.002$ |

$\frac{1.002 - .998}{1.001 - .999} = \frac{.004}{.002} = \underset{\text{slope}}{2}$

$\underset{y}{y - 1} = \underset{m}{2}(\underset{x}{x - 1})$

**EXAMPLE 3** Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, 450 m above the ground. Find the velocity of the ball after 5 seconds.

*Note: distance fallen is modelled by s(t) = 4.9t²*

| L | R |
|---|---|
| $x = 4.999$ | $x = 5.001$ |
| $s(x) = 122.45$ | $s(x) = 122.55$ |

$\frac{y - y}{x - x} = \frac{122.55 - 122.45}{5.001 - 4.999} = \frac{.1}{.002} = 50$ m/s

~~**EXAMPLE 1** Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.~~