Calc

Extra Practice for Q4 Exam

* Also study Quiz 11 Quiz 12

21–47 Differentiate.

21. $f(t) = t^2 \ln t$

22. $g(t) = \dfrac{e^t}{1 + e^t}$

23. $h(\theta) = e^{\tan 2\theta}$

24. $h(u) = 10^{\sqrt{u}}$

~~25.~~ $\cancel{y = \ln|\sec 5x + \tan 5x|}$

26. $y = x \cos^{-1}x$

27. $y = x \tan^{-1}(4x)$

28. $y = e^{mx} \cos nx$

29. $y = \ln(\sec^2 x)$

30. $y = \sqrt{t} \ln(t^4)$

31. $y = \dfrac{e^{1/x}}{x^2}$

32. $y = (\arcsin 2x)^2$

33. $y = 3^{x \ln x}$

34. $y = e^{\cos x} + \cos(e^x)$

35. $H(v) = v \tan^{-1}v$

36. $F(z) = \log_{10}(1 + z^2)$

37. $y = x \sinh(x^2)$

38. $y = (\cos x)^x$

39. $y = \ln \sin x - \frac{1}{2} \sin^2 x$

40. $y = \arctan\left(\arcsin \sqrt{x}\right)$

41. $y = \ln\left(\dfrac{1}{x}\right) + \dfrac{1}{\ln x}$

42. $xe^y = y - 1$

43. $y = \ln(\cosh 3x)$

44. $y = \dfrac{(x^2 + 1)^4}{(2x + 1)^3(3x - 1)^5}$

~~45.~~ $\cancel{y = \cosh^{-1}(\sinh x)}$

~~46.~~ $\cancel{y = x \tanh^{-1}\sqrt{x}}$

47. $y = \cos\left(e^{\sqrt{\tan 3x}}\right)$

92–105 Evaluate the integral.

92. $\displaystyle\int_0^4 \frac{1}{16 + t^2}\, dt$

93. $\displaystyle\int_0^1 ye^{-2y^2}\, dy$

94. $\displaystyle\int_2^5 \frac{dr}{1 + 2r}$

95. $\displaystyle\int_0^1 \frac{e^x}{1 + e^{2x}}\, dx$

96. $\displaystyle\int_0^{\pi/2} \frac{\cos x}{1 + \sin^2 x}\, dx$

97. $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$

98. $\displaystyle\int \frac{\sin(\ln x)}{x}\, dx$

99. $\displaystyle\int \frac{x + 1}{x^2 + 2x}\, dx$

100. $\displaystyle\int \frac{\csc^2 x}{1 + \cot x}\, dx$

101. $\displaystyle\int \tan x \ln(\cos x)\, dx$

102. $\displaystyle\int \frac{x}{\sqrt{1 - x^4}}\, dx$

103. $\displaystyle\int 2^{\tan\theta} \sec^2\theta\, d\theta$

104. $\displaystyle\int \sinh au\, du$

105. $\displaystyle\int \left(\frac{1 - x}{x}\right)^2 dx$

89. A bacteria culture contains 200 cells initially and grows at a rate proportional to its size. After half an hour the population has increased to 360 cells.
   (a) Find the number of bacteria after t hours.
   (b) Find the number of bacteria after 4 hours.
   (c) Find the rate of growth after 4 hours.
   (d) When will the population reach 10,000?

90. Cobalt-60 has a half-life of 5.24 years.
   (a) Find the mass that remains from a 100-mg sample after 20 years.
   (b) How long would it take for the mass to decay to 1 mg?

start

7:40

(22) $g'(t) = \frac{(1+e^t)e^t - e^t e^t}{(1+e^t)^2}$

(~~24~~) $g(t) = e^t(1+e^t)^{-1}$
$e^t(1+e^t)^{-1} + e^t(-1)(1+e^t)^{-2}(e^t)$

(24) $h'(u) = 10^{\sqrt{u}}\left(\frac{1}{2}u^{-1/2}\right)\ln 10$

(26) $y' = 1\cos^{-1}x + x\left(\frac{-1}{\sqrt{1-x^2}}\right)$

(28) $y' = e^{mx}(m)\cos(nx) + e^{mx}(-\sin(nx)n)$

(30) $y' = \frac{1}{2}(t\ln(t^4))^{-1/2}\left(\ln(t^4) + t\left(\frac{4t^3}{t^4}\right)\right)$

(32) $y' = 2(\arcsin(2x))^1\left(\frac{2}{\sqrt{1-(2x)^2}}\right)$

(34) $y' = e^{\cos x}(-\sin x) + -\sin(e^x)e^x$

(36) $F'(z) = \frac{2z}{(1+z^2)\ln 10}$

(38) $\ln y = x\ln(\cos x)$
$\frac{y'}{y} = 1\cdot\ln(\cos x) + x\left(\frac{-\sin x}{\cos x}\right)$
$y' = (\cos x)^x(\ln(\cos x) - x\tan x)$

(40) $y' = \dfrac{\dfrac{\frac{1}{2}x^{-1/2}}{\sqrt{1-x}}}{1+\arcsin^2\sqrt{x}}$

(42) $1e^y + xe^y y' = y'$

$xe^y y' - y' = -e^y$

$y'(xe^y - 1) = -e^y$

$y' = \dfrac{-e^y}{xe^y - 1}$

(44) $\dfrac{(2x+1)^3(3x-1)^5 4(x^2+1)^3(2x) - (x^2+1)^4\left[3(2x+1)^2(2)(3x-1)^5 + (2x+1)^3 5(3x-1)^4(3)\right]}{\left[(2x+1)^3(3x-1)^5\right]^2}$

~~(46)~~

(21) $f'(t) = 2t\ln t + t^2\left(\frac{1}{t}\right)$

(23) $h'(\theta) = e^{\tan 2\theta}(\sec^2 2\theta)(2)$

~~(25)~~ SKIP

(27) $y' = 1\tan^{-1}(4x) + x\left(\dfrac{4}{1+16x^2}\right)$

(29) $y' = \dfrac{2(\sec x)\sec x\tan x}{\sec^2 x} = \dfrac{2\sec^2 x\tan x}{\sec^2 x}$

$(\sec x)^2$

$\boxed{2\tan x}$

(31) $y' = \dfrac{x^2\left(e^{x^{-1}}(-x^{-2})\right) - e^{x^{-1}}(4x^3)}{x^4}$

or $y = e^{x^{-1}}x^{-2}$ $\quad y' = e^{x^{-1}}(-x^{-2})x^{-2} + e^{x^{-1}}(-2x^{-3})$

(33) $y = 3^{x\ln x}$ $\quad y' = 3^{x\ln x}\left(\ln x + x\left(\frac{1}{x}\right)\right)\ln 3$

(35) $H'(v) = 1\cdot\tan^{-1}v + v\left(\frac{1}{1+v^2}\right)$

(37) $y' = 1\cdot\sinh(x^2) + x\cosh(x^2)(2x)$

(39) $y' = \dfrac{\cos x}{\sin x} - 1\sin x(\cos x)$ $\quad \frac{1}{2}(\sin x)^2$

$y' = \cot x - \sin x\cdot\cos x$

(41) $\cancel{y' = \frac{\frac{1}{x^2}}{\frac{1}{x}} + -(\ln x)^{-2}\left(\frac{1}{x}\right)}$

OR $\quad y = \ln(x^{-1}) + (\ln x)^{-1}$

$y' = \dfrac{-x^{-2}}{x^{-1}} + -1(\ln x)^{-2}\left(\frac{1}{x}\right)$

(43) $\dfrac{-\sinh(3x)(3)}{\cosh 3x}$

(47) $y' = -\sin\left(e^{\sqrt{\tan 3x}}\right)e^{\sqrt{\tan 3x}}\left(\frac{1}{2}(\tan 3x)^{-1/2}\sec^2(3x)(3)\right)$

(89)

$$\frac{dy}{dx} = Ky \qquad y = Ce^{kt}$$

$$\frac{1}{y}dy = K \cdot dx$$

$$\ln y = Kx$$

$$y = Ce^{kx} \qquad C = 200$$

$$y = 200e^{kx} \qquad (.5, 360)$$

$$360 = 200e^{k(.5)}$$

$$\frac{360}{200} = e^{k(.5)}$$

$$2\ln\left(\frac{360}{200}\right) = K = 1.175 \qquad y = 200e^{1.175x}$$

a) $y = 200e^{1.175t}$

b) $y = 200e^{1.175(4)} = 21989$ bac

c) $y' = 200e^{1.175t}(1.175)$

$t = 4 \qquad y' = 25838$ bac/hr

d) $10000 = 200e^{1.175t}$

$500 = e^{1.175t}$

$\frac{\ln 50}{1.125} = t \rightarrow 3.329$ hrs

(90) $y = Ce^{kt}$ $\quad C = 100$

$y = 100e^{kt}$

point $\rightarrow (5.24, 50)$

$50 = 100e^{5.24K}$

$.5 = e^{5.24K}$

$\ln(.5) = 5.24K$

$\frac{\ln(.5)}{5.24} = K = -.132$ $\quad y = 100e^{-.132t}$

a) $y = 100e^{-.132(20)}$

$y = 7.136$ mg

b) $1 = 100e^{-.132t}$

$.01 = e^{-.132t}$

$\ln(.01) = -.132t$

$\frac{\ln(.01)}{-.132} = t = 34.9$ years

(92) $\int_0^4 \frac{1}{1+t^2}\,dt = \tan^{-1}(t)\Big|_0^4 = \tan^{-1}(4) - \tan^{-1}(0)$

$$\int_0^4 \frac{1}{16\left(1+\frac{1}{16}t^2\right)}\,dt = \int_0^4 \frac{1}{16}\cdot\frac{1}{1+\left(\frac{t}{4}\right)^2}\,dt$$

$$= \frac{1}{16}\int_0^4 \frac{\frac{1}{4}(4)}{1+\left(\frac{t}{4}\right)^2}\,dt$$

$$= \frac{4}{16}\tan^{-1}\left(\frac{t}{4}\right)\Big|_0^4$$

(93) $\int_0^1 x e^{-2x^2}\,dx$

$u = -2x^2$

$du = -4x\,dx$

$-\frac{1}{4}du = x\,dx$

$-\frac{1}{4}\int e^u\,du$

$-\frac{1}{4}e^u \rightarrow -\frac{1}{4}e^{-2x^2}\Big|_0^1$

(95) $\int_0^1 \frac{e^x}{1+e^{2x}}\,dx = \int_0^1 \frac{e^x}{1+(e^x)^2}\,dx$

$= \tan^{-1}(e^x)\Big|_0^1 =$

(94) $\int_2^5 \frac{1}{1+2r}\,dr = \frac{1}{2}\ln|1+2r|\Big|_2^5$

(96) $\int_0^{\pi/2} \frac{\cos x}{1+(\sin x)^2}\,dx = \tan^{-1}(\sin x)\Big|_0^{\frac{\pi}{2}}$

(97) $y = 2e^{\sqrt{x}} + C$ test: $y' = 2e^{\sqrt{x}}\left(+\frac{1}{2}x^{-1/2}\right) = 2\left(\frac{1}{2}\right)\frac{e^{\sqrt{x}}}{\sqrt{x}}$

(98) $y = -\cos(\ln x) + C$ test: $y' = \sin(\ln x)\,\frac{1}{x}$

(99) ~~y~~ $y = \frac{1}{2}\ln|x^2+2x|$ test: $y' = \frac{1}{2}\left(\frac{2x+2}{x^2+2x}\right)$

$= \frac{x+1}{x^2+2x}$

(100) $y = \ln(1 + \cot x) + C$

$$\text{test: } y' = \frac{-\csc^2 x}{1 + \cot x}$$

(101) $u = \ln(\cos x)$

$du = \frac{\sin x}{\cos x} = \tan x$

so $\ln(\cos x)$ creates $\tan x$

$y = \frac{1}{2}(\ln(\cos x))^2$

$$\text{test: } y' = \frac{1}{2}(2)(\ln(\cos x))^1 \frac{\sin x}{\cos x}$$

$$= \ln(\cos x)\tan x$$

(102) $\int \frac{x}{\sqrt{1-(x^2)^2}}\,dx$

$y = \frac{1}{2}\sin^{-1}(x^2) + C$

$$\text{test: } y' = \frac{\cancel{2}x}{\cancel{2}\sqrt{1-x^4}}$$ ✓

(103) $y = 2^{\tan\theta} + C$? $y' = 2^{\tan\theta}\sec^2\theta\,(\ln 2)$

fix → $y = \left(\frac{1}{\ln 2}\right)2^{\tan\theta} + C$

(104) $y = -\frac{1}{a}\cosh(au) + C$

$$\text{test: } y' = -\frac{1}{a}(-\sinh(au)a)$$

$$= \sinh(au)$$

(105) $\int \left(\frac{1-x}{x}\right)^2 dx \rightarrow \left(\frac{1}{x} - 1\right)\left(\frac{1}{x} - 1\right) = \frac{1}{x^2} - \frac{2}{x} + 1$

$\int \left(x^{-2} - \frac{2}{x} + 1\right) dx$

$y = -x^{-1} - 2\ln|x| + x + C$