Calculus Review for Sinclair Exam 1 Name____________________

***** Quiz 1 Material *****

Problem type 1: Evaluating Limits

11–32 Evaluate the limit, if it exists.

11. $\lim_{x \to 5} \frac{x^2 - 6x + 5}{x - 5}$

18. $\lim_{h \to 0} \frac{(2 + h)^3 - 8}{h}$

20. $\lim_{t \to 1} \frac{t^4 - 1}{t^3 - 1}$

44. $\lim_{x \to -2} \frac{2 - |x|}{2 + x}$

29–39 Determine the infinite limit.

29. $\lim_{x \to 5^+} \frac{x + 1}{x - 5}$

36. $\lim_{x \to \pi^-} \cot x$

38. If $2x \leqslant g(x) \leqslant x^4 - x^2 + 2$ for all x, evaluate $\lim_{x \to 1} g(x)$.

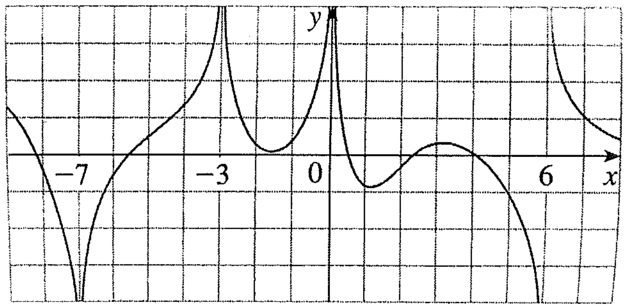
Problem Type 2: Sketching and graphs

9. For the function f whose graph is shown, state the following.

(a) $\lim_{x \to -7} f(x)$ (b) $\lim_{x \to -3} f(x)$ (c) $\lim_{x \to 0} f(x)$

(d) $\lim_{x \to 6^-} f(x)$ (e) $\lim_{x \to 6^+} f(x)$

(f) The equations of the vertical asymptotes.



15–18 Sketch the graph of an example of a function f that satisfies all of the given conditions.

15. $\lim_{x \to 0^-} f(x) = -1, \quad \lim_{x \to 0^+} f(x) = 2, \quad f(0) = 1$

Calculus Review for Sinclair Exam 1

***** Quiz 2 Material ***** Sec 1.4

Problem Type 3: Continuity

47. The *signum* (or sign) *function*, denoted by sgn, is defined by

$$\operatorname{sgn} x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

(a) Sketch the graph of this function.
(b) Find each of the following limits or explain why it does not exist.

(i) $\lim_{x \to 0^+} \operatorname{sgn} x$ (ii) $\lim_{x \to 0^-} \operatorname{sgn} x$

(iii) $\lim_{x \to 0} \operatorname{sgn} x$ (iv) $\lim_{x \to 0} |\operatorname{sgn} x|$

11–14 Use the definition of continuity and the properties of limits to show that the function is continuous at the given number a.

11. $f(x) = (x + 2x^3)^4, \quad a = -1$

For #20, explain why the function is discontinuous at point a. Sketch the graph.

20. $f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases} \qquad a = 1$

23–24 How would you "remove the discontinuity" of f? In other words, how would you define $f(2)$ in order to make f continuous at 2?

23. $f(x) = \dfrac{x^2 - x - 2}{x - 2}$

45. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geqslant 2 \end{cases}$$

Problem type 7: Average and instantaneous rate of change

EXAMPLE 1 Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.

EXAMPLE 3 Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, 450 m above the ground. Find the velocity of the ball after 5 seconds.

Note: distance fallen is modelled by $s(t) = 4.9t^2$

~~**EXAMPLE 1** Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.~~

Calculus Review for Sinclair Exam 1

*** Quiz 3 Material *** Sec 1.4, extra 1.0, 2.0

Problem Type 4: IVT

53–56 Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval.

53. $x^4 + x - 3 = 0, \quad (1, 2)$

Problem Type 5: sigma questions

14. Given that $\lim_{x \to 2}(5x - 7) = 3$, illustrate Definition 2 by finding values of δ that correspond to $\varepsilon = 0.1$, $\varepsilon = 0.05$, and $\varepsilon = 0.01$.

Problem Type 6: Derivative as slope/rate of change

13. If a ball is thrown into the air with a velocity of 40 ft/s, its height (in feet) after t seconds is given by $y = 40t - 16t^2$. Find the velocity when $t = 2$.

20. Find an equation of the tangent line to the graph of $y = g(x)$ at $x = 5$ if $g(5) = -3$ and $g'(5) = 4$.

41. $\lim\limits_{h \to 0} \dfrac{\cos(\pi + h) + 1}{h}$

Problem type 7: Average and instantaneous rate of change

EXAMPLE 1 Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.

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EXAMPLE 1 Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.

Calculus Review for Sinclair Exam 1

*** Quiz 4 Material ***

Problem Type 8: Derivative Graphs

EXAMPLE 1 The graph of a function f is given in Figure 1. Use it to sketch the graph of the derivative f'.

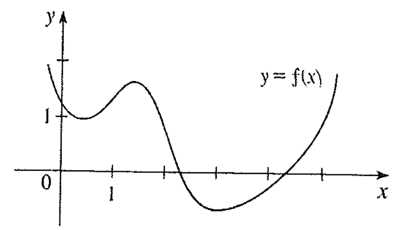
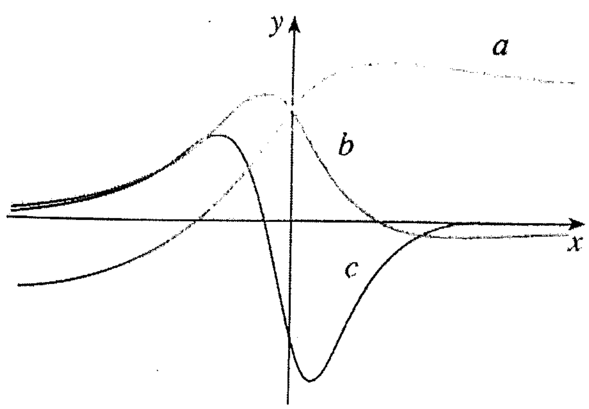


FIGURE 1

47. The figure shows the graphs of f, f', and f''. Identify each curve, and explain your choices.



Problem Type 9: Derivatives as Functions

EXAMPLE 2

(a) If $f(x) = x^3 - x$, find a formula for $f'(x)$.

(b) Illustrate this formula by comparing the graphs of f and f'.

EXAMPLE 3 If $f(x) = \sqrt{x}$, find the derivative of f. State the domain of f'.

Calculus Review for Sinclair Exam 1 **Name**____________________

***** Quiz 1 Material *****

Problem type 1: Evaluating Limits

11–32 Evaluate the limit, if it exists.

11. $\lim_{x\to5} \frac{x^2-6x+5}{x-5}$ $\lim_{x\to5} \frac{(x-5)(x-1)}{x-5}$ $\lim_{x\to5}(x-1) = 5-1=4$ Ans: 4

$(2+h)(4+4h+h^2) = 8+8h+2h^2$
$4h+4h^2+h^3$

18. $\lim_{h\to0} \frac{(2+h)^3-8}{h}$ $\lim_{h\to0} \frac{8+12h+6h^2+h^3-8}{h}$ $\lim_{h\to0} \frac{h}{h}(12+6h+h^2)$

$12+6(0)+0^2$ Ans = 12

20. $\lim_{t\to1} \frac{t^4-1}{t^3-1}$

L	R
$x = .999$	$x = 1.001$
$y = 1.\overline{3}$	$y = 1.\overline{3}$

Ans $= 1.\overline{3}$ or $\frac{4}{3}$

44. $\lim_{x\to-2} \frac{2-|x|}{2+x}$

L	R
$x = -2.001$	$x = -1.999$
$y = 1$	$y = 1$

Ans = 1

29–39 Determine the infinite limit.

29. $\lim_{x\to5^+} \frac{x+1}{x-5}$

R
$x = 5.001$
$y = 6001$

Ans $= \infty$

36. $\lim_{x\to\pi^-} \cot x$ $\lim_{x\to\pi^-}\left(\frac{1}{\tan x}\right)$

Ans $= -\infty$

$x = \pi^- = 3.14 - .001$
$y = -1000$

38. If $2x \leqslant g(x) \leqslant x^4 - x^2 + 2$ for all x, evaluate $\lim_{x\to1} g(x)$.

$2(1) \leq g(x) \leq 1^4 - 1^2 + 2$

$2 \leq g(x) \leq 2$

$\downarrow$

2

$\lim_{x\to1} g(x) = 2$

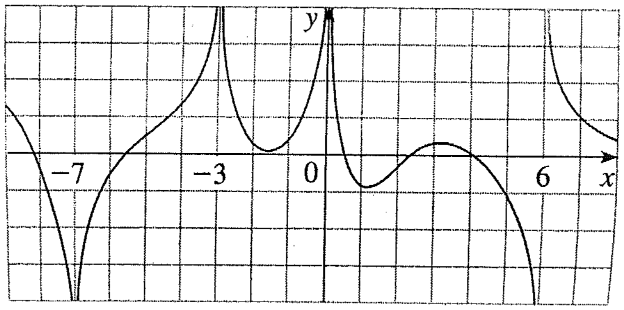
Problem Type 2: Sketching and graphs

9. For the function f whose graph is shown, state the following.

(a) $\lim_{x \to -7} f(x)$ $-\infty$ (b) $\lim_{x \to -3} f(x)$ ∞ (c) $\lim_{x \to 0} f(x)$ ∞

(d) $\lim_{x \to 6^-} f(x)$ $-\infty$ (e) $\lim_{x \to 6^+} f(x)$ ∞ $\lim_{x \to 6} f(x) = DNE$

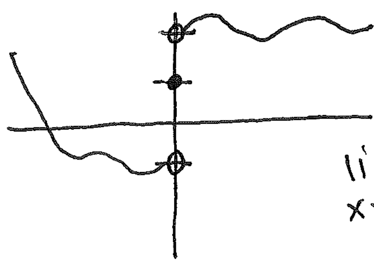
(f) The equations of the vertical asymptotes.



15–18 Sketch the graph of an example of a function f that satisfies all of the given conditions.

15. $\lim_{x \to 0^-} f(x) = -1, \quad \lim_{x \to 0^+} f(x) = 2, \quad f(0) = 1$

$(0, 1)$



$\lim_{x \to 0} f(x) = DNE$

Calculus Review for Sinclair Exam 1

Key

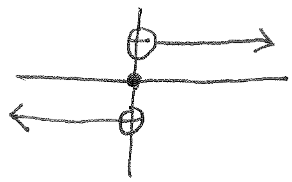
*** Quiz 2 Material ***

Problem Type 3: Continuity

$\lim_{x \to c} f(x)$ exists
$f(c)$ exists
$f(x) = \lim_{x \to c} f(x)$

47. The *signum* (or sign) *function*, denoted by sgn, is defined by

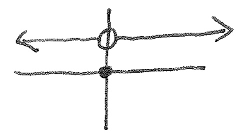
$$\operatorname{sgn} x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$



(a) Sketch the graph of this function.
(b) Find each of the following limits or explain why it does not exist.

(i) $\lim_{x \to 0^+} \operatorname{sgn} x = 1$ (ii) $\lim_{x \to 0^-} \operatorname{sgn} x = -1$

(iii) $\lim_{x \to 0} \operatorname{sgn} x$ DNE (iv) $\lim_{x \to 0} |\operatorname{sgn} x| = 1$



11–14 Use the definition of continuity and the properties of limits to show that the function is continuous at the given number a.

11. $f(x) = (x + 2x^3)^4, \quad a = -1$

$f(-1) = (-1 + 2(-1)^3)^4 = 81$
Limit exist and also equals 81
$\therefore f(x)$ is continuous at $x = -1$

For #20, explain why the function is discontinuous at point a. Sketch the graph.

20. $f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases} \qquad a = 1$

$\dfrac{x(x-1)}{(x+1)(x-1)} \quad \dfrac{x}{x+1} \quad \dfrac{1}{1+1} = \dfrac{1}{2}$

$\lim_{x \to 1} f(x) = \frac{1}{2} \quad \frac{1}{2} \neq 1$

Not cont. at $x = 1$

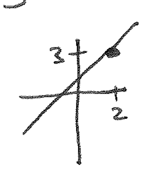
23–24 How would you "remove the discontinuity" of f? In other words, how would you define $f(2)$ in order to make f continuous at 2?

23. $f(x) = \dfrac{x^2 - x - 2}{x - 2}$

$\dfrac{(x-2)(x+1)}{x-2} = x + 1 = 2 + 1 = 3$

$\lim_{x \to 2} f(x) = 3$ need $f(2) = 3$

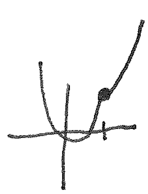
$\begin{cases} \dfrac{x^2 - x - 2}{x - 2} & x \neq 2 \\ 3 & x = 2 \end{cases}$



45. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geqslant 2 \end{cases}$$

$c4 + 4$
$8 - c2$



$4c + 4 = 8 - 2c$
$6c = 4$
$c = 2/3$

Calculus Review for Sinclair Exam 1

Key

*** Quiz 3 Material ***

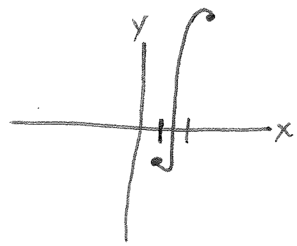
Problem Type 4: IVT

53–56 Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval.

53. $x^4 + x - 3 = 0, \quad (1, 2)$

$x=1$: $1^4+1-3=-1$, $y=-1$

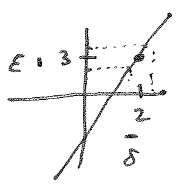
$x=2$: 2^4+2-3, $y=15$



function $f(x)=x^4+x-3$ is cont. on $(1,2)$ and has points $(1,-1)$ and $(2,15)$ there must be a root between $x=1$ and $x=2$ by the I.V.T.

Problem Type 5: sigma questions

14. Given that $\lim_{x\to 2}(5x - 7) = 3$, illustrate Definition 2 by finding values of δ that correspond to $\varepsilon = 0.1$, $\varepsilon = 0.05$, and $\varepsilon = 0.01$.



$3+.1=3.1$
$5x-7=3.1$
$5x=10.1$
$x=2.02$
$\delta=.02$

$3-.1=2.9$
$5x-7=2.9$
$5x=9.9$
$x=1.98$
$\delta=.02$

$\delta=.02$

Problem Type 6: Derivative as slope/rate of change

13. If a ball is thrown into the air with a velocity of 40 ft/s, its height (in feet) after t seconds is given by $y = 40t - 16t^2$. Find the velocity when $t = 2$.

L: $t=1.999$, $y=16.024$

R: $t=2.001$, $y=15.976$

$\frac{y-y}{x-x} = \frac{15.976-16.024}{2.001-1.999} = \frac{-.048 ft}{.002 sec} = -24 ft/sec$

20. Find an equation of the tangent line to the graph of $y = g(x)$ at $x = 5$ if $g(5) = -3$ and $g'(5) = 4$.

$\underset{y}{-3}$ $\underset{\text{slope } m}{4}$

$y - \underset{y}{-3} = \underset{m}{4}\,(x - \underset{x}{5})$

$* \; y+3=4(x-5)$

$y+3=4x-20$

$* \; y=4x-23$

41. $\lim_{h\to 0} \frac{\cos(\pi + h) + 1}{h} = 0$

L: $h=-.001$, $y=-.0005$

R: $h=.001$, $y=.0005$

$\underbrace{\qquad\qquad}_{\text{Agree } y=0}$

Key

Problem type 7: Average and instantaneous rate of change

EXAMPLE 1 Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.

x y

$$y - \underbrace{1}_{y} = \underbrace{2}_{m}(x - \underbrace{1}_{x})$$

L	R
$x = .999$	$x = 1.001$
$y = .998$	$y = 1.002$

$$\frac{1.002 - .998}{1.001 - .999} = \frac{.004}{.002} = \underbrace{2}_{\text{slope}}$$

EXAMPLE 3 Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, 450 m above the ground. Find the velocity of the ball after 5 seconds.

Note: distance fallen is modelled by $s(t) = 4.9t^2$

L	R
$x = 4.999$	$x = 5.001$
$s(x) = 122.45$	$s(x) = 122.55$

$$\frac{y - y}{x - x} = \frac{122.55 - 122.45}{5.001 - 4.999} = \frac{.1}{.002} = 50 \text{ m/s}$$

~~**EXAMPLE 1** Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.~~

Key

Calculus Review for Sinclair Exam 1

*** Quiz 4 Material ***

Problem Type 8: Derivative Graphs

EXAMPLE 1 The graph of a function f is given in Figure 1. Use it to sketch the graph of the derivative f'.

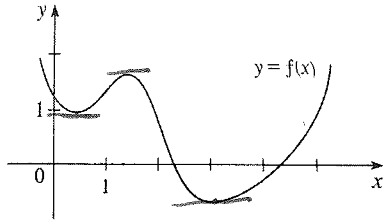
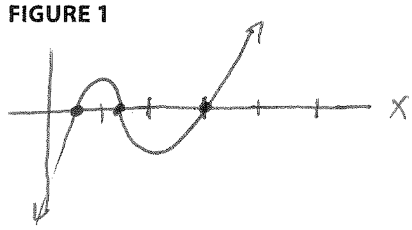
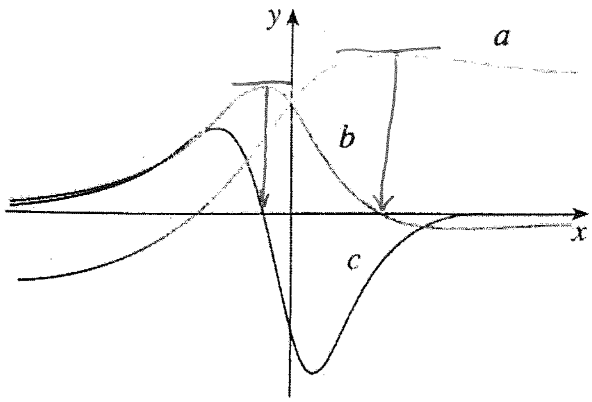


FIGURE 1



47. The figure shows the graphs of f, f', and f''. Identify each curve, and explain your choices.



b is deriv of a
c is deriv of b

f is a
f' is b
f'' is c

Problem Type 9: Derivatives as Functions

EXAMPLE 2
(a) If $f(x) = x^3 - x$, find a formula for $f'(x)$.
(b) Illustrate this formula by comparing the graphs of f and f'.

$f'(x) = 3x^2 - 1$

work on the Back

EXAMPLE 3 If $f(x) = \sqrt{x}$, find the derivative of f. State the domain of f'.

$f'(x) = \frac{1}{2\sqrt{x}}$

$$f(x) = x^3 - x$$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{(x+h)^3 - (x+h) - (x^3 - x)}{h}$$

$$(x^2 + 2xh + h^2)(x+h)$$
$$x^3 + 3x^2h + 3xh^2 + h^3$$

$$\frac{x^3 + 3x^2h + 3xh^2 + h^3 - x - h - x^3 + x}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2 - 1)}{\cancel{h}} = 3x^2 - 1$$

$$f'(x) = 3x^2 - 1$$

$$f(x) = \sqrt{x}$$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right)$$

$$\frac{x+h + \sqrt{x+h}\sqrt{x} - \sqrt{x}\sqrt{x+h} - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$\lim_{h\to 0} \frac{\cancel{h}\cdot 1}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+0} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

$$= \frac{\sqrt{x}}{2x}$$