Lesson 591 – Derivatives and Integrals of Arc-Trig Functions

Hyperbolic Functions:

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{\sinh x}{\cosh x}$$

Hyperbolic Sine aka sinh	f(x) = sinh(g(x))	f'(x) = cosh(g(x))•g'(x)
Hyperbolic Cosine aka cosh	f(x) = cosh(g(x))	f'(x) = sinh(g(x))•g'(x)
Hyperbolic Tangent aka tanh	f(x) = tanh(g(x))	f'(x) = sech^2(g(x))•g'(x) OR f'(x) = 1 – $\tanh^2$(g(x))•g'(x)
Hyperbolic Cosecant aka csch	f(x) = csch(g(x))	f'(x) = -coth(g(x))csch(g(x))•g'(x)
Hyperbolic Cosine aka cosh	f(x) = cosh(g(x))	f'(x) = -tanh(g(x))sech(g(x))•g'(x)
Hyperbolic Cotangent aka tanh	f(x) = coth(g(x))	f'(x) = -csch^2(g(x))•g'(x) OR f'(x) = 1 – $\coth^2$(g(x))•g'(x)

In Exercises 15–28, find the derivative of the function.

15. $y = \sinh(1 - x^2)$

17. $f(x) = \ln(\sinh x)$

19. $y = \ln\left(\tanh \frac{x}{2}\right)$

21. $h(x) = \frac{1}{4}\sinh 2x - \frac{x}{2}$

23. $f(t) = \arctan(\sinh t)$

25. $g(x) = x^{\cosh x}$

27. $y = (\cosh x - \sinh x)^2$

18. $g(x) = \ln(\cosh x)$

20. $y = x\cosh x - \sinh x$

26. $f(x) = e^{\sinh x}$

In Exercises 39–54, find or evaluate the integral.

39. $\int \sinh(1 - 2x)\, dx$

40. $\int \frac{\cosh \sqrt{x}}{\sqrt{x}}\, dx$

41. $\int \cosh^2(x - 1) \sinh(x - 1)\, dx$

42. $\int \frac{\sinh x}{1 + \sinh^2 x}\, dx$

43. $\int \frac{\cosh x}{\sinh x}\, dx$

44. $\int \operatorname{sech}^2(2x - 1)\, dx$