

## Lesson 581 – Derivatives and Integrals of Inverse Trig Functions

Arcsine aka $\sin^{-1}$	$f(x) = \arcsin(g(x))$	$f'(x) = \frac{g'(x)}{\sqrt{1-g(x)^2}}$
Arccosine aka $\cos^{-1}$	$f(x) = \arccos(g(x))$	$f'(x) = \frac{-g'(x)}{\sqrt{1-g(x)^2}}$
Arctangent aka $\tan^{-1}$	$f(x) = \arctan(g(x))$	$f'(x) = \frac{g'(x)}{1+g(x)^2}$
Arccosecant aka $\csc^{-1}$	$f(x) = \operatorname{arccsc}(g(x))$	$f'(x) = \frac{-g'(x)}{ g(x) \sqrt{g(x)^2-1}}$
Arcsecant aka $\sec^{-1}$	$f(x) = \operatorname{arcsec}(g(x))$	$f'(x) = \frac{g'(x)}{ g(x) \sqrt{g(x)^2-1}}$
Arccotangent aka $\cot^{-1}$	$f(x) = \operatorname{arccot}(g(x))$	$f'(x) = \frac{-g'(x)}{1+g(x)^2}$

### Examples

**Differentiate each function with respect to  $x$ .**

1)  $y = \cos^{-1} -5x^3$

2)  $y = \sin^{-1} -2x^2$

3)  $y = \tan^{-1} 2x^4$

4)  $y = \csc^{-1} 4x^2$

Evaluate each indefinite integral.

$$5) \int \frac{8x}{\sqrt{9-16x^4}} dx$$

$$6) \int \frac{3x^2}{x^3\sqrt{x^6-1}} dx$$

$$7) \int \frac{10x}{16+25x^4} dx$$

$$8) \int -\frac{4\sin 4x}{\sqrt{9-\cos^2 4x}} dx$$

## Assignment 582

$$5) y = (\sin^{-1} 5x^2)^3$$

$$6) y = \sin^{-1} (3x^5 + 1)^3$$

$$7) y = (\cos^{-1} 4x^2)^2$$

$$8) y = \cos^{-1} (-2x^3 - 3)^3$$

Evaluate each indefinite integral. Use the provided substitution.

$$1) \int \frac{20x^3}{\sqrt{25 - 25x^8}} dx; u = 5x^4$$

$$2) \int \frac{10x^4}{9 + 4x^{10}} dx; u = 2x^5$$

$$3) \int -\frac{2 \cdot \csc^2 2x}{\cot(2x) \cdot \sqrt{\cot^2 2x - 1}} dx; u = \cot 2x$$

$$4) \int \frac{1}{x\sqrt{25 - (\ln -2x)^2}} dx; u = \ln -2x$$

## Assignment 582

In Exercises 41–60, find the derivative of the function.

41.  $f(x) = 2 \arcsin(x - 1)$

42.  $f(t) = \arcsin t^2$

43.  $g(x) = 3 \arccos \frac{x}{2}$

44.  $f(x) = \operatorname{arcsec} 2x$

45.  $f(x) = \arctan \frac{x}{a}$

46.  $f(x) = \arctan \sqrt{x}$

47.  $g(x) = \frac{\arcsin 3x}{x}$

48.  $h(x) = x^2 \arctan x$

49.  $h(t) = \sin(\arccos t)$

50.  $f(x) = \arcsin x + \arccos x$

51.  $y = x \arccos x - \sqrt{1 - x^2}$

52.  $y = \ln(t^2 + 4) - \frac{1}{2} \arctan \frac{t}{2}$

53.  $y = \frac{1}{2} \left( \frac{1}{2} \ln \frac{x+1}{x-1} + \arctan x \right)$

54.  $y = \frac{1}{2} \left[ x\sqrt{4 - x^2} + 4 \arcsin \left( \frac{x}{2} \right) \right]$

55.  $y = x \arcsin x + \sqrt{1 - x^2}$

56.  $y = x \arctan 2x - \frac{1}{4} \ln(1 + 4x^2)$

57.  $y = 8 \arcsin \frac{x}{4} - \frac{x\sqrt{16 - x^2}}{2}$

58.  $y = 25 \arcsin \frac{x}{5} - x\sqrt{25 - x^2}$

59.  $y = \arctan x + \frac{x}{1 + x^2}$

60.  $y = \arctan \frac{x}{2} - \frac{1}{2(x^2 + 4)}$

In Exercises 1–30, find or evaluate the integral.

1.  $\int \frac{5}{\sqrt{9-x^2}} dx$

3.  $\int_0^{1/6} \frac{1}{\sqrt{1-9x^2}} dx$

5.  $\int \frac{7}{16+x^2} dx$

7.  $\int_0^{\sqrt{3}/2} \frac{1}{1+4x^2} dx$

9.  $\int \frac{1}{x\sqrt{4x^2-1}} dx$

11.  $\int \frac{x^3}{x^2+1} dx$

13.  $\int \frac{1}{\sqrt{1-(x+1)^2}} dx$

15.  $\int \frac{t}{\sqrt{1-t^4}} dt$

17.  $\int_0^{1/\sqrt{2}} \frac{\arcsin x}{\sqrt{1-x^2}} dx$

19.  $\int_{-1/2}^0 \frac{x}{\sqrt{1-x^2}} dx$

21.  $\int \frac{e^{2x}}{4+e^{4x}} dx$

23.  $\int_{\pi/2}^{\pi} \frac{\sin x}{1+\cos^2 x} dx$

25.  $\int \frac{1}{\sqrt{x}\sqrt{1-x}} dx$

27.  $\int \frac{x-3}{x^2+1} dx$

29.  $\int \frac{x+5}{\sqrt{9-(x-3)^2}} dx$

2.  $\int \frac{3}{\sqrt{1-4x^2}} dx$

4.  $\int_0^1 \frac{dx}{\sqrt{4-x^2}}$

6.  $\int \frac{4}{1+9x^2} dx$

8.  $\int_{\sqrt{3}}^3 \frac{1}{9+x^2} dx$

10.  $\int \frac{1}{4+(x-1)^2} dx$

12.  $\int \frac{x^4-1}{x^2+1} dx$

14.  $\int \frac{t}{t^4+16} dt$

16.  $\int \frac{1}{x\sqrt{x^4-4}} dx$

18.  $\int_0^{1/\sqrt{2}} \frac{\arccos x}{\sqrt{1-x^2}} dx$

20.  $\int_{-\sqrt{3}}^0 \frac{x}{1+x^2} dx$

22.  $\int_1^2 \frac{1}{3+(x-2)^2} dx$

24.  $\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x} dx$

26.  $\int \frac{3}{2\sqrt{x}(1+x)} dx$

28.  $\int \frac{4x+3}{\sqrt{1-x^2}} dx$

30.  $\int \frac{x-2}{(x+1)^2+4} dx$